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A new method for solving fully fuzzy linear Bilevel programming problems

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Abstract In this paper, a new method is proposed to find the fuzzy optimal solution of fully fuzzy linear Bilevel programming (FFLBLP) problems by representing all the parameters as triangular fuzzy numbers. In the proposed method, the given FFLBLP problem is decomposed into three crisp linear programming (CLP) problems with bounded variables constraints, the three CLP problems are solved separately and by using its optimal solutions, the fuzzy optimal solution to the given FFLBLP is obtained. The proposed method is easy to understand and to apply for finding the fuzzy optimal solution of FFLBLP occurring in real life situations.

Keywords Fully Fuzzy Linear Programming Problems, Triangular fuzzy Numbers, Bilevel Programming.

1 Introduction

Fuzzy set theory has been applied to many disciplines such as control theory and management sciences, mathematical modeling and industrial applications. The concept of fuzzy mathematical programming on general level was first proposed by Tanaka et al. [1] in the framework of the fuzzy decision of Bellman and Zadeh [2]. The first formulation of fuzzy linear programming (FLP) is proposed by Zimmermann [3].Many researchers adopted this concept for solving fuzzy linear programming problems [1–14]. However, in all of the above mentioned works, those cases of fuzzy linear programming have been studied in which not all parts of the problem were assumed to be fuzzy, e.g., only the right hand side or the objective function coefficients were fuzzy but the variables were not fuzzy.

Allahviranloo et al. [4] solved the fuzzy integer linear programming problem by reducing it into a crisp integer linear programming problem. Allahviranloo et al. [5] proposed a new method for solving fully fuzzy linear programming problems by the use of ranking function. Nasseri [6] proposed a method for solving fuzzy linear programming problems by solving the classical linear programming. Lotfi et al. [7] discussed fully fuzzy linear programming problems by representing all parameters and variables as triangular fuzzy numbers. Ebrahimnejad and Nasseri [8] used the complementary slackness theorem to solve fuzzy linear programming problem with fuzzy parameters without the need of a simplex tableau. Ebrahimnejad et al. [9] proposed a new primal-dual algorithm for solving linear programming

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problems with fuzzy variables by using duality results. Nasseri and Ebrahimnejad [10] proposed a fuzzy primal simplex algorithm for solving the flexible linear programming problem. Kumar et al. [11] proposed a new method for solving fully fuzzy linear programming problems by introducing fuzzy slack and surplus variables. Kumar etal. [12, 13] proposed a new method for solving fully fuzzy linear programming problems with inequality constraints. Kumar et al. [14] proposed a new method for finding the fuzzy optimal solution of fully fuzzy linear programming problems with equality constraints.

This paper is organized as follows: In Section 2 some basic definitions and arithmetics between two triangular fuzzy numbers are reviewed. In Section 3 formulation of FLBLP problems and a new method is proposed for solving FLBLP problems we solve an illustrative numerical example in Section 4.

2 Preliminaries

In this section, some necessary backgrounds and notions of fuzzy set theory are reviewed.

2.1 Basic definitions

Definition 2.1 [15]. The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$.The assigned value indicate the membership grade of the element in the set A.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ iscalled a fuzzy set.

Definition 2.2 [15]. A fuzzy number $\tilde{A} = (a,b,c)$ is said to be a triangular fuzzy number if its membership function is givenby

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & x = b \\ \dfrac{x-c}{b-c} & b \le x \le c \end{cases}$$

Definition 2.3 [15]. A triangular fuzzy number (a,b,c) is said to be non-negative fuzzy number iff $a \ge 0$.

Definition 2.4 [15] A fuzzy number $\tilde{A}$ is said to be non-negative fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0$, for all $x \prec 0$.

Definition 2.5 [15]. Two triangular fuzzy numbers $\tilde{A}=(a,b,c)$ and $\tilde{B}=(e,f,g)$ are said to be equal if and only if $a=e, b=f, c=g$.

2.2 Arithmetic operations

In this subsection, arithmetic operations between two triangular fuzzy numbers, defined on universal set of real numbers, are reviewed [4].

Let $\tilde{A}=(a,b,c)$ and $\tilde{B}=(e,f,g)$ be two triangular fuzzy numbers then

(i) $\tilde{A}\oplus\tilde{B}=(\mathrm{a,b,c})\oplus(e,f,g)=(a+e,\mathrm{b}+f,\mathrm{c}+g)$

(ii) $-\tilde{A}=-(\mathrm{a,b,c})=(-c,-b,-a)$

(iii) Let $\tilde{A}=(a,b,c)$ be any triangular fuzzy number and Let $\tilde{B}=(x,y,z)$ be a non-negative triangular fuzzy number then

$$\tilde{A}\otimes\tilde{B}=\begin{cases}(ax,by,cz) & a\geq 0\\ (az,by,cz) & a\prec 0, c\geq 0\\ (az,by,cx) & c\prec 0\end{cases}$$

3 Fully fuzzy linear bilevel programming problem

Consider a FlBLPP of maximization-typ objective functions at each level. Suppose that DM_i denotes the DM at the $i-th$ level $(i=1,2)$ who controls the decision vector

$$\begin{aligned}
&\underset{\tilde{x}_1}{Max}\quad \tilde{Z}_1\approx\tilde{C}_{11}{}^{T}\tilde{x}_1\oplus\tilde{C}_{12}{}^{T}\tilde{x}_2\oplus\tilde{\alpha}_1\\
&where\ \tilde{x}_2 solve\\
&Max\quad \tilde{Z}_2\approx\tilde{C}_{21}{}^{T}\tilde{x}_1\oplus\tilde{C}_{22}{}^{T}\tilde{x}_2\oplus\tilde{\alpha}_2\\
&s.t.\\
&\qquad \tilde{A}\tilde{x}_1\oplus\tilde{B}\tilde{x}_2\{\preccurlyeq,\approx,\succcurlyeq\}\tilde{t}\\
&\qquad \tilde{x}_1,\tilde{x}_2\succcurlyeq\tilde{0}
\end{aligned}$$

Where $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are fuzzy number, $\tilde{A}=(\tilde{a}_{ij})_{m\times n_1},\tilde{B}=(\tilde{b}_{ij})_{m\times n_2},\tilde{t}=(\tilde{t}_i)_{m\times 1}$ and $\tilde{C}_{11}=(\mathrm{C}_{11_j})_{1\times n_1},\tilde{C}_{12}=(\mathrm{C}_{21_j})_{1\times n_2},\tilde{C}_{21}=(\mathrm{C}_{21_j})_{1\times n_1},\tilde{C}_{22}=(\mathrm{C}_{22_j})_{1\times n_2}, n_1+n_2=n,\tilde{a}_{ij},\tilde{b}_{ij},\tilde{x}_1,\tilde{x}_2,\tilde{t}_i\in F(\mathrm{R})$ $for\ all\ 1\leq i\leq m, 1\leq j\leq n.$

Let the parameters $\tilde{C}_{11},\tilde{C}_{12},\tilde{C}_{21},\tilde{C}_{22},\tilde{a}_{ij},\tilde{b}_{ij},\tilde{x}_1,\tilde{x}_2$ and $\tilde{t}_i$ be the triangular fuzzy number $(p_{1j},\mathrm{q}_{1j},r_{1j}),(p_{2j},\mathrm{q}_{2j},r_{2j}),(p_{3j},\mathrm{q}_{3j},r_{3j}),(p_{4j},\mathrm{q}_{4j},r_{4j}),(a_{ij}{}^1,a_{ij}{}^2,a_{ij}{}^3),(b_{ij}{}^1,b_{ij}{}^2,b_{ij}{}^3),(x_{1j},y_{1j},t_{1j}),(x_{2j},y_{2j},t_{2j})$ and (m_i,n_i,p_i) respectively. Then, the problem can be written as follows:

$$\underset{x_{2j},y_{2j},t_{2j}}{Max}\left(Z_{1L},Z_{1M},Z_{1U}\right)=\sum_{j=1}^{n_1}\left(p_{1j},q_{1j},r_{1j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{2j},q_{2j},r_{2j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)$$

where x_{2j},y_{2j},t_{2j} *solves*

$$Max\ \left(Z_{2L},Z_{2M},Z_{2U}\right)=\sum_{j=1}^{n_1}\left(p_{3j},q_{3j},r_{3j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{4j},q_{4j},r_{4j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)$$

$s.t.$

$$\sum_{j=1}^{n_1}\left(a_{ij}{}^1,a_{ij}{}^2,a_{ij}{}^3\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(b_{ij}{}^1,b_{ij}{}^2,b_{ij}{}^3\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\{\le,=,\ge\}\left(m_i,n_i,p_i\right)$$

$for\ all\ i=1,\dots,\mathrm{m}.$

and all decision variables are non-negative.

Now, using the arithmetic operations and partial ordering relations, we decompose the given FLBLPP as follows:

$$\underset{x_{2j}}{Max}\ Z_{1L}=lower\ value\ of\ \left[\sum_{j=1}^{n_1}\left(p_{1j},q_{1j},r_{1j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{2j},q_{2j},r_{2j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

$$\underset{y_{2j}}{Max}\ Z_{1M}=middle\ value\ of\ \left[\sum_{j=1}^{n_1}\left(p_{1j},q_{1j},r_{1j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{2j},q_{2j},r_{2j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

$$\underset{t_{2j}}{Max}\ Z_{1U}=upper\ value\ of\ \left[\sum_{j=1}^{n_1}\left(p_{1j},q_{1j},r_{1j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{2j},q_{2j},r_{2j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

where x_{2j},y_{2j},t_{2j} *solves*

$$Max\ \ Z_{2L}=lower\ value\ of\ \left[\sum_{j=1}^{n_1}\left(p_{3j},q_{3j},r_{3j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{4j},q_{4j},r_{4j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

$$Max\ \ Z_{2M}=middle\ value\ of\ \left[\sum_{j=1}^{n_1}\left(p_{3j},q_{3j},r_{3j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{4j},q_{4j},r_{4j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

$$Max\ \ Z_{2U}=upper\ value\ of\ \left[\sum_{j=1}^{n_1}\left(p_{3j},q_{3j},r_{3j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{4j},q_{4j},r_{4j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

$s.t.$

$$lower\ value\ of\ \left[\sum_{j=1}^{n_1}\left(a_{ij}{}^1,a_{ij}{}^2,a_{ij}{}^3\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(b_{ij}{}^1,b_{ij}{}^2,b_{ij}{}^3\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\{\le,=,\ge\}\,m_i\right]$$

$$middle\ value\ of\ \left[\sum_{j=1}^{n_1}\left(a_{ij}{}^1,a_{ij}{}^2,a_{ij}{}^3\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(b_{ij}{}^1,b_{ij}{}^2,b_{ij}{}^3\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\{\le,=,\ge\}\,n_i\right]$$

$$upper\ value\ of\ \left[\sum_{j=1}^{n_1}\left(a_{ij}{}^1,a_{ij}{}^2,a_{ij}{}^3\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(b_{ij}{}^1,b_{ij}{}^2,b_{ij}{}^3\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\{\le,=,\ge\}\,p_i\right]$$

$for\ all\ i=1,\dots,\mathrm{m}.$

and all decision variables are non-negative.

From the above decomposition problem, we construct the following CLP problems namely, middle level bilevel problem (MLBLP), upper level bilevel problem (ULBLP) and lower level bilevel problem (LLBLP) as follows:

$$(MLBLP)$$

$$\underset{y_{2j}}{Max}\ Z_{1M} = middle\ \ value\ \ of\ \ \left[\sum_{j=1}^{n_1}\left(p_{1j},\mathrm{q}_{1j},r_{1j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{2j},\mathrm{q}_{2j},r_{2j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

$where\ y_{2j}\ solve$

$$Max\ \ Z_{2M} = middle\ \ value\ \ of\ \ \left[\sum_{j=1}^{n_1}\left(p_{3j},\mathrm{q}_{3j},r_{3j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{4j},\mathrm{q}_{4j},r_{4j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

$s.t.$

Constraints in the decomposition problem in which at least one decision variable of the MLBLP occurs and all decision variables are non – negative.

$$(LLBLP)$$

$$\underset{x_{2j}}{Max}\ Z_{1L} = lower\ \ value\ \ of\ \ \left[\sum_{j=1}^{n_1}\left(p_{1j},\mathrm{q}_{1j},r_{1j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{2j},\mathrm{q}_{2j},r_{2j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

$where\ x_{2j}\ solve$

$$Max\ \ Z_{2L} = lower\ \ value\ \ of\ \ \left[\sum_{j=1}^{n_1}\left(p_{3j},\mathrm{q}_{3j},r_{3j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{4j},\mathrm{q}_{4j},r_{4j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

$s.t.$

$$x_{1j} \le y^*_{1j},\ x_{2j} \le y^*_{2j}$$

Constraints in the decomposition constraints in which at least one decision variable of the LLBLP occurs which are not used in MLBLP and ULBLP; all variables in the constraints and objective function in LLBLP must satisfy the bounded constraints; replacing all values of the decision variables which are obtained in the MLBLP and all decision variables are non – negative. Where y^*_{1j}, y^*_{2j} *is the optimal value of MLBLP.*

$$(ULBLP)$$

$$\underset{x_{2j},y_{2j},t_{2j}}{Max}\ Z_{1U} = upper\ \ value\ \ of\ \ \left[\sum_{j=1}^{n_1}\left(p_{1j},\mathrm{q}_{1j},r_{1j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{2j},\mathrm{q}_{2j},r_{2j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

$where\ t_{2j}\ solve$

$$Max\ \ Z_{2U} = upper\ \ value\ \ of\ \ \left[\sum_{j=1}^{n_1}\left(p_{3j},\mathrm{q}_{3j},r_{3j}\right)\oplus\left(x_{1j},y_{1j},t_{1j}\right)\oplus\sum_{j=1}^{n_2}\left(p_{4j},\mathrm{q}_{4j},r_{4j}\right)\otimes\left(x_{2j},y_{2j},t_{2j}\right)\right]$$

$s.t.$

$$\mathrm{t}_{1j} \ge y^*_{1j},\ \mathrm{t}_{2j} \ge y^*_{2j}$$

Constraints in the decomposition problem in which atleast one decision variable of the ULBLP occurs and are not used in MLBLP; all variables in the constraints and objective function in ULBLP must satisfy the bounded constraints ; replacing all values of the decision variables which are obtained in MLBLP *and LLBLP and all decision variables are non – negative. Where* y^*_{1j}, y^*_{2j} *is the optimal value of MLBLP.*

4 Numerical example

Example : Consider the following fully fuzzy linear Bilevel programming problem:

$$
\begin{aligned}
&\underset{\tilde{x}_1}{Max} \quad \tilde{Z}_1 \approx (3,5,7)\otimes\tilde{x}_1 \oplus (2,4,8)\otimes\tilde{x}_2 \\
&\textit{where } \tilde{x}_2 \textit{solve} \\
&Max \quad \tilde{Z}_2 \approx (3,5,10)\otimes\tilde{x}_1 \oplus (1,7,8)\otimes\tilde{x}_2 \\
&s.t. \\
&\qquad (4,5,9)\otimes\tilde{x}_1 \oplus (2,7,8)\otimes\tilde{x}_2 \preccurlyeq (4,10,20) \\
&\qquad (0,3,7)\otimes\tilde{x}_1 \oplus (1,2,10)\otimes\tilde{x}_2 \preccurlyeq (2,5,18) \\
&\qquad \tilde{x}_1, \tilde{x}_2 \succcurlyeq \tilde{0}
\end{aligned}
$$

Now, the decomposition problem of the given FLPP is given below:

$$
\begin{aligned}
&Max \quad Z_{1L} = 3x_1 + 2x_2 \\
&Max \quad Z_{1M} = 5y_1 + 4y_2 \\
&Max \quad Z_{1U} = 7t_1 + 8t_2 \\
&\textit{where } x_2, y_2 \textit{ and } t_2 \textit{ solves} \\
&Max \quad Z_{2L} = 3x_1 + x_2 \\
&Max \quad Z_{2M} = 5y_1 + 7y_2 \\
&Max \quad Z_{2U} = 10t_1 + 8t_2 \\
&s.t. \\
&\qquad 4x_1 + 2x_2 \le 4, 5y_1 + 7y_2 \le 10, 9t_1 + 8t_2 \le 20 \\
&\qquad x_2 \le 2, 3y_1 + 2y_2 \le 5, 7t_1 + 10t_2 \le 18 \\
&\qquad x_i, \mathrm{y}_i, t_i \ge 0, \quad i = 1, 2.
\end{aligned}
$$

Now, the Middle Level problem is given below:

(MLBLP)

$$
\begin{aligned}
&Max \quad Z_{1M} = 5y_1 + 4y_2 \\
&\textit{where } y_2 \textit{ solve} \\
&Max \quad Z_{2M} = 5y_1 + 7y_2 \\
&s.t. \\
&\qquad 5y_1 + 7y_2 \le 10, 3y_1 + 2y_2 \le 5 \\
&\qquad y_1, \mathrm{y}_2 \ge 0
\end{aligned}
$$

Now, solving the problem(MLBLP)using simplex method, we obtain the optimal solution $y_1 = 1.36, \mathrm{y}_2 = 0.45$ and $Z_{1M} = 8.6$.
Now, the lower Level problem is given below:

(LBLLP)

$$\begin{aligned}
&Max \quad Z_{1L} = 3x_1 + 2x_2 \\
&where\ x_2\ solve \\
&Max \quad Z_{2L} = 3x_1 + x_2 \\
&s.t. \\
&\qquad 4x_1 + 2x_2 \le 4, x_2 \le 2 \\
&\qquad x_1 \le 1.36, x_2 \le 0.45 \\
&\qquad x_1, \mathrm{x}_2 \ge 0
\end{aligned}$$

Now, solving the problem LLP with $y_1 = 1.36, \mathrm{y}_2 = 0.45$ by simplex method, we obtain the optimal solution $x_1 = 0.39, \mathrm{x}_2 = 0.45$ and $Z_{1L} = 2.07$.
Now, the Upper Level problem is given below:

(ULBLP)

$$\begin{aligned}
&Max \quad Z_{1U} = 7t_1 + 8t_2 \\
&where\ t_2\ solves \\
&Max \quad Z_{2U} = 10t_1 + 8t_2 \\
&s.t. \\
&\qquad 9t_1 + 8t_2 \le 20, 7t_1 + 10t_2 \le 18 \\
&\qquad t_1 \ge 1.36, t_2 \ge 0.45 \\
&\qquad t_1, t_2 \ge 0
\end{aligned}$$

Now, solving the problem LLPwith $y_1 = 1.36, \mathrm{y}_2 = 0.45$ by simplex method, we obtain the optimal solution $t_1 = 1.65, t_2 = 0.65$ and $Z_{1U} = 16.75$.

Therefore, an optimal fuzzy solution to the given fully fuzzy linear Bilevel programming problem is
$\tilde{x}_1 = (\mathrm{x}_1, \mathrm{y}_1, \mathrm{t}_1) = (0.39, 1.36, 1.65), \tilde{x}_2 = (\mathrm{x}_2, \mathrm{y}_2, \mathrm{t}_2) = (0.45, 0.45, 0.45), \tilde{\mathrm{Z}}_1 = (\mathrm{Z}_{1L},$
$\mathrm{Z}_{1M}, \mathrm{Z}_{1U}) = (2.07, 8.6, 16.75)$ and $\tilde{\mathrm{Z}}_2 = (\mathrm{Z}_{2L}, \mathrm{Z}_{2M}, \mathrm{Z}_{2U}) = (1.62, 9.95, 21.7)$.

5 Conclusions

In this paper, fuzzy linear Bilevel programming problems with fuzzy variables and fuzzy constraints are discussed. The significant of this paper is solving fuzzy linear Bilevel programming problem when the variables triangular fuzzy numbers without using ranking method. Thus the method is very useful in the real world problems where the product is uncertain.

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