

Constrained Renewable Resource Allocation in Fuzzy Metagraphs via Min-Slack

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Abstract This paper discusses that the fuzzy metagraphs can be used as a tool for scheduling and control of fuzzy projects. Often, available resources for executing projects may be limited. It is assumed the resources required to accomplish each activity of project (metagraph edges) is renewable. One of the common methods for scheduling projects is using the min-slack. So, first, the forward and backward computations for deterministic metagraphs are described. Then, assuming that the completion time of activities are positive trapezoidal fuzzy numbers, the forward and backward computations are generalized for fuzzy metagraphs. Consequently, completion time of project, earliest and latest time of start and end of activities and floating time of them are obtained as fuzzy numbers. Also, critical activities and critical paths are defined. Then, for scheduling, by using the selected ranking method, the activities of project (metagraph edges) based on min-slack ascending were ordered. Through some numerical examples, calculation steps and the results are illustrated.

Keywords Metagraph, Fuzzy, Constrained Renewable Resource, Project Scheduling.

1 Introduction

In many real world projects, the duration of activities is non-deterministic. A non-deterministic character may be stochastic or fuzzy. Recent works define the fuzzy characters for the project networks because the fuzzy models are closer to reality and simpler to use [1-7]. On the other hand, completion of a project on time has significant effects on its cost, revenue and usefulness. Therefore, the main objective of project managers is to avoid any delay. However, due to the limitation, to achieve this goal, optimal project scheduling is vital.

In recent years, many researchers used the metagraphs for analysis of systems. One of these applications is project planning and control using the metagraph. Metagraph and some of its applications are described in [8]. Metagraph and its specifications have been described in [9-11]. Metagraphs may have other applications. Applications of metagraph in decision support systems are shown in [12, 13]. Metagraphs are used in workflow management [14-16]. Hierarchical modeling by metagraphs is done in [17]. Model management using the Petri net and metagraphs has been illustrated in [18]. Reference [19] describes the model integration using metagraphs. By using the metagraphs, enterprise modeling can be executed [20]. Implicit integrity constraints using metagraphs has been described in [21]. Trapezoidal fuzzy number is proposed for estimation of activity time of project [2, 22]. A project can be

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shown as metagraph [16].

To the best of our knowledge, there is not any paper addressing the fuzzy metagraph scheduling. So, this paper discusses the constrained renewable resource allocation in fuzzy metagraphs. The aim is the completion of project with lowest delay subject to resource limitation. Fuzzy min-slack as a criterion is developed in order to reach the aim. The proposed algorithm is illustrated through an example.

This paper has the following structure. Section 2 introduces the metagraphs. Section 3 describes the kinds of paths in a metagraph. Section 4 determines the critical path method in deterministic metagraphs. Section 5 generalizes the critical path method for fuzzy metagraphs. By using the selected method and definition of new base for comparison of fuzzy trapezoidal numbers, Section 6 schedules the metagraph edges based on minimum fuzzy slack time when the available resource is renewable and constrained. Section 7 is devoted to conclusions and recommendations.

2 Metagraph

Definition 1. A metagraph is identified with $F = (X, E, D)$. $X = \{x_i, i = 1, 2, \dots, I\}$ is called the generating set. x_i is called the element of X . $E = \{e_j, j = 1, 2, \dots, n\}$ is the set of edges. Each edge is a ordered pair as (V_j, W_j) . $V_j \subset X$ is called the invertex of e_j and $W_j \subset X$ is called the outvertex of e_j such that $\forall j, V_j \cap W_j = \emptyset$. It is supposed that the duration of edge e_j is known and represented by d_j such that $d_j \in D$. Figure 1 shows a metagraph. In this metagraph $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$, $e_1 = (\{x_1\}, \{x_3\})$, $e_2 = (\{x_3, x_4\}, \{x_7\})$, $e_3 = (\{x_1, x_2\}, \{x_7\})$, $e_4 = (\{x_2\}, \{x_6\})$, $e_5 = (\{x_5, x_6\}, \{x_7\})$.

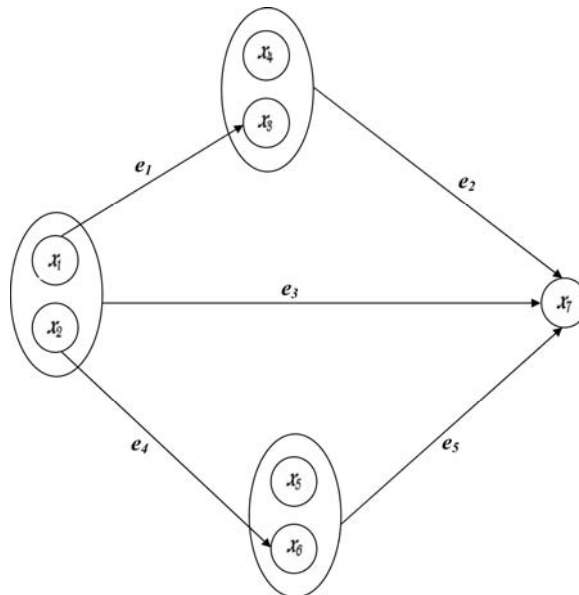


Fig. 1 A metagraph

Definition 2. If $a_k \in V_j$ then $V_j - \{a_k\}$ is the coinput of element a_k .

Definition 3. If $b_k \in W_j$ then $W_j - \{b_k\}$ is the cooutput of element b_k .

3 Paths

3.1 Simple Path

An element $x \in X$ is connected to element $x' \in X$ if the sequence of edges $(e'_k, k = 1, 2, \dots, K')$ exists such that, $x \in V'_1, x' \in W'_{K'}$ and $\forall k = 1, 2, \dots, K' - 1$ $W'_k \cap V'_{k+1} \neq \emptyset$. This sequence of edges is called a simple path from x to x' . x is called source and x' is called target. K' is called the length of simple path.

In figure 1 $e_4 e_5$ is a simple path from x_2 to x_7 .

3.2 Metapath

A metapath is the set of edges that is shown with $M(B, C)$. B is the source set and C is the target set. B is the invertex elements that are not also outvertex elements. C is the outvertex elements that are not also invertex elements.

Metagraph in figure 1 is a metapath with $B = \{x_1, x_2, x_4, x_5\}$ and $C = \{x_7\}$.

4 Critical path method for deterministic metagraph

Suppose that the duration of edge e_j be deterministic and known. It is shown by d_j . Then, by using the following algorithm [16], completion time of metapath and its critical path and other specifications such as the earliest and latest times of starting and finishing and critical edges can be determined.

Stage 1. Forward computations

For each element $x_i \in B$, set $Q_i = 0$ and mark x_i . Let $Q_i = 0$ for all other elements. Let $E = M(B, C)$. While $E \neq \emptyset$ for each edge e_j in E such that all elements in the invertex of e_j are mark set

$$ES_j = \max_{x_i \in V_j} \{Q_i\}$$

Such that ES_j is the earliest start time of edge e_j . Then, for each $x_k \in W_j$ set

$$Q_k = \max\{Q_k, (ES_j + d_j)\}, E \leftarrow E - \{e_j\} \text{ and mark it.}$$

Repeat the above operations while $E \neq \emptyset$. Then, T^e (earliest completion time of metapath) is obtained.

Set $T^e = T^l = \max\{Q_i\}, \forall x_i \in X$.

Stage 2. Backward computations

For each element $x_i \in C$, set $L_i = T^l$ and mark x_i . Let $L_i = T^l$ for all other elements. Let $E_0 = M(B, C)$. While $E_0 \neq \emptyset$ for each edge e_j in E_0 such that all elements in the outvertex of e_j are mark set

$$LF_j = \min_{x_i \in W_j} \{L_i\}$$

Such that LF_j is the latest finish time of edge e_j . Then, for each $x_k \in V_j$ set

$$L_k = \min\{L_k, (LF_j - d_j)\}, E_0 \leftarrow E_0 - \{e_j\} \text{ and mark it.}$$

Repeat the above operations while $E_0 = \emptyset$.

So, ordered pair (Q_i, L_i) can be obtained for each element of metagraph.

Example 1. Consider the metagraph of figure 2. d_j for each edge e_j is written under the edge.

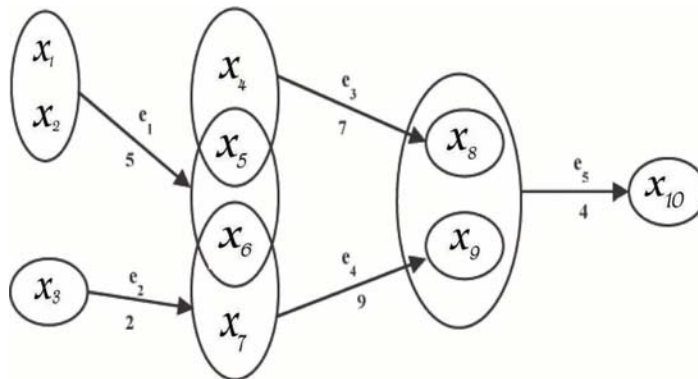


Fig. 2 Metagraph of example 1

By using the algorithm, results are obtained as figure 3.

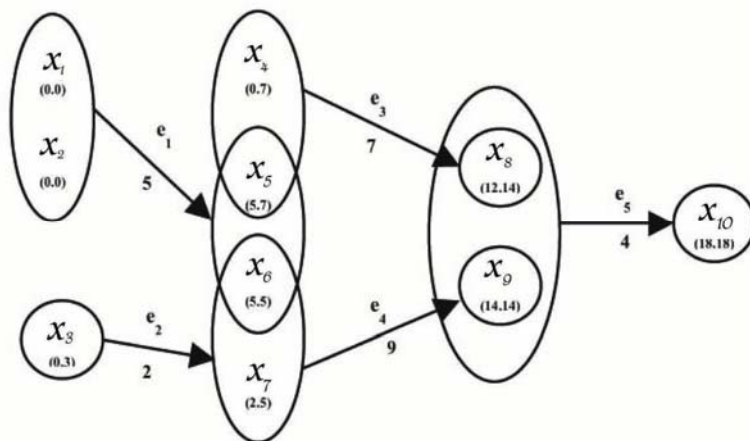


Fig. 3 Results for example 1

Definition 4. Element x_i is critical if and only if $Q_i = L_i$.

In example 1 elements $x_1, x_2, x_6, x_9, x_{10}$ are critical.

Definition 5. V is the critical invertex if and only if $\max_{x_i \in V}(Q_i) = \min_{x_i \in V}(L_i)$.

So, in example 1, invertex of edges e_1, e_4, e_5 are critical.

Note that, $\max_{x_i \in V}(Q_i) \leq \min_{x_i \in V}(L_i)$.

Theorem 1. An invertex V is critical if it contains any critical elements.

Proof: Assume that V contains two elements. Element a is critical and element b is non-critical. So, $Q_a = L_a$ and $\max_{x_i \in V}(Q_i) \leq \min_{x_i \in V}(L_i)$ or $\max(Q_a, Q_b) \leq \min(L_a, L_b)$ or

$$\max(Q_a, Q_b) \leq \min(Q_a, L_b).$$

Case 1. $Q_a \leq Q_b \leq L_b$

In this case, $\max\{Q_i\} = Q_b$ and $\min\{L_i\} = Q_a$. Therefore, $Q_b \leq Q_a$ and consequently, $Q_b = Q_a$ and invertex V is critical.

Case 2. $Q_b \leq Q_a \leq L_b$

In this case, $\max\{Q_i\} = Q_a$ and $\min\{L_i\} = Q_a$. So, invertex V is critical.

Case 3. $Q_b \leq L_b \leq Q_a$

In this case, $\max\{Q_i\} = Q_a$ and $\min\{L_i\} = L_b$ and $Q_a \leq L_b$. Consequently, $Q_a = L_b$. So, invertex V is critical.

Definition 6. The slack time of edge e_j is defined as $Slack(e_j) = \min_{x_i \in W_j}(L_i) - \max_{x_i \in V_j}(Q_j) - d_j$

Definition 7. Edge e_j is critical if and only if $Slack(e_j) = 0$

Theorem 2. If the invertex of edge e contains a critical element a with completion time T_a and the outvertex of this edge contains a critical element b with completion time T_b such that $T_b - T_a = d_e$, then e is the critical edge.

Proof: Since a, b are critical elements, then $\min_{x_i \in W_e}(L_i) \leq T_b$, $\max_{x_j \in V_e}(Q_j) \geq T_a$. So,

$$\min_{x_i \in W_e}(L_i) - \max_{x_j \in V_e}(Q_j) \leq T_b - T_a = d_e \text{ and finally, } Slack(e) \leq d_e - d_e = 0. \text{ Consequently, } Slack(e) = 0.$$

5 Critical path method in fuzzy metagraph

If the completion time of edges is not deterministic, it may be stochastic or fuzzy. Here, it is supposed that the completion time of edge is a positive trapezoidal fuzzy number. Some of researchers support this belief [2, 3, 4, 5, 6, 7, 12].

References [22, 23] introduce the operations on fuzzy numbers. Some of them are as follows.

If $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ are two arbitrary trapezoidal fuzzy numbers then

$$\begin{aligned}\tilde{A} + \tilde{B} &= (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2) \\ \tilde{A} - \tilde{B} &= (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2) \\ \text{m}\tilde{\text{a}}\text{x}(\tilde{A}, \tilde{B}) &= (\max(a_1, a_2), \max(b_1, b_2), \max(c_1, c_2), \max(d_1, d_2)) \\ \text{m}\tilde{\text{i}}\text{n}(\tilde{A}, \tilde{B}) &= (\min(a_1, a_2), \min(b_1, b_2), \min(c_1, c_2), \min(d_1, d_2))\end{aligned}$$

Stage 1. Forward computations

For each element $x_i \in B$, set $Q_i = (0,0,0,0)$ and mark x_i . Let $Q_i = (0,0,0,0)$ for all other elements. Let $E = M(B, C)$. While $E \neq \phi$ for each edge e_j in E such that all elements in the invertex of e_j are mark set

$$ES_j = \max_{x_i \in V_j} \{\tilde{Q}_i\}$$

Such that ES_j is the earliest start time of edge e_j . Then, for each $x_k \in W_j$ set

$$\tilde{Q}_k = \max\{\tilde{Q}_k, (ES_j + \tilde{d}_j)\}, E \leftarrow E - \{e_j\} \text{ and mark it.}$$

Repeat the above operations while $E = \phi$. Then, \tilde{T}^e (the earliest completion time of metapath) is obtained.

$$\text{Set } \tilde{T}^e = \tilde{T}^l = \max\{\tilde{Q}_i\}, \forall x_i \in X .$$

Stage 2. Backward computations

For each element $x_i \in C$, set $\tilde{L}_i = \tilde{T}^l$ and mark x_i . Let $\tilde{L}_i = \tilde{T}^l$ for all other elements. Let $E_0 = M(B, C)$. While $E_0 \neq \phi$ for each edge e_j in E_0 such that all elements in the outvertex of e_j are mark set

$$LF_j = \min_{x_i \in W_j} \{\tilde{L}_i\}$$

Such that LF_j is the latest finish time of edge e_j . Then, for each $x_k \in V_j$ set

$$\tilde{L}_k = \min\{\tilde{L}_k, (LF_j - \tilde{d}_j)\}, E_0 \leftarrow E_0 - \{e_j\} \text{ and mark it.}$$

Repeat the above operations while $E_0 = \phi$.

So, ordered pair $(\tilde{Q}_i, \tilde{L}_i)$ can be obtained for each element of metagraph.

In backward computations, we must compute the subtraction of two positive trapezoidal fuzzy numbers. This subtraction must be positive trapezoidal fuzzy number but based on subtraction definition in this section, the subtraction of two positive trapezoidal fuzzy numbers may be non-positive and this result in backward computations is not feasible. So, we apply the other suitable subtraction operator that supposed in [24]. This subtraction operator is defined as follows:

$$\text{If } LF_j = (lf_{j1}, lf_{j2}, lf_{j3}, lf_{j4}),$$

$$L\tilde{S}_j = (ls_{j1}, ls_{j2}, ls_{j3}, ls_{j4})$$

$$\text{and } \tilde{d}_j = (d_{j1}, d_{j2}, d_{j3}, d_{j4}) \text{ then}$$

$$L\tilde{S}_j = L\tilde{F}_j - \tilde{d}_j = (ls_{j1}, ls_{j2}, ls_{j3}, ls_{j4})$$

$$ls_{j4} = \max(0, \min(lf_{j4} - d_{j4}))$$

$$ls_{j3} = \max(0, \min(lf_{j4}, \min(lf_{j3} - d_{j3})))$$

$$ls_{j2} = \max(0, \min(lf_{j3}, \min(lf_{j2} - d_{j2})))$$

$$ls_{j1} = \max(0, \min(lf_{j2}, \min(lf_{j1} - d_{j1})))$$

Example 2. consider the fuzzy metagraph shown in figure 4.

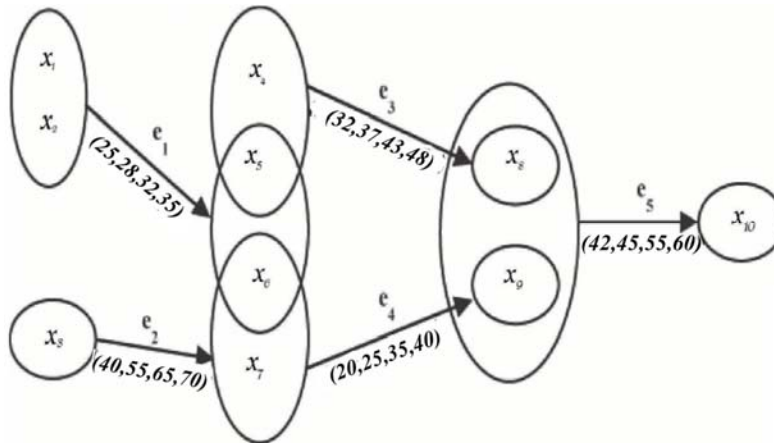


Fig. 4 The metagraph of example 2

After the forward and backward computations, results are shown in figure 5.

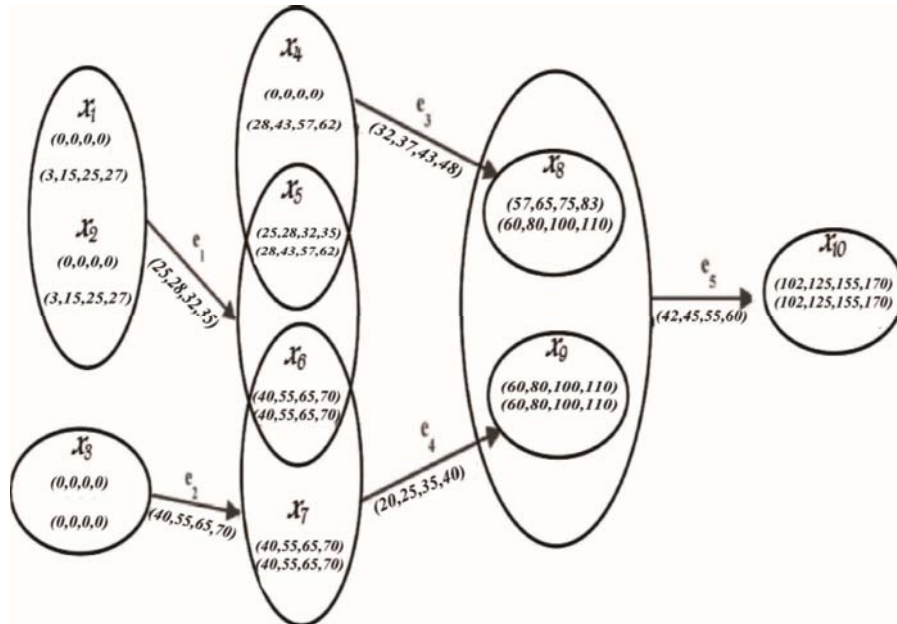


Fig. 5 Result of computations of critical path method in example 2

Now, we can generalize the previous definitions and theorems for fuzzy metagraphs.

Definition 8. If $\tilde{Q}_i = \tilde{L}_i$ then element x_i is critical.

Definition 9. If $\max_{x_i \in V}(\tilde{Q}_i) = \min_{x_i \in V}(\tilde{L}_i)$ then V is the critical invertex.

Theorem 3. An invertex V is critical if it contains any critical elements.

Theorem 4. If the invertex of edge e contains a critical element a with completion time \tilde{T}_a and the outvertex of this edge contains a critical element b with completion time \tilde{T}_b such that $\tilde{T}_b - \tilde{T}_a = \tilde{d}_e$ then e is the critical edge.

Consider that the inverse of theorems 3, 4 may be incorrect.

Definition 10. The slack time of edge e_j is defined as $Slack(e_j) = \min_{x_i \in W_j}(\tilde{L}_i) - \max_{x_i \in V_j}(\tilde{Q}_i) - \tilde{d}_j$

Definition 11. Edge e_j is critical if and only if $Slack(e_j) = (0,0,0,0)$

In example 2, completion time of metagraph is (102, 125, 155, 170) and edges e_2, e_4, e_5 are critical. So, $Slack(e_2) = Slack(e_4) = Slack(e_5) = (0,0,0,0)$.

Edges e_1, e_3 are non-critical and slack times of them are

$$Slack(e_1) = (3,15,25,27), \quad Slack(e_3) = (3,15,25,27)$$

6 Constrained renewable resource allocation in fuzzy metagraphs

Suppose that available resource for executing the metagraph be renewable and constrained. Therefore, the main objective of project managers is to avoid any delay. So, first, critical edges must be executed. Also, other edges must be in ascending order based on slack time of edges. However, we need a suitable ranking method for ordering. A method that described in [25] is selected for achieving this purpose.

In the selected method, it is supposed that, A, B are any two fuzzy numbers with arbitrary continuous membership functions $\mu_A(x), x \in \Omega_A$, $\mu_B(x), x \in \Omega_B$. Also, suppose that $\Omega = \Omega_A \cup \Omega_B$ and $\alpha \in \Omega$ is a given number. $G_A(\alpha)$ and $G_B(\alpha)$ for fuzzy numbers A, B are defined as

$$G_A(\alpha) = \frac{\int_{\alpha}^{U_A} \mu_A(x) dx}{\int_{L_A}^{U_A} \mu_A(x) dx}$$

$$G_B(\alpha) = \frac{\int_{\alpha}^{U_B} \mu_B(x) dx}{\int_{L_B}^{U_B} \mu_B(x) dx}$$

where

$$U_A = \max(x | x \in \Omega_A), L_A = \min(x | x \in \Omega_A)$$

$$U_B = \max(x | x \in \Omega_B), L_B = \min(x | x \in \Omega_B)$$

Definition 12. $A \geq B$ if and only if $G_A(\alpha) \geq G_B(\alpha)$.

The above definition can be extended for more than two fuzzy numbers. Suppose that the slack times of non-critical edges are shown with $Slack(e_j) = (s_{j1}, s_{j2}, s_{j3}, s_{j4})$. In this paper, the following value is proposed for α . Notice that the selected fuzzy ranking method can order the slack times based on α .

$$\alpha = \frac{\sum_{j \in NC} \frac{1}{4} \sum_{k=1}^4 s_{jk}}{|NC|}$$

Where NC is the set of non-critical edges and $|NC|$ is the number of NC members.

Example 3. Consider the fuzzy metagraph shown in figure 6.

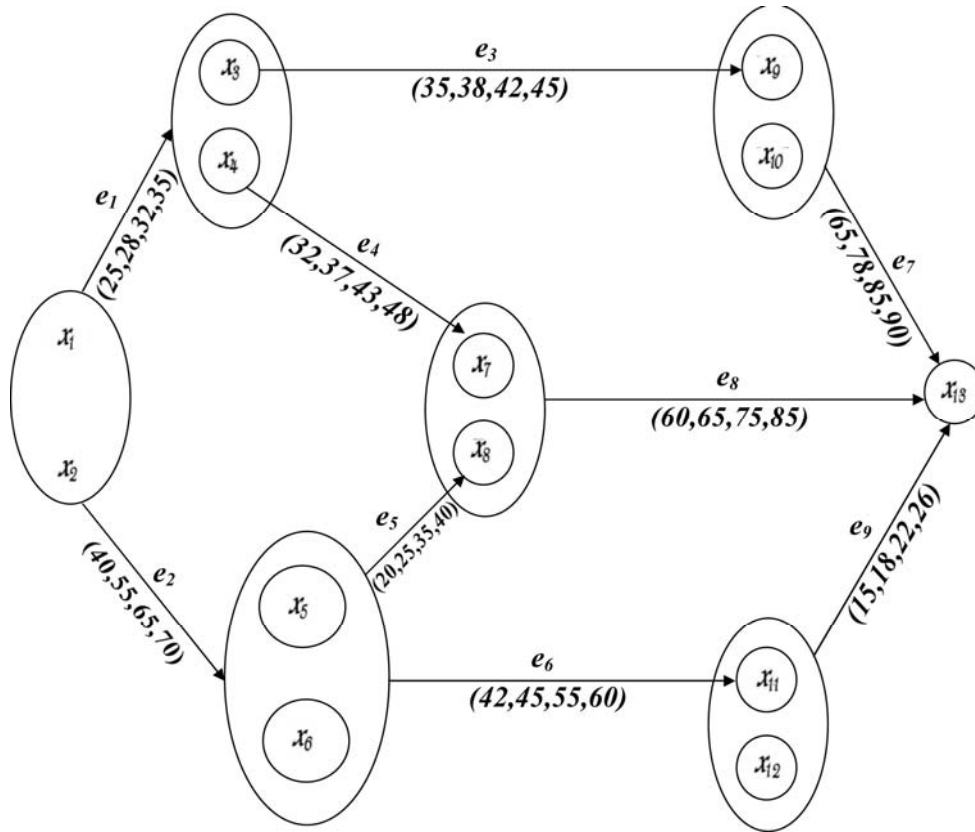


Fig. 6 Metagraph of example 6

By using the proposed method described in section 5, fuzzy slack time of edges are obtained. Then, fuzzy slack times are ordered based on selected method and proposed α in section 6. α in this example is $\frac{77}{6}$. Results are summarized in table 1.

Table 1 Fuzzy slack times in example 3

e_j	Ordered fuzzy slack time of e_j
e_2	(0,0,0,0)
e_5	(0,0,0,0)
e_8	(0,0,0,0)
e_1	(0,1,16,25)
e_3	(0,1,16,25)
e_7	(0,1,16,25)
e_4	(8,15,25,27)
e_6	(27,27,33,39)
e_9	(27,27,33,39)

7 Conclusions and recommendations

In this paper, it is shown that the uncertain projects can be defined as metagraphs with fuzzy edge times. This paper by generalizing the critical path method for fuzzy metagraphs and computing the fuzzy slack times and ranking them based on new criteria gives the primal base for scheduling fuzzy metagraphs when the available resource is renewable and constrained.

In future studies, combination of fuzzy min-slack with other known criteria such as min-latest finish time in fuzzy environment or using the other ranking method may be useful.

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