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A New Method in Bankruptcy Assessment Using DEA Game Theory

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Abstract Among the most important economic concepts are evaluation and bankruptcy prediction; financial problems which can expose an organization to serious risks and lead it to bankruptcy. Therefore, bankruptcy assessment of an organization enables it to be getting to know the financial risks and resolve financial deficiencies. So in this paper, we discuss one of the most important economic concepts and a new model is presented in the game theory of data envelopment analysis (DEA) and in other words. Thereby, the proposed model can assess the demands of an organization in the form of a player and distinguish net estate or the remaining estate after payment of the demands. The advantage of this model as a model of bankruptcy in games theory could be an easier concept and more practical tool for understanding bankruptcy in case of using imprecise and qualitative concepts and important indexes in the economic as financial ratios. Indeed, DEA has the comparative property that makes it suitable in presented model using game theory.

Keywords: DEA, Game Theory, Bankruptcy, Allocation.

1 Introduction

One of the most significant concepts of an economy is bankruptcy and its assessment. DEA has been generally used to assess the best relative efficiency of decision-making units (DMUs), but there are models that locate the worst performing DMUs and determine an inefficient frontier. The additive DEA method of bankruptcy assessment developed by Premachandra, *et al*., [1] takes a set of financial ratios as output variables in such a way that lower value of a ratio indicates better prosperity of the firm and another set of financial ratios as input variables; in such a way higher value of these ratios assessment is in agreement with economic theory of bankruptcy. Inefficient firms are most often eliminated through bankruptcy. Also, economic theory suggests that bankruptcy should serve as a screening process designed to eliminate only those firms that are economically inefficient. Therefore, bankruptcy is the worst position DMU; in such cases bankruptcy value should be assessed in the direction of anti-ideal DMU.

In bankruptcy prediction models, bankruptcy will be based on selected financial ratios. Altman's [2] work of predicting bankruptcy uses financial ratio. In another research, Premachandra, *et al*. [3] used DEA as an alternative method for bankruptcy measurement by

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applying Charne’s additive DEA model based on positive or non-positive objective function introduced bankruptcy and non-bankruptcy.

Game theory is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. Game theory is mainly used in economics, political science, and psychology, as well as logic, computer science, and biology.

Therefore, game theory helps us to consider bankruptcy problem in the form of a game, and we assess their performance viewpoint of the game theory. A bankruptcy problem occurs when a company goes to the bankruptcy owing money to some investors, but the company has only an amount E to cover debts [4, 5]. We use concepts above and combine game theory and the linear programming. At the end, we will suggest a new bankruptcy model.

The paper is organized as follows. We first provide the framework of bankruptcy model in the DEA in section 2; bankruptcy problems as well as cooperative games in characteristic function form are discussed in section 3 that appropriately summarizes the situation of bankruptcy problems. The study of a new bankruptcy assessment using DEA and game theory is discussed in section 4. Finally, we study a numerical example for bankruptcy game in the economics in section 5, and we discuss the results of the new model and its advantages in section 6.

2 Bankruptcy Assessment Model with DEA Concepts

In this section, we give a brief review of some concepts and notation related to bankruptcy problem and DEA as well as preliminaries related to *n*-person cooperative games in characteristic function form [6, 7].

2.1 Directional Distance Function

The DEA method is a linear programming model based on the measurement of the efficiency of DMUs. Suppose that there are a set of ‘n’ DMUs with’ inputs and‘s’ outputs. Let y_{rj} be the amount of r the output from the DMU_j and x_{ij} be the amount of i the input in the j unit. All data are assumed to be non-negative but at least one component of every input and output vector is positive and is given by $x_{ij} \geq 0, x_j \neq 0$, $y_{rj} \geq 0, y_j \neq 0$ for $j = 1,...,n$.

The DEA models can be used as a directional distance function. In fact, direction distance formulation has been used in the measurement of efficiency [3].

$$
\begin{aligned}
& \theta^* = Min \quad \theta \\
& s.t. \\
& \qquad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1,...,m, \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1,...,s, \\
& \qquad \lambda_j \geq 0, \qquad j = 1,...,n.
\end{aligned}
\tag{1}
$$

In the represented model x_{io}', y_{ro} show the input and output have been studied DMU.

This model has a feasible solution $\theta = 1,\ \lambda_o = 1,\ \lambda_j = 0\left(j \neq 0\right)$. Thus θ^* isn't greater than 1. Now, DEA models can be considered as a directional distance function for measurement of performance. Now we are using the directional distance function to bankruptcy assessment.

The variable returns to scale (VRS) DEA model for the directional distance function developed by Chambers as follows. In this method, the directional distance function can be calculated for each input and output vectors associated with production possibility set [8]:

$$
\begin{aligned}
& Max \quad \beta \\
& s.t. \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \beta g_{y_{ro}}, \\
& \qquad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \beta g_{x_{io}}, \\
& \qquad \lambda_j \geq 0,\ j = 1, 2, \ldots, n, \\
& \qquad g_{y_{ro}}, g_{x_{io}} \geq 0, \\
& \qquad g_x = \max_j \left\{x_{ij}\right\} - x_{io} \quad i = 1, \ldots, m, \\
& \qquad g_y = y_{ro} - \min\left\{y_{rj}\right\} \quad r = 1, \ldots., \mathrm{s}.
\end{aligned}
\tag{2}
$$

The above model β can be unrestricted. In fact, the technical inefficiency is measured by β in the decision-making units. So $1-\beta$ will indicate efficiency and increase in input and outputs reduction simultaneously be done by the amount of β. In model (2), g_x and g_y are direction vectors.

2.2 Bankruptcy Assessment Using Modified Directional Distance Function

Here, for the bankruptcy assessment, a modified version of DEA with worst relative efficiency as a model (3) is given. We examine model output oriented. At the first worst position of a DMU, introduced as anti-ideal DMU that the input and output of this DMU are as follows[9]:

$$x_i^{\max} = \max_j (x_{ij}) \quad i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$$

$$y_r^{\min} = \min_j (y_{rj}) \quad r = 1, 2, \ldots, s;\ j = 1, 2, \ldots, n$$

We assume that each 'n' DMU has'm' inputs and's' outputs. DMU_o is the under evaluation unit. For bankruptcy assessment $\left(x_o, y_o\right)$ will change the output of DMU_o in the direction output vector g_y. In this case, the production possibility set is defined as follows:

$$T_{CRS}^{BR} = \left\{(x, y): x \leq \sum_{j=1}^{n} \lambda_j x_{ij}\ ,\ y \geq \sum_{j=1}^{n} \lambda_j y_{rj}\ ;\ \lambda_j \geq 0\right\}$$

So, presented oriented-output bankruptcy model as follows:

$$\begin{aligned}
&Max \quad \beta^{BR} \\
&s.t. \\
&\qquad \sum_{j=1}^{n} \lambda_j^{BR} y_{rj} \le y_{ro} - \beta^{BR} g_y; \quad r = 1,2,\dots,s, \\
&\qquad \sum_{j=1}^{n} \lambda_j^{BR} x_{ij} \ge x_{io}; \qquad i = 1,2,\dots,m, \\
&\qquad g_y, \beta^{BR} \ge 0 \\
&\qquad g_y = y_{ro} - \min_j \{y_{rj}\} \qquad r = 1,2,\dots,s.
\end{aligned} \tag{3}$$

Worst relative efficiency can be calculated by the model (3) in the direction of anti-ideal DMU and the bankruptcy value of $1-\beta$ is a distance measure between the observed point and possible worsening point with reference to the anti-ideal point i.e. maximum possible worsening point.

3 Bankruptcy Problem and the Allocation

Now, we interpret problem of bankruptcy as follows. So $E \in R$ is a number of estates that may be returned. $d \in R^n$ As $d_i \ge 0$ for all $0 \le i \le n$ is established and represents the amount that demands i the creditor. $N = \{1,\dots,n\}$ is a number of creditors and. It can be said that the problem of bankruptcy is a pair(E,d). An answer the problem of bankruptcy is(E,d)or shorter an allocation n-tuple.

So it is $E = \sum_{j\in N} x_j$ where x_j represents the allocated amount to the jth creditor, are calculated by the method of Shapely value. In fact, when we want to choose an estate that leads to the most efficient then use allocation in all directions is calculated and it is one of the most versatile topics optimization. The using of optimization models allows us all the different aspects of the allocation according to objective function is evaluated. Here it can be said that the allocation is a function which assigns a unique allocation to each bankruptcy problem. By an n-person cooperative game in characteristic function form, we mean a pair (N,c).

Where $N = \{1,\dots,n\}$ is a finite set of players and $c: 2^n \to R$ (that 2^n denotes the set of subsets N) and assumed $c(\varnothing) = 0$?

We usually refer to subsets S of N as coalitions and $c(S)$ to the number as the worth S [10].

The allocation may be interpreted of estates to each player, as maximum profit or cost minimum. Now a fixed set of players by a game (N,c) that c is a characteristic function [11]. Thus they define the bankruptcy game (E,D) corresponding to the bankruptcy problem by:

$$c_{E,d}(S) = \max\left\{E - \sum_{j\in N\setminus S} d_j, 0\right\}$$

In this case, bankruptcy value would be zero. If an estate's value is less than or equal to the demands of his creditors then obtained value in $E - \sum_{j\in N\setminus S} d_j$ is negative and all estates are paid to creditors as claims which this indicates that the player is bankrupt and nonzero value would indicate bankruptcy.

The above implies that obtained value in $E - \sum_{j \in N \setminus S} d_j$ is positive and the amount of net estate remaining after payment of the claims, it is intended for players which can then be used as a tool to generate even more destinations that this can be interpreted as indicating the desired player non-bankruptcy.
With this description and combining all the above concepts, we propose the following model.

4 The Proposed Model

Bankruptcy assessment using data envelopment analysis game

Using the concepts in the previous section, express a new model (4) for bankruptcy and combining of this new model by using of rough set theory as the other new method are discussed. Finally, this model by using of fuzzy rough and interval rough in the application example are studied [12].

4.1 A New Method in Bankruptcy Assessment Using Game Theory

Suppose that E_j is the initial value of the total estates of j[th] organization and $N = \{1, \dots, n\}$ represents the total number of organizations, x_{ij} and y_{rj} is j[th] organization input and output, x_{io} and y_{ro} show the input and output under review organization and d_i as the amount that the ith creditor demands and g_y is as production direction.

$$
\begin{aligned}
\max\ & E_j - \sum_{j \in N} d_j \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \le y_{ro} - (E_j - \sum_{j \in N} d_j) g_y, \qquad r = 1, \dots, s, \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \ge x_{io}, \qquad\qquad\qquad i = 1, \dots, m, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& E_j - \sum_{j \in N} d_j \ge 0, \\
& g_y = y_{ro} - \min\{y_{rj}\}, \\
& i = 1, \dots, m;\ r = 1, \dots, s.
\end{aligned} \tag{4}
$$

A change in the model (2) can be shown a more practical form of bankruptcy problem with using the game theory that offers a different interpretation and bankruptcy economic aspects will be considered in further. Description of the model expressed as follow.

We noted that $N = \{1, \dots, n\}$ represents the total number of players. The above model would be a controlled model. In this model consider firms as a set of players. Of course, in here every player that it is a studied organization has an initial estate value. Moreover, to investigate the factors affecting the failure of an organization, the organization demands are remarkable. As an organization with relatively acceptable estate may be lead to the bankrupt. In this regard, we offer a model that we determine how best to help bankrupt using the total estate and demands.

If we want to describe the model, we explain the concepts as follows:

The model g_y indicates the production direction, d_j the demands for j^{th} players E_j is the initial value of the total estate for the players (organizations) and-and y_{rj} is input and output for the j^{th} organization and, y_{ro} is input and output of the player under review.

In the model, the value of solving this model, as the net estate, in other words, be interpreted the estate remaining after payment of claims to the jth player. As positive value obtained from the model are represented as non-bankruptcy. Interpretation of this model is that the players after payment of liabilities and remaining equity are interpreted as a net estate if this value is equal to some non-zero then net estate value remaining for the players so keep partly to their distance to the bankruptcy. And through the remainder of its estate that it is net estate the amount produce of the other uses or purposes. We consider zero value as bankruptcy. That means that the amount of total estate equal to demands value and have been paid all estate as demands to the jth player then don't have a distance to bankruptcy so the players or organizations are declared as bankrupt.

However, we offer the general algorithm for this model, the following. It is noteworthy that the model to receive inputs and outputs of the organizations or the named players and evaluation of the input and output of the studied player with the production direction g_y .

4.2 Algorithm

Initial steps

Consider the set of `players $N = \{1,...,n\}$. Receive inputs and outputs all players associated with the production direction. If we deal with negative data (input or output), production direction (g_y) should be negative, as we deal with the positive data.

Main steps

1- Solve models (1). Put efficient units in Eff the set. Organize $N - Eff$ For an inefficient player and determine bankruptcy with the model (4).

2- If g_y is not available, put it aside and, go to the next step.

3- If the solution was positive will be non-bankrupt. If the value is zero then the player is bankrupt.

4- Finish.

It is noteworthy, this algorithm solves model (1) all of the efficient players and model (4) solve for all inefficient players. Total evaluation is done after solving the model for all of the players. We show which these algorithms study bankruptcy problem in four stages easily, which it can help us in the more understanding of the bankruptcy problem. In this model, $\max \ E_j - \sum_{j \in N} d_j$ as least right that can have a creditor (In fact, this value is the value that should be returned). In other words, $\max \ E_j - \sum_{j \in N} d_j$ be equal to the allocated amount in accordance with the bankruptcy problem $c_{E,d}$ in this game, mean to the least right, which can be considered for a creditor.

5 Numerical Example

Let's consider eight medicinal firms as objects and indexes of debt, sale and performance assessment, and their data are given in Table 1:

Table 1 Indicators and firms values (features or objects)

index / firms	estate (X1)	debt (X2)	sale (X3)	Performance assessment (X4)
DMU 1	123	46	11	30=weak
DMU 2	234	78	3	10=Very weak
DMU 3	176	68	164	70=good
DMU 4	28	17	165	90=Very good
DMU 5	530	71	171	50=middle
DMU 6	69	69	76	30=weak
DMU 7	109	85	37	70=good
DMU 8	135	72	60	70=good

Now, we form financial ratios, estate to debt, estate to sales, debt-to-sales, sales and performance to sales and we calculate efficiency and bankruptcy of firms. The performance evaluations are qualitative and imprecise as an interval scale [0,100] and also we consider ratios a_2, a_3, a_4 as input values and ratios a_1, a_5 as output values in DEA models.

$$a_1 = \frac{x_1}{x_2} \qquad a_2 = \frac{x_1}{x_3} \qquad a_3 = \frac{x_2}{x_3} \qquad a_4 = \frac{x_4}{x_3} \qquad a_5 = \frac{x_4}{x_1}$$

Table 2 Firms and financial ratios

Ratios / firms	a_1	a_2	a_3	a_4	a_5	efficiency	bankruptcy
DMU 1	2/76	11/18	4/18	2/72	0/24	0.89	0.18
DMU 2	3	78	26	3/33	0/04	0.35	0
DMU 3	2/59	1/07	0/41	0/43	0/39	0.77	0.40
DMU 4	1/65	0/17	0/1	0/55	3/21	1	-
DMU 5	7/46	3/09	0/42	0/29	0/09	1	-
DMU 6	1	0/09	0/90	2/95	2/29	0/34	0
DMU 7	1/28	2/95	2/29	1/89	0/64	0/11	0
DMU 8	1/88	2/25	1/2	1/17	0/52	0/24	0.40

5.1 Analysis of Results

Above example show, DMU 1 (first firm), with relatively significant estate and having almost 1/3 debt to the estate, is an amount of the equal to 0/18. This indicates that the net estate value next claims pay is not significant and now it is on the frontier of the bankruptcy. It is noteworthy that the sale in these issues is important especially given the circumstances of this table with qualification because they have relatively significant estate but having low sales and relatively low debt goes to the bankruptcy and the remaining net estate cannot be used in other productions.

However, it is well-recognized efficiency due to the relatively high investment. Similarly, DMU 2 with high estate and as well as relatively low debt has poor efficiency. Efficiency this firm is poor because its sale is very low toward estate and so goes to the bankrupt group. This means that the remaining estate can be used to produce a vector. And so check next firms. DMU 4 can be seen that with the low estate is efficient and this means the optimal use of estate even with the usual amount of debt and so on, goes to the non-bankrupt group. And bankruptcy does not need to be calculated and next DMUs are discussed the same trends and bankrupt and non-bankrupt firms easily be diagnosed using a combination of these two methods.

6 Conclusions

The main purpose of this paper is studying bankruptcy as one of the most important economic problems. So, we use modified directional distance function for estimation and assessment of the bankruptcy. Also, we introduced a new model for bankruptcy assessment using DEA games theory and allocation. The objective function of the new model is demand and total estate, and interpretation of this model is better than the models of the modified directional distance function in DEA.

The numerical results have shown using financial ratios (corresponding to every problem bankruptcy) and imprecise data can be “good” bankruptcy assessment in performance evaluation in the new model. The main advantage of this model is an objective function which has a simpler form and economic for bankruptcy problem. The objective function of the new model represents a more practical discussion of bankruptcy which interpretation it is easier. This model describes the remaining estate after payment of demands as a net estate that could have a better interpretation of the bankruptcy. This model distinguishes bankrupt or non-bankrupt simply. It is noteworthy; this model is a new model in the DEA and game theory (DEA game).

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