International Journal of Applied Operational Research
Vol. 9, No. 4, pp. 1-19, Autumn 2019

Journal homepage: ijorlu.liau.ac.ir

Mathematical modeling for an integrated inventory system with two-level trade credit and random defectiveness in transport

A. Thangam*

Received: 27 May 2019 ; **Accepted:** 2 September 2019

Abstract Modern business environment focuses on improving the operational efficiency of supplier, retailer and customers through integrating their inventory. Although a smoothly running integrated inventory system is ideal, the reality is to deal with imperfectness in transportation. In actual production environments, inventory items are not perfect and defectiveness occurs in a random process. In this paper, we propose an integrated supplier–retailer inventory model in which both supplier and retailer have adopted trade credit policies, and the retailer receives an arriving lot which contains random defectiveness in quality of items. This paper proposes a mathematical model considering two situations such as (a) risk neutral and (b) risk-averse case and the solution procedures are described with computational algorithm. The optimization procedures are discussed for determining optimal cycle time and the optimal number of shipments by minimizing the expected joint total cost in the integrated inventory system. Numerical examples are provided to illustrate the theoretical results, and sensitivity analysis is made for major inventory parameters.

Keyword: Inventory; Defective Items; Trade Credit; Delay in Payments.

1 Introduction

The integrated inventory control model in the supply chain has received a great deal of attention because of the modern business situations which are focusing on the integration of inventory between supplier and retailer. In supply chain management, the long term strategic partnerships are established between supplier and retailer. It is advantageous for the two parties regarding their costs and so their profits, since they cooperate and share information with each other to achieve improved benefits. Several researchers have shown that the integrated inventory system between supplier and retailer can increase their mutual benefits through strategic cooperation with each other.

In most of the early literature dealing with integrated inventory problems, the random effect in mishandling of items due to transport, were not considered in mathematical modeling; but in practice, the extraordinary circumstances (such as earthquake, mishandling in transport, shipping damage, and misplacing products) that may result a risk in delivery from a supplier to a retailer. The supply disruptions take the form of high-impact and low-probability contingencies which can threaten decision makers of a supply chain. Mathematical modeling of inventory problems under the extra-ordinary situations, helps decision makers to

* **Corresponding Author.** (✉)
E-mail: thangamgri@yahoo.com (A. Thangam)

A. Thangam
Department of Mathematics, Pondicherry University – Community College, Lawspet , Pondicherry - 605 008, India

evaluate optimal ordering policies against an incredibly complex and dynamic set of risks and constraints.

Mathematical modeling of an integrated inventory system in a supply chain associated with trade credit is a popular topic in manufacturing technology. Modern business competition in today's markets emphasizes us to build a high level coordination between supplier and retailer in order to satisfy the customer's demand. The concept of joint optimal decision making is more profitable than the independent inventory control of any supply chain system.

In the classical logistics models, it was assumed that the retailers and their customers must pay for the items as soon as the items are received. However, in practice, the supplier/ retailer would allow a specified credit period (say 30 days) to their retailers/customers for payment without penalty to stimulate the demand of the consumable products. This credit term in financial management is denoted as "net 30." The benefits of a trade credit policy are: (1) it attracts new customers who consider the trade credit policy to be a type of price reduction, and (2) it should cause a reduction in sales outstanding, since some established customer will pay more promptly in order to take advantage of trade credit more frequently.

This paper investigates an integrated inventory model in which the supplier is willing to provide the retailer a full trade credit period for payments and the retailer offers the full trade credit to his/her customer. This is called two-echelon (or two-level) trade credit financing. In practice, this two-level trade credit financing at a retailer is more matched to real-life situations in a supply chain. Companies, like TATA and Toyato, can delay the full amount of purchasing cost until the end of the delay period offered by his suppliers. These companies offer delay payment to his dealership on the permissible credit period.

Although a smooth running supply chain is ideal, the reality is to deal with imperfectness in transportations. To manage the risk in delivery, the retailer arranges some alternatives to rework those defective items which involve defective costs. The retailer replenishes his inventory non-instantaneously and faces probabilistic risks due to supply disruptions. According to risk management in operations research, two situations such as (a) risk neutral and (b) risk-averse are considered. The solution procedures are described for the retailer in both cases. This paper tries to propose an optimal solution procedure for the supply chain under the effect of unexpected disruptions in transport from supplier to retailer. Supplier offers the retailer a trade credit period t_1 and the retailer in turn offers his customers a permissible delay period t_2, and he receives the revenue from t_2 to $T + t_2$, where T is the cycle time at the retailer.

The rest of this paper is organized as follows. The literature review is presented in Section 1.1. Notations and assumptions are described in Section 2. Mathematical model is obtained in Section 3. In Section 4, optimal solutions are derived with computational algorithm. Numerical results are presented in Section 5 and conclusions are drawn in Section 6.

1.1 Literature Review

The integrated inventory management model in the supply chain has received a great deal of attention since more than three decades ago. Since Goyal [1] introduced the integrated inventory model consisting of a vendor and a buyer. Many researchers have developed the models under various cases, such as [2-9]. Further, Goyal [10] developed a model of vendor-buyer with unequal-sized shipment. Some researchers, including [11-15] proposed vendor-buyer model under unequal-sized shipment and proved that the proposed policy gives an impressive cost reduction in comparison to equal-sized policy. The above mentioned papers assumed that the product produced by the vendor is always in perfect quality. However, in real situations, the production process may produce a certain number of defective items. Porteus [16] was among the first researchers who introduced an EPQ model considering defective items and showed a significant relationship between quality and lot size. Some researchers are interested in developing an inventory model considering the imperfect quality. Salameh and Jaber [17] developed an EOQ model assuming that the lot contains a random proportion of defective items. The model assumed that there is no error caused by human in the inspection process. Then, Raouf et al. [18] studied human errors in the inspection. Yoo et al. [19] proposed a model that considered both imperfect production and two-way imperfect inspection. The model considered the situation in which the inspector may incorrectly classify a non-defective item as defective (Type I inspection error), or incorrectly classify a defective item as non-defective (Type II inspection error). Lin [20] developed a model for a simple supply chain system based on [19] and assumed that both Type I and Type II inspection errors are known constants. Hsu and Hsu [21] then developed an integrated vendor-buyer inventory model for items with imperfect quality and inspection errors. This model assumes that the defective items are sold to a secondary market at a discounted price. Furthermore, Darwish et al. [22] examined the effect of imperfect quality in the vendor-buyer system under vendor managed inventory model. Other relative inventory control financing issue studies were Pamudji et al. [23], Hill and Riener [24], Abad and Jaggi [25], Chen and Kang [26], Huang and Hsu [27], Ho et al. [28], Thangam and Uthayakumar [29], Su [30], Kim et al.,[31], Mosca et al., [32], Hu et al., [33], Lin et al., [34], Li et al., [35], Chan et al. [36,43], Das et al. [37,40], AlDurgam, et al. [38], Ouyang et al. [39], Bhunia et al. [41], Li and Wang [42], Jha and Shankar [44], Chung et al. [45], Firouz et al. [46], and Rad et al. [47].

2 Notations and Assumptions

The following notations and assumptions are used throughout this paper.

P Supplier's production rate,

q delivery quantity from the supplier to the retailer,

A_s set up cost at the supplier,

C_T fixed shipment cost per delivery,

α value added shipment cost,

β transportation cost per unit item,

c_p supplier's production cost per unit item,

h_s supplier's unit stock holding cost per unit time,

n number of shipments from the supplier to the retailer per production run, positive integer (decision variable),

I_s supplier capital opportunity cost, per unit time,

λ demand rate at the retailer,

A_r retailer's ordering cost per order,

h_r retailer's unit stock holding cost per unit time, excluding interest charges,

c_r retailer's unit purchasing price

I_k interest charged per dollar in stock per year at the retailer

I_e interest earned per dollar per year at the retailer

s the retailer unit selling price for the items of perfect quality,

Q retailer's order quantity,

T length of cycle time at the retailer, (decision variable),
t_1 retailer's trade credit period offered by the supplier,
t_2 customer's trade credit period offered by the retailer,
t_c time when contingency occurs,
x percentage of imperfect quality items,
π defective cost, the unit cost per item due to disruption in transport,
R delivery rate the retailer, at time T_R.

Assumptions

1. The present model considers single supplier and single retailer.
2. The inventory system deals with only one type of item, supplied to multiple customers.
3. Shortages are not allowed.
4. Demand rate at the retailer is known and Production rate at the supplier is also known.
5. Lead time is zero at the supplier and retailer.
6. Inventory horizon period is infinite.
7. The supplier offers full trade credit period t_1 to the retailer and he in turn offers trade credit period t_2 to his customers.
8. Each batch is dispatched from supplier to the retailer in 'n' equal sized shipments, where 'n' is positive integer (decision variable)
9. As soon as the lot comes to the warehouse of retailer, 100% inspection is done. The inspection time is negligible.
10. The retailer orders $Q = nq$, of good quality items.
11. An arrival of 'q' units, contains imperfect items with rate 'x' percentage.
12. Transport cost is $C_T = \alpha + \beta q$.
13. The retailer earns interest at the rate I_e on the deposit over his credit period. At the end of his credit period, he settles the payment and he starts paying interest for the item in stock at the rate I_k.
14. In every replenishment cycle, the supplier incurs a opportunity cost finance rate I_s for offering trade credit.
15. If the products are defective due to contingency in delivery, the retailer need to find supply sources to recover these items, it accounts for the cost π.
16. Elapsed time (t_c) until contingency occurs, is probabilistic continuous random variable. According to birth-death process in queueing theory, t_c follows an exponential distribution with mean $1/\mu$.

3 Mathematical model formulation

An integrated inventory system is considered with a single supplier and single retailer who delivers good quality of items to many customers. In a production cycle, the supplier produces a batch quantity of $Q/_n = q$ units. The occurrence of defectiveness (x) due to mishandling in transport follows a random process. To manage the risk in delivery, the retailer arranges some alternatives to rework those defective items which involve defective cost π. The retailer replenishes his inventory instantaneously and faces probabilistic risks due to supply disruptions. According to risk management in operations research, two situations such as (a) risk neutral and (b) risk aversion are considered.

To encourage sales revenue and market share, the supplier offers trade credit period t_1 to the retailer and the retailer offers credit period t_2 to his customers. To formulate the integrated inventory model, the supplier's total cost per unit time is discussed first, and then the retailer's total cost per unit time is discussed.

3.1 Supplier's total cost per unit time

The supplier's total cost per production cycle consists of the following parts:

(a) **Set up cost:** Supplier set up cost is A_s per production cycle.

(b) **Production cost:** The supplier delivers 'n' batches shipment of quantity 'q' units to the retailer and therefore the production cost is

$$c_p \times Q = c_p \times (nq) = c_p n \times (\lambda T)$$

(c) **Holding cost:** When the supplier produces the first of 'q' units, he delivers them to the retailer . After that the supplier will make the delivery of q units on every cycle time T= q/λ until the inventory level falls to zero. These situations are illustrated in figure 1,

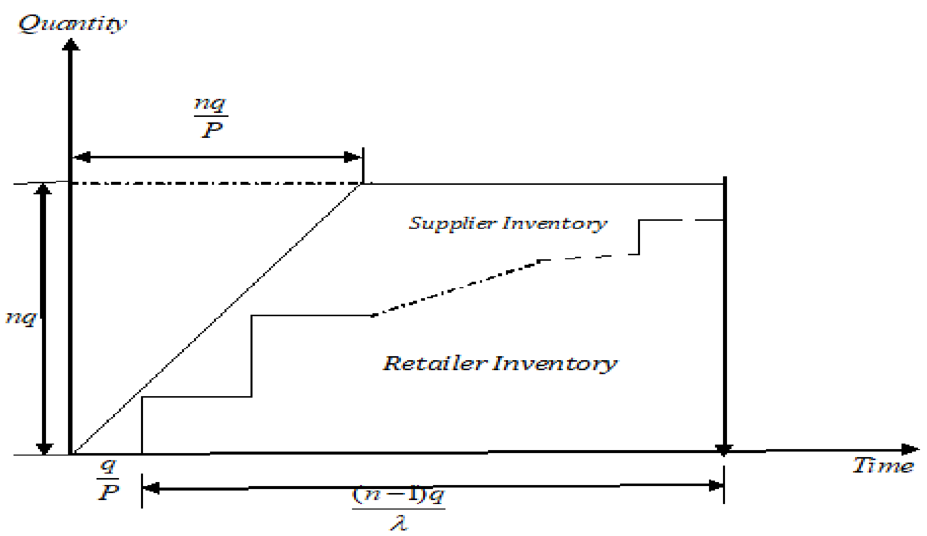

Fig. 1 Supplier's inventory system

With unit stock holding cost h_s per unit time, the supplier's stock holding cost can be calculated as follows:

$$= h_s \times \left\{ nq\left[\frac{q}{P} + \frac{(n-1)q}{\lambda}\right] - \frac{1}{2} nq \times \frac{nq}{P} - [1+2+3+\ldots+(n-1)]q \times \frac{q}{\lambda} \right\}$$

$$= h_s \times \frac{[n(n-1)P - n\lambda(n-2)]q^2}{2\lambda P} \tag{1}$$

$$= \frac{h_s n\lambda}{2}\left[(n-1) - \frac{\lambda}{P}(n-2)\right]T^2$$

(d) **Opportunity cost due to trade credit offered to retailer**

Supplier will not receive payment until time t_1, so he incurs the interest payable cost of

$$I_s \times \left[c_r n(qt_1)\right] = I_s c_r n t_1 \lambda T \quad (2)$$

Therefore, the supplier's total cost per unit time is

$$TC_s(n) = \frac{1}{nT}\left\{A_s + c_p n \times (\lambda T) + \frac{h_s n\lambda}{2}\left[(n-1) - \frac{\lambda}{P}(n-2)\right]T^2 + I_s c_r n t_1 \lambda T\right\} \quad (3)$$

3.2 Retailer's total cost

(a) Retailer's ordering cost: the retailer orders Q quantity with an ordering cost A_r and he returns in 'n' batches of shipment. So the ordering cost is A_r/n.

(b) Cost of transport: $C_T = \alpha + \beta q = \alpha + \beta(\lambda T)$.

(c) Excluding interest charges, stock holding cost is $\frac{h_r}{2} \times \left[\lambda(1 - \frac{\lambda}{R})\right] T^2$

(d) Defective cost due to disruption in supply (transport),

If the number of defective items in each replenishment cycle is

$$\gamma = \begin{cases} 0 & \text{if } t_c \geq T_R \\ xR\left(\frac{\lambda T}{R} - t_c\right) & \text{if } t_c < T_R \end{cases}$$

where T_R is delivery time and R. T_R = λT

then the expected number of defective products in each cycle,

$$E[\gamma] = \int_0^\infty \gamma \;\mu e^{-\mu t_c} \;\mathrm{d}t_c$$

$$= Rx\left[\frac{\lambda T}{R} + \frac{1}{\mu}\left(e^{-\mu(\lambda T/R)} - 1\right)\right]$$

The annual defective cost is

$$\frac{\pi \lambda E[\gamma]}{R\left(\frac{\lambda T}{R}\right)} = \pi \lambda x \left(1 + \frac{e^{-\mu(\lambda T/R)} - 1}{\mu(\lambda T/R)}\right)$$

Using the approximation $e^{-x} \approx 1 - x + \frac{x^2}{2}$, the annual defective cost can be rewritten as $\frac{\pi \lambda x \mu}{2}\left(\frac{\lambda T}{R}\right)$ when μ is small.

(e) Cost of interest charges for unsold items and the interest earned are calculated as follows:

Case 1: $T \leq t_1 \leq T + t_2$

Annual Interest earned = $\frac{sI_e}{T}\left[\frac{\lambda(t_1-t_2)^2}{2}\right]$

$= \frac{sI_e\lambda}{2T}(t_1-t_2)^2$

Annual Interest payable = $\frac{c_r I_k}{T}\left[\frac{\lambda(T+t_2-t_1)^2}{2}\right]$

Case 2: $T \le T+t_1 \le t_2$

Annual Interest earned = $\frac{sI_e}{2T}\left[\frac{\lambda T^2}{2}+\lambda T(t_1-t_2-T)\right]$

There is no interest payable for the retailer.

Case 3: $t_1 \le t_2$

There is no interest earned for the retailer since retailer's credit period is prior to customer's credit period.

Interest Payable = $\frac{c_r I_k}{T}\left[(t_1-t_2)\lambda T+\frac{\lambda T^2}{2}\right]$

The annual total cost incurred at the retailer, is

TC(T) = Annual ordering cost + Annual Transport Cost+ Annual stock holding cost + Annual interest payable – Annual interest earned + Annual defective cost.

Therefore, the total cost incurred at the retailer $TC_r(T)$ is

$$TC_r(T)=\begin{cases} TC_{r1}(T) & \text{if} \quad T \le t_1 \le T+t_2 \\ TC_{r2}(T) & \text{if} \quad T \le T+t_1 \le t_2 \\ TC_{r3}(T) & \text{if} \quad t_1 \le t_2 \end{cases}$$

where

$$TC_{r1}(T)=\frac{A_r}{nT}+\frac{\alpha+\beta\lambda T}{T}+\frac{h_r}{2}\left[\lambda\left(1-\frac{\lambda}{R}\right)\right]T+\frac{\pi\lambda x\mu}{2}\left(\frac{\lambda T}{R}\right)+\frac{c_r I_k\lambda}{2T}\left[T^2+(t_1-t_2)^2+2T(t_1-t_2)\right]$$
$$-\frac{sI_e\lambda}{2T}\left[(t_1-t_2)^2\right] \tag{4}$$

$$TC_{r2}(T)=\frac{A_r}{nT}+\frac{\alpha+\beta\lambda T}{T}+\frac{h_r}{2}\left[\lambda\left(1-\frac{\lambda}{R}\right)\right]T+\frac{\pi\lambda x\mu}{2}\left(\frac{\lambda T}{R}\right)-\frac{sI_e\lambda}{2T}\left[\frac{\lambda T^2}{2}+\lambda T(t_1-t_2-T)\right]. \tag{5}$$

$$TC_{r3}(T)=\frac{A_r}{nT}+\frac{\alpha+\beta\lambda T}{T}+\frac{h_r}{2}\left[\lambda\left(1-\frac{\lambda}{R}\right)\right]T+\frac{\pi\lambda x\mu}{2}\left(\frac{\lambda T}{R}\right)+\frac{c_r I_k}{T}\left[\lambda(t_1-t_2)T+\frac{\lambda T^2}{2}\right] \tag{6}$$

3.3 Total cost in the integrated inventory system

The supplier and the retailer will collaborate and share information through strategic alliance to achieve improved benefits and minimize their total cost. Under this situation, the joint total cost per unit time for the supplier and the retailer is

$$JTC(n,T)=\begin{cases} JTC_1(n,T)=TC_s(n)+TC_{r1}(T) & \text{if} \quad T\le t_1\le T+t_2 \\ JTC_2(n,T)=TC_s(n)+TC_{r2}(T) & \text{if} \quad T\le T+t_1\le t_2 \\ JTC_3(n,T)=TC_s(n)+TC_{r3}(T) & \text{if} \quad t_1\le t_2 \end{cases}$$

After simplification from Eq. (3) and Eq.(4) , we get

$$JTC_1(n,T)=\frac{1}{T}\left[\frac{A_s+A_r+n\alpha}{n}+\frac{(c_rI_k-sI_e)\lambda}{2}(t_1-t_2)^2\right]+T\left[h_s\lambda\left[(n-1)-\frac{\lambda}{P}(n-2)\right]+h_r\left[\lambda(1-\frac{\lambda}{R})\right]+c_rI_k\lambda+\pi\lambda x\mu\left(\frac{\lambda}{R}\right)\right]\Big/2$$
$$+\left[c_rI_k\lambda(t_1-t_2)+\lambda c_p+\beta\lambda+I_sc_rt_1\lambda\right] \tag{7}$$

After simplification from Eq. (3) and Eq.(5) , we get

$$JTC_2(n,T)=\frac{1}{T}\left[\frac{A_s+A_r+n\alpha}{n}\right]+T\left[h_s\lambda\left[(n-1)-\frac{\lambda}{P}(n-2)\right]+h_r\left[\lambda(1-\frac{\lambda}{R})\right]+\pi\lambda x\mu\left(\frac{\lambda}{R}\right)+sI_e\lambda\right]\Big/2$$
$$+\left[\lambda c_p+\beta\lambda+I_sc_rt_1\lambda-sI_e\lambda(t_1-t_2)/2\right] \tag{8}$$

$$JTC_3(n,T)=\frac{1}{T}\left[\frac{A_s+A_r+n\alpha}{n}\right]+T\left[h_s\lambda\left[(n-1)-\frac{\lambda}{P}(n-2)\right]+h_r\times\left[\lambda(1-\frac{\lambda}{R})\right]+c_rI_k\lambda+\pi\lambda x\mu\left(\frac{\lambda}{R}\right)\right]\Big/2$$
$$+\left[\lambda c_p+\beta\lambda+I_sc_rt_1\lambda+c_rI_k\lambda(t_1-t_2)\right] \tag{9}$$

4. Optimal solutions

Here, two situations, namely (a) risk – neutral and (b) risk averse are considered.

4.1. Risk Neutral situation

In this section, a solution procedure is given to find an optimal replenishment policy without limiting the expected number of defective in transport.

To find optimal solutions, say (n^*, T^*) that minimizes the above integrated total cost, the following procedures are taken. First, for fixed T, we check the effect of 'n' on the joint total cost per unit time $JTC(n,T)$. With the fact that,

$$\frac{\partial^2 JTC_i(n,T)}{\partial n^2}=\frac{2(A_s+A_r)}{n^3T}>0 \quad , \quad \text{for i =1,2,3.} \tag{10}$$

$JTC(n,T)$ is a convex function of 'n'. Therefore, the search for the optimal shipment number 'n' is reduced to find a local optimal solution.

Next, in order to obtain the solutions for minimum joint total cost function $JTC_i(n,T)$, i=1,2,3 for 'fixed n'. The following conditions are necessary:

$$\frac{\partial JTC_1(n,T)}{\partial T}=\frac{-1}{T^2}\left[\frac{A_s+A_r+n\alpha}{n}+\frac{(c_rI_k-sI_e)\lambda}{2}(t_1-t_2)^2\right]+\left[h_s\lambda\left[(n-1)-\frac{\lambda}{P}(n-2)\right]+h_r\times\left[\lambda(1-\frac{\lambda}{R})\right]+c_rI_k\lambda+\pi\lambda x\mu\left(\frac{\lambda}{R}\right)\right]\Big/2=0 \quad (11)$$

$$\frac{\partial JTC_2(n,T)}{\partial T}=\frac{-1}{T^2}\left[\frac{A_s+A_r+n\alpha}{n}\right]+\left[h_s\lambda\left[(n-1)-\frac{\lambda}{P}(n-2)\right]+h_r\times\left[\lambda(1-\frac{\lambda}{R})\right]+\pi\lambda x\mu\left(\frac{\lambda}{R}\right)+sI_e\lambda\right]\Big/2=0 \quad (12)$$

and

$$\frac{\partial JTC_3(n,T)}{\partial T}=\frac{-1}{T^2}\left[\frac{A_s+A_r+n\alpha}{n}\right]+\left[h_s\lambda\left[(n-1)-\frac{\lambda}{P}(n-2)\right]+h_r\times\left[\lambda(1-\frac{\lambda}{R})\right]+c_rI_k\lambda+\pi\lambda x\mu\left(\frac{\lambda}{R}\right)\right]\Big/2=0 \quad (13)$$

Let

$$Z_1=\left[\frac{A_s+A_r+n\alpha}{n}\right]$$

$$Z_2=h_s\lambda\left[(n-1)-\frac{\lambda}{P}(n-2)\right]+h_r\left[\lambda(1-\frac{\lambda}{R})\right]+\pi\lambda x\mu\left(\frac{\lambda}{R}\right).$$

$JTC_1(n,T)$ is convex function of T, for fixed 'n' if $\left[2Z_1+(c_rI_k-sI_e)\lambda(t_1-t_2)^2\right]>0$.

Clearly, $JTC_2(n,T)$ and $JTC_2(n,T)$ are convex functions of T for fixed value 'n' since $\frac{\partial^2 JTC_i(n,T)}{\partial T^2}>0$ for i=2,3.

By solving the Eq. (11), the unique solution of T can be found and it is as follows:

$$T_1^*(n)=\sqrt{\left[\frac{2Z_1+(c_rI_k-sI_e)\lambda(t_1-t_2)^2}{Z_2+c_rI_k\lambda}\right]} \text{ for n=1,2,3……} \quad (14)$$

By solving the Eq. (12), the unique solution of T can be found and it is as follows:

$$T_2^*(n)=\sqrt{\left[\frac{2Z_1}{Z_2+sI_e\lambda}\right]} \quad \text{for} \quad n=1,2,3……. \quad (15)$$

By solving the Eq.(13), the unique solution of T can be found and it is as follows:

$$T_3^*(n)=\sqrt{\left[\frac{2Z_1}{Z_2+c_rI_k\lambda}\right]} \quad \text{for} \quad n=1,2,3,……. \quad (16)$$

To ensure the condition that $T_1^*(n)\le t_1\le T_1^*(n)+t_2$, we substitute Eq. (14) into this inequality and we get:

$$\Delta_1=2Z_1-Z_2t_1^{\,2}+c_rI_k\lambda t_2(t_2-2t_1)-sI_e\lambda(t_1-t_2)^2\le 0$$

and

$$\Delta_2=2Z_1-\left(Z_2+sI_e\lambda\right)(t_1-t_2)^2\ge 0.$$

To ensure the condition that $T_2^*(n)+t_1\le t_2$, we substitute Eq. (15) into this inequality and we get:

$$\Delta_2=2Z_1-\left(Z_2+sI_e\lambda\right)(t_1-t_2)^2\le 0$$

From the above discussions the following theorem is obtained.

Theorem 1. For a fixed value of n,

(1) when $t_1 \leq t_2$, one has the following
(a) If $\Delta_1 \leq 0$ and $\Delta_2 \geq 0$ then there exists an optimal replenishment cycle time $T_1^*(n)$ as in Eq.(14).
(b) If $\Delta_2 \leq 0$ then exists an optimal replenishment cycle time $T_2^*(n)$ as in Eq.(15).
(2) when $t_1 > t_2$, there exists an optimal solution for cycle time $T_3^*(n)$ as in Eq.(16)

Summarizing the above results, we can establish the following algorithm:

Algorithm to find optimal solution (n^*, T^*) in Risk neutral situation:

Step 1: Set n =1
Step 2: If **$t_1 \leq t_2$ then**
(a) Determine $T_1^*(n)$ by Eq.(14). If $\Delta_1 \leq 0$ and $\Delta_2 \geq 0$ then substituting T_1^* in to Eq.(7) to get $JTC_1(n, T_1^*)$; otherwise let $JTC_1(n, T_1) = \infty$.
(b) Determine $T_2^*(n)$ by Eq.(15). If $\Delta_2 \leq 0$ then substituting T_2^* in to Eq.(8) to get $JTC_2(n, T_2^*)$; otherwise let $JTC_2(n, T_2) = \infty$.
Step 3 : If $t_1 > t_2$, then
Determine $T_3^*(n)$ by Eq.(16). Substituting T_3^* in to Eq.(9) to get $JTC_3(n, T_3^*)$; otherwise let $JTC_3(n, T_3) = \infty$.
Step 4 : Find min $\{JTC_1(n,T_1), JTC_2(n,T_2), JTC_3(n,T_3)\}$.
Set $JTC\ (n, T^{(n)}) = \min\{JTC_1(n,T_1), JTC_2(n,T_2), JTC_3(n,T_3)\}$ then $T^{(n)}$ is the optimal solution
for the given n.
Step 5 :let n = n+1, repeat the steps from 2 to 4 for finding out $JTC\ (n, T^{(n)})$.
Step 6 : If $JTC\ (n, T^{(n)}) \leq JTC\ (n-1, T^{(n-1)})$, go to step 5; otherwise go to step 7.
Step 7 : Set $(n^*, T^*) = (n-1, T^{(n-1)})$ and hence (n^*, T^*) is the optimal solution.
Once the optimal solutions (n^*, T^*) is obtained, the optimal delivery quantity per cycle $q^* = \lambda T^*$ and order quantity $Q^* = n^* q^*$.

4.2 Risk –averse solution

In this section, a solution procedure is given to find an optimal replenishment policy by limiting the expected number of defective items (up to D_{max}) due to supply disruptions in transport. Now, the optimization problem is to minimize

$$JTC(n,T) = \begin{cases} JTC_1(n,T) & \text{if} \quad T \leq t_1 \leq T + t_2 \\ JTC_2(n,T) & \text{if} \quad T \leq T + t_1 \leq t_2 \\ JTC_3(n,T) & \text{if} \quad t_1 \leq t_2 \end{cases} \tag{17}$$

subject to

$$E[\gamma] \leq D_{\max}$$

Case 1: When $T \le t_1 \le T + t_2$

Kuhn-Tucker conditions are used to solve the constrained optimization as in Eq. (17). In this case, the following are the Kuhn-Tucker conditions:

$$\frac{\partial JTC_1}{\partial T} - \eta_1 \frac{\partial}{\partial T}\left[E[\gamma] - D_{\max}\right] - \eta_2 \frac{\partial}{\partial T}\left[(t_1 - t_2) - T\right] = 0$$
$$\frac{\partial JTC_1}{\partial n} - \eta_1 \frac{\partial}{\partial n}\left[E[\gamma] - D_{\max}\right] - \eta_2 \frac{\partial}{\partial n}\left[(t_1 - t_2) - T\right] = 0$$
$$E[\gamma] - D_{\max} \le 0,$$
$$\left[(t_1 - t_2) - T\right] \le 0,$$
$$\eta_1\left[E[\gamma] - D_{\max}\right] = 0,$$
$$\eta_2\left[(t_1 - t_2) - T\right] = 0,$$
$$\eta_1 \ge 0, \eta_2 \ge 0.$$

If (n^*, T_1^*) is the optimal replenishment cycle time, then there exists values η_1^* and η_2^* such that (n^*, T_1^*), η_1^* , and η_2^* satisfy the above conditions.

Case 2: when $T \le T + t_1 \le t_2$

In this case, the following are the Kuhn-Tucker conditions:

$$\frac{\partial JTC_2}{\partial T} - \eta_1 \frac{\partial}{\partial T}\left[E[\gamma] - D_{\max}\right] - \eta_2 \frac{\partial}{\partial T}\left[(T - t_2) + t_1\right] = 0$$
$$\frac{\partial JTC_2}{\partial n} - \eta_1 \frac{\partial}{\partial n}\left[E[\gamma] - D_{\max}\right] - \eta_2 \frac{\partial}{\partial n}\left[(T - t_2) + t_1\right] = 0$$
$$E[\gamma] - D_{\max} \le 0,$$
$$\left[(T - t_2) + t_1\right] \le 0,$$
$$\eta_1\left[E[\gamma] - D_{\max}\right] = 0,$$
$$\eta_2\left[(T - t_2) + t_1\right] = 0,$$
$$\eta_1 \ge 0, \eta_2 \ge 0.$$

If (n^*, T_2^*) is the optimal replenishment cycle time, then there exists values η_1^* and η_2^* such that (n^*, T_2^*), η_1^* , and η_2^* satisfy the above conditions.

Case 3: when $t_1 \le t_2$

In this case, the following are the Kuhn-Tucker conditions:

$$\frac{\partial JTC_3(T)}{\partial T} - \eta_1 \frac{\partial}{\partial T}\left[E[\gamma] - D_{\max}\right] - \eta_2 \frac{\partial}{\partial T}\left[t_1 - t_2\right] = 0$$
$$\frac{\partial JTC_3(T)}{\partial n} - \eta_1 \frac{\partial}{\partial n}\left[E[\gamma] - D_{\max}\right] - \eta_2 \frac{\partial}{\partial n}\left[t_1 - t_2\right] = 0$$

$$E[\gamma]-D_{\max}\le 0,$$
$$\left[t_1-t_2\right]\le 0,$$
$$\eta_1\left[E[\gamma]-D_{\max}\right]=0,$$
$$\eta_2\left[t_1-t_2\right]=0,$$
$$\eta_1\ge 0,\eta_2\ge 0.$$

If (n^*,T_3^*) is the optimal replenishment cycle time, then there exists values η_1^* and η_2^* such that (n^*,T_3^*), η_1^*, and η_2^* satisfy the above conditions.

4 Numerical examples and sensitivity analysis

To illustrate the theoretical results in the proposed model, several examples are considered below.

Example 1.
We Consider the below inventory parametric values.

A_s=$5200/order; A_r=$3600/order; α = $277/delivery ; β =$0.5/unit; c_r =$ 10/unit; I_k = $0.12/unit; s = $15/unit; I_e=$0.1/unit; t_1 = 0.1yr; t_2=0.25yr; h_s=$3/unit; h_r =$7/unit; λ = 4500 units/year; P =10000 units/year, R= 6000 units/year; π=$6\unit; x=0.4\unit; μ = 0.1; c_p=$8\unit; I_s=$0.08\unit.

The following solutions are obtained as in Table 1 using the proposed computational algorithm:

Table 1 Numerical solutions for numerical example 1

Value of n	$T^{(n)}$	$JTC(n,T^{(n)})$ in $
1	0.9489	56931
2	0.5823	53863
3	0.4282	52793
4	0.3417	52299
5	0.2858	52052
6	0.2467	51934
n^*=7**	**0.2178 ****	**51891****
8	0.1954	51895
9	0.1776	51930
10	0.1631	51987
11	0.1510	52060
12	0.1409	52145
13	0.1321	52238
14	0.1246	52237

The optimal number of shipment n^*=7, the optimal replenishment cycle time T_3^* = 0.2178, JTC^* = 51891. The supply quantity from the supplier to the retailer in a production cycle is
$q^* = \lambda T^* = 980$ units per delivery. The retailer's ordering quantity $Q^* = nq^* = 6860$ units. Expected number of defective items = E[ϒ]=14.

The convexity of JTC with respect to T and n is shown in Fig 2. The convexity of JTC with respect to n is shown in Fig. 3.

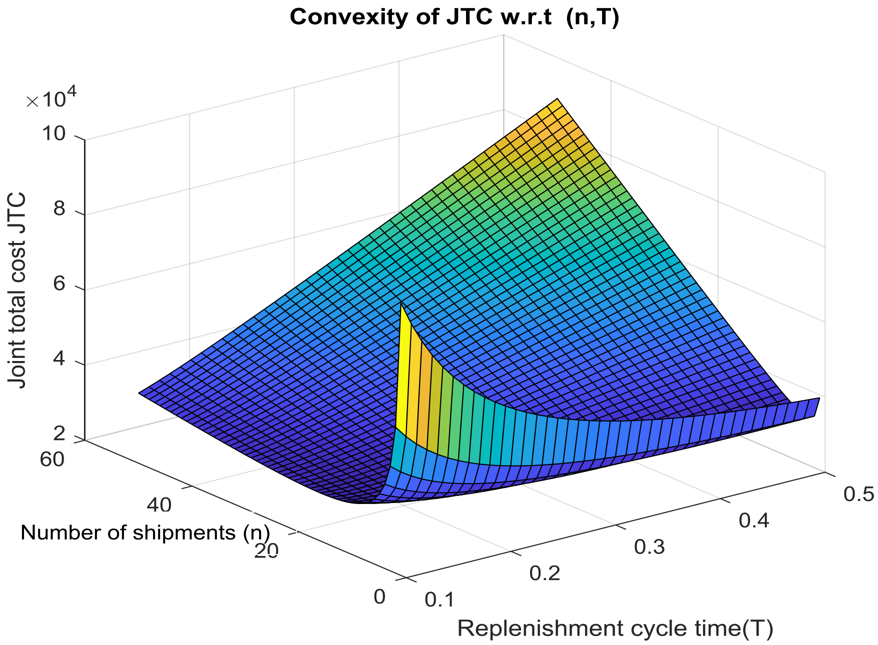

Fig. 2 The convexity of JTC (n,T)

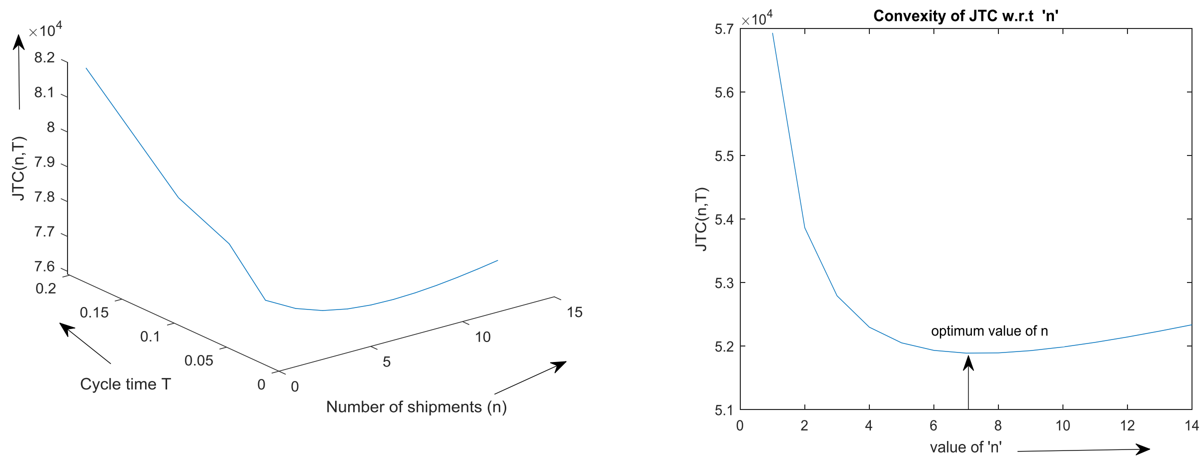

Fig. 3 The convexity of JTC (n,T) with respect to n .

For the risk averse solution, we limit the expected number of defective items at most D_{max} = 1. Using the solution procedure in section 4.2, we obtain: n = 7, $T_3^* = 0.2$, $JTC\ (T_3^*) = 52287$. The total cost of the risk-averse solution is greater than that in the risk-neutral solution. It is because that the retailer will increase the cost to reduce the product's defectiveness which is bound to be within the upper limit D_{max}. If the number of defective items D_{max} = 2, the total cost of the risk-averse solution is JTC=51993, n = 7; if the number of defective items =3, the total cost in the risk-averse case is JTC=51575, n = 7.

This means that when the number of defective products is larger than 3, the risk-averse solution gives a total cost lesser than in the risk-neutral solution. On the contrary, when the number of defective items is less than or equal to 2, the risk neutral solution is better than the risk-averse solution.

Example 2. Sensitivity analysis

We are considering the below parametric values for sensitivity analysis,

----

A_s =\$1900/order; A_r =\$1900/order; α = \$110/delivery ; β =\$1/unit; c_r =\$ 10/unit; I_k = \$0.12/unit; s = \$20/unit; I_e=\$0.1/unit; t_1 = 0.2yr; t_2=0.1yr; h_s=\$10/unit; h_r =\$7/unit; λ = 4500 units/year; P =10000 units/year, R= 6000 units/year; π=\$6\unit; x=0.4\unit; μ = 0.1; c_p=\$8\unit; I_s=\$0.08\unit.

Optimum solutions	n^*=4	T^*=0.1394	JTC^* = 54590

For various values of t_1, the following optimum solutions are obtained in Table 2,

Table 2

t_1 change	n^*	T^*	$JTC^*(n^*,T^*)$
0.2	4	0.1394	54590
0.25	4	0.1135	54560
0.3	5	0.1130	54443
0.35	5	0.1126	54359
0.4	5	0.1121	54142

It is observed from the Table 2 that: as the credit period t_1 increases, the number of shipments in transport increases, replenishment cycle time and the total system cost are decreasing. So (*) when the supplier extends his credit terms, the retailer will shorten the cycle time to take advantage of credit period more frequently.

For various values of t_2, the following optimum solutions are obtained in Table 3,

Table 3

t_2 change	n^*	T^*	$JTC^*(n^*,T^*)$
0.05	4	0.1419	53627
0.1	4	0.1394	54590
0.15	4	0.1127	54677
0.2	4	0.0917	54884
0.25	4	0.0845	54914

From the above results, sensitivity analysis of t_2 is similar to the case as in the sensitivity analysis of t_1.

Sensitivity analysis for other parameters

A sensitivity analysis is conducted to study the effect of changes on the optimal solutions and the joint total cost. To carry out the analysis, the values of one parameter are changed by -10%, -25%, +10%,+25%, while the other parameters are unchanged. The results are shown in table 4. The percentage of cost savings is derived.

Table 4

% change in the Parameter		(n^*, T^*)	JTC*	% of Saving cost
λ	-10%	(3, 0.1860)	50235	-8.66%
	-25%	(3, 0.1968)	43525	-25.42%
	+10%	(4, 0.1370)	58768	+7.10%
	+25%	(5, 0.1109)	64863	+15.83%
C_p	-10%	(3, 0.1345)	51890	-5.2%
	-25%	(4, 0.1631)	44765	-21%
	+10%	(4, 0.1352)	54662	+3%
	+25%	(4, 0.1372)	60765	+10%
A_s	-10%	(4, 0.1075)	53754	-1.5%
	-25%	(5, 0.1363)	54216	-0.68%
	+10%	(4, 0.1425)	54897	0.55%
	+25%	(4, 0.1471)	55389	1.44%
P	-10%	(4, 0.1459)	52765	-3.4%,
	-25%	(4, 0.1393)	51432	-6.14%
	+10%	(5, 0.1337)	54884	+0.53%
	+25%	(5, 0.1646)	57067	+4,.30%
h_s	-10%	(4, 0.1459)	53886	-1.30%
	-25%	(5, 0.1293)	52806	-3.37%
	+10%	(4, 0.1337)	55205	+1.11%
	+25%	(3, 0.1646)	56091	+2.67%
h_r	-10%	(4, 0.1339)	54505	-0.16%
	-25%	(3, 0.1831)	54409	-0.33%
	+10%	(4, 0.1389)	54615	+ 0.05%
	+25%	(4, 0.1382)	54697	+0.20%
I_k	-10%	(4, 0.1397)	54491	-0.18%
	-25%	(4, 0.1402)	54386	-0.37%
	+10%	(4, 0.1391)	54630	+0.07%
	+25%	(4, 0.1387)	54734	+0.26%
I_e	-10%	(4, 0.1395)	54675	+0.26%
	-25%	(4, 0.1396)	54602	+0.07%
	+10%	(5, 0.1139)	54580	-0.01%
	+25%	(5, 0.0965)	54568	-0.04%
c_r	-10%	(4, 0.1397)	54401	-0.35%
	-25%	(4, 0.1402)	54161	-0.79%
	+10%	(4, 0.1391)	54720	+0.24%
	+25%	(4, 0.1387)	54959	+0.67%

s	-10%	(4, 0.1395)	54640	+0.09%
	-25%	(4, 0.1396)	54592	+0.03%
	+10%	(4, 0.1394)	54580	-0.02%
	+25%	(4, 0.1392)	54568	-0.04%
π	-10%	(4, 0.1395)	54554	-0.06%
	-25%	(4, 0.1396)	54560	-0.05%
	+10%	(5, 0.1139)	54617	+0.05%
	+25%	(5, 0.1139)	54624	+0.06%
α-	-10%	(4, 0.1387)	54481	-0.20%
	-25%	(4, 0.1376)	54362	-0.41%
	+10%	(4, 0.1402)	54639	+0.08%
	+25%	(4, 0.1827)	54739	+0.27%
μ -	-10%	(4, 0.1394)	54554	-0.06%
	-25%	(4, 0.1396)	54546	-0.08%
	+10%	(4, 0.1394)	54596	+0.01%
	+25%	(4, 0.1139)	54624	+0.06%
x	-10%	(4, 0.1395)	54054	-0.9%
	-25%	(4, 0.1396)	54046	1%
	+10%	(4, 0.1394)	55496	1.63%
	+25%	(4, 0.1393)	55512	1.66%

Comparison between the independent and integrated policies

When the supplier and the retailer do not cooperate with each other, both them will determine their own policy independently. First, the supplier makes his/her own individual optimal decision. In response, the retailer formulates his/her own policy. To explore the advantage of coordination among the integrated model, we use the same data in example 1. The optimal solutions for the supplier and retailer can be obtained and further results shown in Table 5,

Table 5 Comparison between the independent and integrated policies

	Independent	**Integrated**
Optimal length of replenishment cycle time (T^*)	0.2445	0.2178
Optimal Order Quantity Q^*	8802	6860
Optimal number of shipments from supplier to retailer in one production run n^*	8	7
Supplier total cost	32675	27509
Retailer's total cost	27866	24382
Sum of total cost	60541	51891

Table 5 shows that adopting the integrated inventory policy minimizes the total cost of the supply chain system.

5.1 Discussion

- When the number of defective products is larger than 3, the risk-averse solution gives a total cost lesser than in the risk-neutral solution. On the contrary, when the number of defective items is less than or equal to 2, the risk neutral solution is better than the risk-averse solution.

- It is observed from the table 2 that: as the credit period t_1 increases, the number of shipments in transport increases, replenishment cycle time and the total system cost are decreasing. So
- When the supplier extends his credit terms, the retailer will shorten the cycle time to take advantage of credit period more frequently.
- Sensitivity analysis of t_2 is similar to the case as in the sensitivity analysis of t_1.
- From the results in Table 4, we obtain the following observations:
  The sensitivity of JTC (n, T) to parameter changes:
- JTC (n, T) is highly sensitive to λ, c_p, x
- JTC (n, T) is moderately sensitive to A_s , P, c_r, h_s , h_r,α,π,s,I_k and I_e
- JTC (n, T) is slightly sensitive to h_r, μ.
- The value of total system cost Joined total cost (JTC) is more sensitive to market demand λ. The other parameters, supplier production cost c_p and defective item's percentage x influence on the JTC.
- The inventory parameters Interest earned rate $\mathbf{I_e}$, and selling price**'s'** are negatively correlated to the JTC. And other parameters are positively correlated with JTC.
- From Table 5, it shows that adopting the integrated inventory policy minimizes the total cost of the supply chain system.

6 Conclusions

In this paper, an integrated supplier-retailer inventory model is proposed that accounts of random defectiveness in transport of the retailer's ordering quantity and also defective items are repaired. For the study, a two-level trade credit inventory model is built mathematically and the average joint total cost function is minimized. By applying the proposed computational algorithm, the optimal replenishment cycle and optimal number of shipments are obtained. Additionally, sensitivity analysis is conducted on the main parameters of the model. It is shown that the extension of supplier's trade credit terms will allow the retailer to take advantage of trade credit more frequently. The retailer may order a smaller quantity to shorten the ordering cycle length. The results in this paper illustrate that to improve the system effectively, both retailer and supplier are required to work carefully to improve market demand through increased perfect product quality.

If the customers are sensitive to length of credit period offered by the retailer, then the trade credit strategy is very effective in earning the profit or minimizing the total cost. Therefore, before adopting trade credit strategy, the retailer should have a complete understanding of the choice of customers for trade credit policy. Normally, a business credit period is dictated by an industry standard or competition. Therefore, in future research on this problem, it would be interesting to allow that the credit period treated as one of the decision. Also, one can extend this present model by considering multiple items.

References

1. Goyal, S.K., (1976). An Integrated Inventory Model for A Single Supplier – Single Customer Problem. International Journal of Production Research., 15, 107-111.

2. Banerjee, A., (1986). A joint economic-lot-size model for purchaser and vendor", Decision Sciences, 17 (3), 292-311.
3. Goyal, S.K., (1988). A Joint Economic-Lot-Size Model for Purchaser and Vendor: A Comment, Decision Sciences, 19, 236-241.
4. Goyal, S.K. and Gupta, Y.P., (1989). Integrated inventory models: the buyer-vendor Coordination", European Journal of Operational Research, 41, 261-269.
5. Jauhari, W.A., Pujawan, I.N., Wiratno. S.E. and Priyandari, Y., (2011). Integrated inventory model for single-vendor single-buyer with probabilistic demand", International Journal of Operational Research, 11 (2), 160-178.
6. Gupta, O.K., Shah, N.H., Patel, A., R., (2011). An integrated deteriorating inventory model with permissible delay in payments and price sensitive stock-dependent demand", International Journal of Operational Research, 11 (4), 452-442.
7. Madhavi, N., Rao, K.S., Lakshminarayana, J., (2011). Optimal pricing policies of an inventory model for deteriorating items with discounts, International Journal of Operational Research, 12 (4), 464-480.
8. Jauhari, W.A., (2012). Integrated inventory model for three-layer supply chain with stochastic demand, International Journal of Operational Research, 13 (3), 295-317.
9. Gupta, V. and Singh, S.R., (2013). An Integrated inventory model with fuzzy variables, three-parameter Weibull deterioration and variable holding cost under inflation, International Journal of Operational Research, 18 (4), 434-451.
10. Goyal, S.K., (1995). A one-vendor multi-buyer integrated production inventory model: A comment, European Journal of Operational Research, 82, 209-210.
11. Hill, R.M., (1999). The optimal production and shipment policy for the single-vendor single-buyer integrated production-inventory problem, International Journal of Production Research, 37, 2463-2475.
12. Hoque, M.A. and Goyal, S.K., (2000). An optimal policy for a single-vendor single-buyer integrated production-inventory system with capacity constraint of the transport equipment, International Journal of Production Economics, 65, 305-315.
13. Zhou, Y.W. and Wang, S.D., (2007). Optimal production and shipment models for a single-vendor-single-buyer integrated system, European Journal of Operational Research, 180, 309-328.
14. Hill, R.M., and Omar, M., (2006). Another look at the single-vendor single-buyer integrated production-inventory problem, International Journal of Production Research, 44, 791-800.
15. Giri, B.C. and Roy, B., (2013). A vendor-buyer integrated production-inventory model with quantity discount and unequal sized shipment, International Journal of Operational Research, 16 (1), 1-13.
16. Porteus, E.L., (1986). Optimal lot sizing, process quality improvement and setup cost reduction. Operations Research, 34, 137-144.
17. Salameh, M.K. and Jaber, M.Y., (2000). Economic production quantity model for items with imperfect quality, International Journal of Production Economics, 64 (1), 59-64.
18. Raouf, A., Jain, J.K., and Sathe, P.T., (1983). A cost-minimization model for multicharateristic component inspection, HE Transactions, 15, 187-194.
19. Yoo, S.H., Kim, D., and Park, M.S., (2009). Economic production quantity model with imperfect-quality items, two-way imperfect inspection and sales return, International Journal of Production Economics, 121, 255-265.
20. Lin, T.Y., (2009). Optimal policy for a simple supply chain system with defective items and returned cost under screening errors, Journal of the Operations Research Society of Japan, 52, 307-320.
21. Hsu, J.T. and Hsu, L.F., (2012). An integrated single-vendor single-buyer production-inventory model for items with imperfect quality and inspection errors", International Journal of Industrial Engineering Computations, 3,, 703-720.
22. Darwish, M.A., Odah, O.M, Goyal, S.K., (2013). Vendor-managed inventory models for items with imperfect quality, International Journal of Operational Research, 18 (4), 401-433.
23. Pamudji, A.S., Jauhari, W.A., and Rosyidi, C.N., (2013). Joint Economic Lot Size Model for Vendor-Buyer System with defects and Deterministic Demand, Proceeding of Industrial Engineering and Service Science, 1-7.
24. H.C. Hill, K.D. Riener, (1979). Determining the cash discount in the firm's credit policy, Financial Management 8, 68–73.
25. P.L. Abad, C.K. Jaggi, (2003). A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive, International Journal of Production Economics 83, 115–122.
26. L.H. Chen, F.S. Kang, (2007). Integrated vendor-buyer cooperative inventory models with variant permissible delay in payments, European Journal of Operational Research 183, 658–673.

27. Y.F. Huang, K.H. Hsu, (2008). An EOQ model under retailer partial trade credit policy in supply chain, International Journal of Production Economics 112, 655–664.
28. C.H. Ho, L.Y. Ouyang, C.H. Su, (2008). Optimal pricing, shipment and payment policy for an integrated supplier-buyer inventory model with two-part trade credit, European Journal of Operational Research 187, 496–510.
29. A. Thangam, R. Uthayakumar, (2009). Two-echelon trade credit financing for perishable items in a supply chain when demand depends on both selling priceand credit period, Computers and Industrial Engineering 57, 773–786.
30. Su, C.H., (2012). Optimal replenishment policy for an integrated inventory system with defective items and allowable shortages under trade credit., Int.J.Production Economics., 139, 247-256.
31. Kim, M.S., Kim, J.S., Sarkar, B., Sarkar,M., Iqbal, M.W., (2018). An improved way to calculate imperfect items during long-run production in an integrated inventory model with backorders Journal of Manufacturing Systems, 47, 153-167.
32. Mosca,A., Vidyarthi,N., Satir, A., (2019). Integrated transportation – inventory models: A review Operations Research Perspectives, 6, Article 100101.
33. Hu,W., Toriello,A., Dessouky, M., (2018). Integrated routing and freight consolidation for perishable goods. European Journal of Operational Research, 271, 548-560.
34. Lin, F., Jia, T., Wu,F., Yang,Z., (2019). Impacts of two-stage deterioration on an integrated inventory model under trade credit and variable capacity utilization European Journal of Operational Research, 272, 219-234.
35. Li, M., Wang, Z., (2017). An integrated replenishment and production control policy under inventory inaccuracy and time-delay Computers & Operations Research, 88, 137-149.
36. Chan,C.K., Wong,W.H., Langevin,A., Lee,Y.C.E., ., Iqba, M.W., (2017). An integrated production integrated model for deteriorating items with consideration of optimal production rate and deterioration during delivery. International Journal of Production Economics, 189, 1-13.
37. Das, B.C., Das,B., Mondal, S.K., (2014). Optimal transportation and business cycles in an integrated production- inventory model with a discrete credit period. Transportation Research Part E: Logistics and Transportation Review, 68, 1-13.
38. AlDurgam,M., Adegbola,K., Glock,C.H., (2017). A single-vendor single-manufacturer integrated inventory model with stochastic demand and variable production rate. International Journal of Production Economics, 191, 335-350.
39. Ouyang, L.H., Ho, C.H., Su,C.H., Yang, C.T., (2015). An integrated inventory model with capacity constraint and order-size dependent trade credit. Computers & Industrial Engineering, 84, 133-143.
40. Das,B.C., Barun Das, Mondal.,S.K., (2017). An integrated production-inventory model with defective item dependent stochastic credit period. Computers & Industrial Engineering, 110, 255-263.
41. Bhunia,A.K., Shaikh,A.A., Cárdenas-Barrón., L.D., (2017). A partially integrated production-inventory model with interval valued inventory costs, variable demand and flexible reliability. Applied Soft Computing, 55, 491-502
42. Li,.M., Wang.,Z., (2017). An integrated replenishment and production control policy under inventory inaccuracy and time-delay.Computers & Operations Research, 88, 137-149.
43. Chan, C.K., Wong, W.H., Langevin, A., Lee., Y.C.E., (2017). An integrated production-inventory model for deteriorating items with consideration of optimal production rate and deterioration during delivery. International Journal of Production Economics, 189, 1-13.
44. Jha, J. K.., Shanker., K., (2014). An integrated inventory problem with transportation in a divergent supply chain under service level constraint. Journal of Manufacturing Systems, 33, 462-475.
45. Chuang, C.J., Ho, C.H., Ouyang, L.Y., Wu, C-W., (2013). An Integrated inventory Model with Order-Size-Dependent Trade Credit and Quality Improvement. Procedia Computer Science, 17, 365-372.
46. Firouz, Keskin, B.,B., Melouk, S.H., (2017). An integrated supplier selection and inventory problem with multi-sourcing and lateral transshipments. Omega, 70, 77-93.
47. Rad, R.H., Razmi,J., Sangari,M.S., Ebrahimi,Z.F., (2014). Optimizing an integrated vendor-managed inventory system for a single-vendor two-buyer supply chain with determining weighting factor for vendor′s ordering cost. International Journal of Production Economics, 153, 295-308.