Forecasting Stock Market Using Wavelet Transforms and Neural Networks and ARIMA (Case study of price index of Tehran Stock Exchange)

N. Mousazadeh Abbasi*, M. A. Aghaei, M. Moradzadeh fard

Received: 19 December 2014 ; Accepted: 6 May 2015

Abstract The goal of this research is to predict total stock market index of Tehran Stock Exchange, using the compound method of ARIMA and neural network in order for the active participations of finance market as well as macro decision makers to be able to predict trend of the market. First, the series of price index was decomposed by wavelet transform, then the smooth's series predicted by using the ARIMA and low-frequency parts of the signal was predicted by using neural network method then this predicted was compound with inverse wavelet transform. The main attention of this paper is investors and traders to achieve a method for predict stock market. Concerning the results of previous researches which confirm the relative superiority of non-linear models in price index prediction, an appropriate model has been offered in this research by compounding the non-linear method and linear method such as neural network and ARIMA with using wavelet transform, The results indicate superiority of the designed system in predicting price index of Tehran Stock Exchange. This paper by compounding the linear and non-linear method issues pattern to predict stock market, to encourage further investigation by academics and practitioners in the field.

Key words: Artificial neural network, Wavelet Transform, ARIMA and Tehran Stock Exchange.

Introduction

Mining stock market tendencies is a challenging task. Numerous factors influence stock market performance, including political events, general economic conditions, and trader expectations. Though stock and futures traders rely heavily on various types of intelligent systems to make trading decisions, to date their success has been limited [1]. Even financial experts find it difficult to make accurate predictions, because stock market trends tend to be nonlinear, uncertain, and non-stationary. No consensus exists among experts as to the effectiveness of forecasting a financial time series. One model that may be more efficient than others in stock prediction is the artificial neural network [2]. Several studies have shown
that the ANN outperforms statistical regression models [3] and discriminant analysis [4]. Generally, two different methodologies exist for stock price prediction using ANNs [5]. The first methodology considers stock price variations as a time series and predicts future prices using past data. This approach uses ANNs as predictors [6]. These prediction models suffer limitations owing to the enormous noise and high dimensionality of stock price data. Consequently, none of the existing prediction models has satisfactory performance, as Zadeh [2] and Marmer [7] have observed. A second approach for stock price prediction has been proposed that considers technical indices and qualitative factors, such as political effects in stock market forecasting and trend analysis. The idea here is that merging technical indicators permits the exploitation of tolerance for imprecision, uncertainty, and partial truth to achieve tractability, robustness, and low-cost solutions.

Other attempts have been made to forecast financial markets that range from traditional time series approaches to artificial intelligence techniques, including, ARIMA, ARCH-GARCH models [8], ANNs [2], and evolutionary computation methods [9]. However, the main disadvantage of both ANNs and these other black-box techniques is the enormous difficulty of interpreting the results.

This study diverges from previous attempts at forecasting stock prices by proposing a method that uses the Wavelet transforms combined with the ANN and ARIMA, this process create a transparent architecture.

Wavelet analysis is a relatively new field in signal processing [10]. Wavelets are mathematical functions that decompose data into different frequency components, after which each component is studied with a resolution matched to its scale, where a scale denotes a time horizon [11]. Wavelet filtering is closely related to the volatile and time varying characteristics of the real-world time series and is not limited by the stationarity assumption [12]. The wavelet transform decomposes a process into different scales, making it useful in distinguishing seasonality, revealing structural breaks and volatility clusters, and identifying local and global dynamic properties of a process at specific timescales [13]. Wavelet analysis has been shown to be particularly useful in analyzing, modeling, and predicting the behavior of financial instruments as diverse as stocks and exchange rates [14, 15]. This study applies wavelet transform using the Daubechies wavelet to decompose the time series. The framework combines several statistical methods and soft computing techniques such as RNN, wavelet transform, and ARIMA. Besides applying wavelet-based data representation, using prior study to determining data processing by the RNN, and using ARIMA model for forecasting liner decomposed time series by wavelet transform.

We tested our method on the Tehran Price Exchange Index (TEPIX) for the period 2005–2011, and found that it was more predictable than other methods.

Methodology

Previous studies have used statistics, technical analysis, fundamental analysis, and linear regression to predict market direction [5]. However, price forecasting is generally conducted using technical analysis or fundamental analysis. Technical analysis concentrates on market action, while fundamental analysis concentrates on the forces of supply and demand that drive price movements. The basic assumption of this study is supported by studies of the financial time series, which indicate that price movements are closely related to market returns during periods of volatility, but also to fundamental factors. The output of these factors is stock price. To study the relations among the financial time-series variables, this work presents a hybrid
method that integrates a wavelet and the ARIMA-RNN-based forecasting scheme. Fig. 1 shows the main procedures of this approach.

Wavelet theory is applied for data preprocessing, since the representation of a wavelet can deal with the non-stationarity involved in the economic and financial time series [14]. The key property of wavelets for economic analysis is decomposition by time scale. Economic and financial systems contain variables that operate on various time scales simultaneously; thus, the relations between variables may differ across time scales. One of the benefits of the wavelet approach is that it is flexible in handling highly irregular data series [12].

This study applies the Daubechies wavelet as the main wavelet transform tool. A wavelet not only decomposes the data in terms of times and frequency, but also significantly reduces the processing time. Let n denote the time series size, then the wavelet decomposition used in this study can be determined in O (n) time [16].

Fig. 1 Framework of our method (WNN-ARIMA)

Wavelets theory is based on Fourier analysis, which represents any function as the sum of the sine and cosine functions. A wavelet \( \psi(t) \) is simply a function of time \( t \) that obeys a basic rule, known as the wavelet admissibility condition [17]:

\[
c_{\psi} = \int_{-\infty}^{\infty} \frac{1}{f} |\psi(f)| df < \infty
\]

where \( \psi(f) \) is the Fourier transform and a function of frequency \( f \), of \( \psi(t) \). The wavelet transform (WT) is a mathematical tool that can be applied to numerous applications, such as image analysis and signal processing. It was introduced to solve problems associated with the Fourier transform as they occur. This occurrence can take place when dealing with non-stationary signals, or when dealing with signals that are localized in time, space, or frequency. Depending on the normalization rules, there are two types of wavelets within a given function/family. Father wavelets describe the smooth and low-frequency parts of a signal, and mother wavelets describe the detailed and high-frequency components. In the following equations, (2a) represents the father wavelet and (2b) represents the mother wavelet, with \( j=1, ..., J \) in the J-level wavelet decomposition: [14]

\[
\phi_{j,k} = 2^{-j/2} \phi(t - 2^{j} k / 2^{j})
\]

\[
\psi_{j,k} = 2^{-j/2} \psi(t - 2^{j} k / 2^{j})
\]
Where J denotes the maximum scale sustainable by the number of data points and the two types of wavelets stated above, namely father wavelets and mother wavelets, and satisfies:

\[ \int f(t) dt = 1 \text{ and } \int \psi(t) dt = 0 \]  

(3)

Time series data, i.e., function f(t), is an input represented by wavelet analysis, and can be built up as a sequence of projections onto father and mother wavelets indexed by both \( \{k\}, k = \{0, 1, 2, \ldots\} \) and by \( \{s\} = 2^j, \{j=1,2,3,\ldots J\} \). Analyzing real discretely sampled data requires creating a lattice for making calculations. Mathematically, it is convenient to use a dyadic expansion, as shown in equation (3). The expansion coefficients are given by the projections:

\[
\begin{align*} 
    s_{j,k} &= \int \phi_{j,k} f(t) dt \\
    d_{j,k} &= \int \psi_{j,k} f(t) dt (j=1,2,\ldots,J) 
\end{align*} 
\]

(4)

The orthogonal wavelet series approximation to f(t) is defined by:

\[
F(t) = \sum_k s_{j,k} \phi_{j,k}(t) + \sum_k d_{j,k} \psi_{j,k}(t) + \sum_k d_{j-1,k} \psi_{j-1,k}(t) + \cdots + \sum_k d_{1,k} \psi_{1,k}(t) 
\]

(5)

Another brief form can also be represented:

\[
\begin{align*} 
    S_j(t) &= D_j(t) + D_{j-1}(t) + \cdots + D_1(t) \\
    S_j(t) &= \sum_k s_{j,k} \phi_{j,k}(t) \\
    D_j(t) &= \sum_k d_{j,k} \psi_{j,k}(t) 
\end{align*} 
\]

(6)

The WT is used to calculate the coefficient of the wavelet series approximation in Eq. (5) for a discrete signal \( f_1, f_2, \ldots, f_n \) with finite extent. The WT maps the vector \( f = (f_1, f_2, \ldots, f_n) \) to a vector of \( n \) wavelet coefficients \( w = (w_1, w_2, \ldots, w_n) \), which contains both the smoothing coefficient \( s_{j,k} \) and the detail coefficients \( d_{j,k} \). Generally, \( s_{j,k} \) describes the underlying smooth behavior of the signal at coarse scale \( 2^j \), while \( d_{j,k} \) provides the coarse scale deviations from the smooth behavior, and \( d_{j-1,k}, \ldots, d_{1,k} \) provides progressively finer scale deviations from the smooth behavior (Adel et al, 2006).

When \( n \) is divisible by \( 2^j \), \( d_{1,k} \) contains \( n/2 \) observations at the finest scale \( 2^1 = 2 \), and \( n/4 \) observations in \( d_{2,k} \) at the second finest scale, \( 2^1 = 2 \). Likewise, each of \( d_{j,k} \) and \( s_{j,k} \) contain \( n/2^j \) observations, where

\[
\text{Let } f(t) \text{ denote the original data, } s_1, \text{ represents an approximation signal, and } D_1 \text{ is a detailed signal. This study defines the multi-resolution decomposition of a signal by specifying: } S_j \text{ is the coarsest scale and } S_{j-1} = S_j + D_j. \text{ Generally, } S_{j-1} = S_j + D_j \text{ where } \{S_j, S_{j-1}, \ldots, S_1\} \text{ is a sequence of multi-resolution approximations of the function } f(t), \text{ with ever increasing levels of refinement. The corresponding multi-resolution decomposition of } f(t) \text{ is given by } \{S_j, D_j, D_{j-1}, \ldots, D_j, \ldots, D_1\}. 
\]

The sequence of terms \( S_j, D_j, D_{j-1}, \ldots, D_j, \ldots, D_1 \) represents a set of orthogonal signal components that represent the signal at resolutions 1 to J. Each \( D_{j-k} \) provides the orthogonal increment to the representation of the function \( f(t) \) at the scale (or resolution) \( 2^{j-k} \).

When the data pattern is very rough, the wavelet process is repeatedly applied. The aim of preprocessing is to minimize the Root Mean Squared Error (RMSE) between the signal before and after transformation. The noise in the original data can thus be removed. Importantly, the adaptive noise in the training pattern may reduce the risk of overfitting in training phase [18]. Thus, we adopt WT twice for the preprocessing of training data in this study.
Literature review

Tang et al [19] offered a model for the prediction of stock prices, using a compound of wavelet transform, recurrent neural network and bee colony algorithm. First, they disintegrated the price time series using Haar wavelet then the prediction was done by recurrent neural network and the obtained weights of neural network were optimized by bee colony algorithm. The offered model was examined on data of Dow Jones Industrial Average (DJIA), FTSE 100 index, London Stock Exchange (FTSE), Nikkei 225, Tokyo Stock Exchange (Nikkei) and Taiex index, Taiwan Stock Exchange. The given model was compared to compound model of neural network and bee colony algorithm, Fuzzy time series and Fuzzy neural network (ANFIS). The suggested model had less error than the other models in all examined cases.

Hadavandi et al [20] offered a model for the prediction of stock price, using a compound of neural network and Fuzzy genetic. They examined the mentioned model on gathered information for IT and airline industry of Newyork Stock Exchange. The suggested model was compared to ARIMA and genetic algorithm and neural network which were used in prediction and in all cases it resulted better than the previous models.

In 2008, Chung and Lu [21] used the fuzzy rule of TSK type to predict stock price. TSK fuzzy model considers technical index as input variables and the obtained result is a linear compound of input variables. This model is examined on the data of electronic Stock Exchange Corporation of Taiwan and the obtained results indicate a precision close to 97.6% in TSE index and 98.08% in Media Tek.

Since in stock market the intellectual investors revise their predictions according to the most recent prediction errors, Chung et al used a new time series model in 2008 to decrease the prediction error in Taiwan stock market. The results indicate the superiority of this model to Chen and Yu model.

In 2007 Chung et al [22] used a Fuzzy time series model for short-term prediction of Taiwan and Hong Kong stock market price. The obtained experimental results of this research indicate the fact that the traditional statistical method and offered model both makes it clear that stock price patterns are short-term in these two markets.

Chung et al [21] used the two element Fuzzy time series model for stock index prediction. Stock index and the amount of trade are considered (in this article) as elements which are effective in price index prediction. The results indicate the good capability of this model in stock index prediction.

Lin et al [23] used the genetic algorithm to predict stock market. The significant factor in a trading rule success is the selection of degrees for all parameters and their combinations. However, the range of parameters changes in a large area and the problem is to find the optimum parameter combinations. Genetic algorithm is used in this article to solve the problem.

Chen et al in 2007 [22] used Fuzzy time series based on Fibonacci sequence to predict stock price. A time period of five years of data for TSMC and a time period of 13 years for TAIEX was taken in this research. The obtained model is superior to the prevalent Fuzzy time series model.

M.T.Sung and his colleagues [24] analyzed the usage of NN5 in stock price prediction of Hong Kong in 2007. This system has been examined on stock data of two banking stock
corporations of Hong Kong and Shanghai. The system indicates a total success rate of more than 70%.

Salcedo-sanz et al [25] indicate the usage of genetic planning in predicting bankruptcy of companies with no life insurance in 2005. Genetic planning has been compared to other classifying methods in this article.

Farnsworth et al in 2004 [26] used genetic planning to predict daily output of S&P500 index amount which indicates the hypothesis of market efficiency. S&P500 is one of the most common indexes which has been studied worldwide.

Refenes, Zapranis and Francis [27] have compared the function of stock price treatment model with regression models through its modeling by neural network. Neural networks are used as a substitution for classical statistic techniques in this research and these networks have been used to predict large companies stocks. The results indicate that neural network function better than statistic techniques and they present superior models.

Research statistical population

The daily price index of Tehran Stock Exchange from 2005 to 2011 has been selected as the statistical population, 1442 data were accumulated for each variable from related databases in the mentioned period. The above-mentioned data were divided to two groups of training and experimental regarding the structure of neural network. Each group is respectively consisted of 1400 and 42 data and it should be also noted that mainly the accomplished prediction for the experimental period was selected as the comparison criterion for models used in prediction.

Methods of price index modeling

Methods of price index modeling will be explained in this section.

Neural Network method:

Preparing data is one of the complicated steps of neural network application, since the best condition for neural network is when all the inputs and outputs are between 0 and 1. One of the reasons which emphasize inserting inputs in range of 0 and 1 is the fact that transfer functions (such as sigmoid function) are unable to differential between large amounts. Therefore, the whole data were normalized using this formula: \[ X_{n} = X / X_{max} \]

Then, variables of S&P500 index of Newyork Exchange Stock, world gold price, Iran basket crude oil price and state dollar value were determined as neural network concerning studies of effective elements on price index. Prediction using MLP is in such a way that the best output weight with the least prediction error is selected by imposing training and learning on the network. After normalizing data, they were given to the network. On the other hand, data were delivered to two groups in order to examine the consistency of the output weight in such a way that first acquiring is done on training data of prediction in order to examine the accuracy of network prediction. About %97 of the total used data were considered as training data and the rest were used to examine the network. The amount of network learning by purpose functions was continuously examined during the learning process and finally a
network with the least error was selected. The parameters of final neural network were determined as is shown in the following table1.

<table>
<thead>
<tr>
<th>Character</th>
<th>Activation function</th>
<th>Learning rate</th>
<th>Epoch</th>
<th>Hidden</th>
</tr>
</thead>
<tbody>
<tr>
<td>mom</td>
<td>Non-linear Sigmoid</td>
<td>0/1</td>
<td>90000</td>
<td>3</td>
</tr>
</tbody>
</table>

In the Figure2, the prediction done by neural network has been compared to real values for experimental data:

![Figure 2 Neural Network Prediction](image)

In addition, values of examination criterion of functionality which is used for this model is as table2.

<table>
<thead>
<tr>
<th>Examination criterion of MLP</th>
<th>ADJR²</th>
<th>R²</th>
<th>MAPE</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0/9541</td>
<td>0/9586</td>
<td>0/0055</td>
<td>0/0048</td>
</tr>
</tbody>
</table>

Concerning the amount of R² and adjusted R² obtained from fitting model, it can be concluded that the presented neural networks model is an appropriate one for price index prediction. In such a way that the model with R² has been able to predict correctly the trend of price index of Tehran Stock Exchange for %95.

**Wavelet based model (WNN-ARIMA)**

At first using Daubechies wavelet we analyzed the first set for two levels so that the smooth's set was separated from low-frequency parts of a signal. Then keeping this in mind that linear models in prevision of smooth's series are more powerful, we used the ARIMA model for forecasting of this set and after that for low-frequency part forecasting, we used the neural networks, by according to affective elements of price index.

Before anything we investigated the stationary of the set and determined the total of Autoregressive sentences and total of moving average sentences. At this point the smooth's set with one level of difference was station and to determine the total of Autoregressive sentences and moving average sentences Akaike and Schwarz criterion was being used, in a way that ARIMA model was optimized with total Autoregressive sentences and the moving average sentences and at last the optimized model was determined as ARIMA(2,1,2).
In order that we could compare these models that have been used and for having a prevision of low-frequency, neural network was being used so that it has the same characters with the previous neural network and therefore the adequate parameters from optimized neural networks were gained (table 3).

Table 3 Neural network character

<table>
<thead>
<tr>
<th>mom</th>
<th>Epoch</th>
<th>Hidden</th>
<th>Learning rate</th>
<th>Activation function</th>
</tr>
</thead>
<tbody>
<tr>
<td>0/15</td>
<td>17000</td>
<td>3</td>
<td>0/15</td>
<td>Non-linear Sigmoid</td>
</tr>
</tbody>
</table>

As for investigating the accuracy prevision of the recombinant model, we used performance evaluation methods and the results are as table 4.

Table 4 Examination criterion of WNN-ARIMA

<table>
<thead>
<tr>
<th>ADJR²</th>
<th>R²</th>
<th>MAPE</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0/9680</td>
<td>0/9711</td>
<td>0/0049</td>
<td>0/0033</td>
</tr>
</tbody>
</table>

In the Figure 3, the prediction done by this model has been compared to real values for experimental data:

![Figure 3 WNN-ARIMA Prediction](image)

Regarding the value of $R^2$ and the adjusted $R^2$ obtained from fitting the model, it can be concluded that the presented WNN-ARIMA model is an appropriated model for price index prediction, so that this model with $R^2$ has been able to predict correctly the trend of price index of Tehran Stock Exchange for %95.

**Conclusion**

The general goal of the research is to offer an appropriate model for price index prediction of Tehran Stock Exchange. Concerning the results of previous researches which confirm the relative superiority of non-linear models in price index prediction, an appropriate model has been offered in this research by compounding the non-linear method and linear method such as neural network and ARIMA with using wavelet transform, in order to predict price index of Tehran Stock Exchange. The wavelet based method used for price index prediction of Tehran Stock Exchange has resulted better as shown in the figure 4:
Regarding the $R^2$ examination criterion, these techniques also indicates its relative superiority when compared to neural network technique. The WNN-ARIMA method can help the investors in their decision making related to investment.

**Suggestions for further researches:**

The following topics are suggested for further research regarding the significant of prediction for managers and investors:

- Dividend per Share (DPS) and P/E ratio using neural network.
- Predicting the rest of important financial factors such as Earning per Share (EPS),
- Comparing this method with Fuzzy regression methods in price index prediction.

**References**