A novel cooperative game between client and subcontractors based on technical characteristics

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Abstract Large projects often have several activities which are performed by some subcontractors with several skills. Costs and time reduction and quality improvement of the project are very important for client and subcontractors. Therefore, in real large projects, subcontractors join together and form coalitions for improving the project profit. A key question is how an extra profit of cooperation among subcontractors should be assigned to them. This paper tries to address this question by proposing a cooperative game model based on technical characteristics of subcontractors. Technical characteristics of each contractor specify its value in the coalitions that it may join. Fair and appropriate allocation of profit among subcontractors is suggested by adopting cooperative game theory methods such as the Core method, Shapley method, Equal Profit Method (EPM) and the t-value.

Keyword: Project Management, Cooperative Games, Project Utilities.

1 Introduction

The relationships between the client and subcontractors in large construction projects have usually been subjected to adversarial relationships [1]. The construction industry, with its adversarial culture, can use relational contracting to improve project payments by improving social relationships between its key players, specially clients and subcontractors. To develop and maintain relationships, at first, the clients and subcontractors must focus on special factors that motivate them to perform relational contracting [2]. Nowadays, some projects require employing different subcontractors to perform specialized activities by using a good planning for reduction of the project time [3]. The most important factors, transaction limitations, further increase the requirement of cooperation in construction [4]. The main client objective of the project is to make the decision to invest in a construction project. Clients with high experience may have the essential expertise to provide their project program. Clients with low experience may need help. The important role of the project manager is to manage, motivate, coordinate and preserve the belief of the project team [5]. A project manager concerns about the customer satisfaction, and the subcontractors are the most important provider of this satisfaction [6].

In situations that multiple players decide to form a coalition, the question of how to allocate the outcome shares plays a dominant role. Cooperative game theory defines several concepts for allocating outcome shares in a joint project with transferable utilities such as money [7].

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Cooperative game theory and cooperative organization share the idea that companies work together for mutual benefits, but cooperative organizations are controlled democratically [8]. Logistics costs have increased due to intensified competition, lower inventory levels and higher service levels for the customers. Horizontal cooperation among companies is an effective way to reduce these costs. Horizontal cooperation is identifying the win–win situation among companies at the same level in the supply chain for improving performance [9]. In many countries, the government penalizes the company in charge legally when a public project is delayed. Therefore, subcontractors avoid the fine by forming coalition with others [3].

Efficient utilization of all contractor resources in the duration of the project time reduces the project costs and it is a major issue that every contractor wants to improve it. Since any project has a specific start and end time, the project time is specified and the subcontractors try to use their resources efficiently; but sometimes subcontractors may fail in one day. Most projects are done by several subcontractors whose many resources may remain unused. Unemployment days lead to increase of project costs and time.

The benefit of empty coalition is zero and the benefit of grand coalition $N$ (consisting of all the players) should be at least the sum of the benefits of individual players in the case of no coalition formation. This means that the players joining together should do better than each one independently [10]. Since the subcontractors have a same purpose and similar activities, they can trade their resources together to reduce the unemployment days. Moreover, the profit improvement may also happen. This paper tries to model the trading resources and considers the technical characteristics of subcontractors (such as human resources, finance (credit), expertise, financial resources, equipment and etc.) in calculation of the improved profit. Then, cooperative game theory methods such as the Core, Shapley, EPM, t-value are used for allocating the improved profit. Finally, these questions are answered: Is the project profit in the grand coalition more than the sum of the project profit of individual subcontractors? How much technical characteristics make it different? How the profit of a coalition of subcontractors should be distributed among members?

This paper is organized in six sections. The literature review is studied in Section 2. Section 3 describes the model assumptions. The proposed model is presented in Section 4. In Section 5, the proposed model is used in a case study and finally, the conclusions are discussed in Section 6.

2 Literature review

The two approaches of cost sharing in delayed joint projects are presented by Branzei et al. [11] in 2002. The both approaches used the activity graph to describe joint projects. The first approach is activity oriented, and the second one is a path (in the activity graph) oriented. Jia et al. (2003) discussed the cooperation possibility of independent power producers (IPPs) in the retail market and proposed an approach to calculate the allocation of their profits based on the Game theory. Fernández et al. [12] studied situations in which a project consisting of several activities is not performed as planned. Their study is divided into three sections. The first section analyzes the activities may be delayed. The second section considers the activities may be expedited. The third section studies some activities may be delayed and some activities may be expedited. They developed their work by considering non decreasing reward functions and by assuming that the activities can be started before their planned starting time. Asgari et al. [13] approved that consideration of time-efficiency function and time of
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subproject affects the total cost. By fairly allocating the benefits of coalition, all subcontractors have a good reason for the coalition. Branzei et al. [14] presented mutual help solutions which is a new family of compensations–penalties solutions which help project managers to determine fair shares of penalties for subcontractors. Lozano et al. [15] used a linear model to study the cost savings that several companies may obtain when they merge their transportation requirements. Cooperative game theory was used for allocating the joint cost savings of the cooperation. Hafezalkotob et al. [16] presented a new mathematical programming model for the maximum flow problem with multiple owners under uncertainty of the arcs’ capacity. Moreover, the benefits of collaboration among different owners were evaluated so that the expected value of flow is increased and variance of flow is reduced. On this basis, this paper analyzed several collaborative game based methods, including Shapley value, $s$-value, least core, core center, and equal utility method on a numerical example.

Asgari et al. [17] presented the agreement of subcontractors to trade their resources in a coalition for a fixed duration of time by cost-effective plan. Cooperative game theory is applied for fair allocation of the benefits of cooperation among the subcontractors. Finally, the results showed that considerable cost savings by grand coalition justify the cooperation.

According to previous sections, the majority of these studies was about project time and recommended subcontractors to form a grand coalition for saving total cost and finally allocated profits to subcontractors by the cooperative game theory. In this paper, synergy of subcontractor’s resources such as human, expertise, financial, equipment resources and etc. are considered as important factors for subcontractors’ motivation to form a grand coalition. By these factors and their effects on the project, project profit function for grand coalition can be identified. This idea is not considered in the previous papers. Finally, this profit is allocated by the cooperative game theory methods such as Core, Shapley and $\tau$ value.

Hlodversdottiret et al. [18] clarified that how project management might be to increase the cooperation between offices and departments, and to improve project management consciousness and skills.

Cooperative game theory methods can prepare useful insights into how parties use environmental resources and allocate benefits of cooperation. Madani [19] developed Nash and Nash–Harsanyi bargaining solutions to study the Federal Energy Regulatory Commission (FERC) relicensing process. He suggested a method that how the lack of incentive for cooperation results in the long delay in FERC relicensing. Usually, managing construction projects contains some conflicts that occur between the stakeholders and subcontractors and/or among subcontractors themselves. So, achieving a win-win situation is the most desirable. The game theory approach can be used as an efficient solution in decision making about conflicts in construction projects. The aim of this paper is to find the best outcome in conflicts for every player (party) according to its opponent’s decision. LechKrus et al. [20] described the cost allocation problem in the cooperation of economic agents implementing a joint project. Their model takes the form of a multi-item cooperative game. Barough et al. [21] discussed two game theory structures, prisoner’s dilemma and chicken game which were so useful for analyzing construction management problems. The players are willing to cooperate if a system can guarantee to allocate the part of benefits obtained from the cooperation to cover the losses of players [22]. The organizational theme of projects is not always conducive for grand collaboration. Priorities of sub-teams are different from central team. Klimkeit [23] found that the organizational theme can prepare important resources such as policies, authorities, procedures and systems which are appropriate to enable the collaboration. Madani et al. [19] evaluated the proposed alternatives for sharing the Caspian Sea resources with respect to the stakeholders’ utilities. Several multi-criteria decision-
making methods such as dominance, maximin, lexicography, simple additive weighting, and TOPSIS are applied to determine the social planner’s ranking of these alternatives. Bankruptcy rules and cooperative game theory solutions can be considered for the conflict of sharing the Caspian Sea energy resources among its five littoral countries.

In many projects, clients have the authority to select and organize a set of subcontractors in order to lower the project costs. This is emphasizing the importance of client capability in contracting with independent subcontractors. To the best of the authors’ knowledge, no research has been found that considers the effects of technical characteristics of contractors on their cooperation by the game theory models.

3 Prerequisites and assumptions

Each contractor obtains some implementation characteristics of the project separately by its technical characteristics as follows:

![Diagram](https://via.placeholder.com/150)

Fig. 1 Factors of a project (contractor’s technical characteristics and obtained implementation characteristics of the project for client).

3.1 Notations

Before the project profit function is described, the parameters and variables are explained. 

$K = \{1, 2, \ldots, k\}$ is the set of subcontractors who are active in a project and $C_l$ denotes the coalition $l$ ($C_l \subseteq K$). Set of $\{1, 2, \ldots, i, \ldots, n\}$ represents the index set of technical characteristics of subcontractors.
Table 1 Explanation of the parameters and variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{ki}$</td>
<td>The technical characteristic $i$ of contractor $k$ such as human resources, finance (credit), expertise, financial resources, equipment and etc.</td>
</tr>
<tr>
<td>$y_i$</td>
<td>The implementation characteristics of the project for each subcontractors such as the cost and the time of implementation, quality of productions and etc.</td>
</tr>
<tr>
<td>$x_{ci}$</td>
<td>The technical characteristic $i$ of coalition $l$.</td>
</tr>
<tr>
<td>$Z = f(y_1, y_2, ..., y_n)$</td>
<td>The project profit function.</td>
</tr>
</tbody>
</table>

The following assumptions are introduced to specify the scope of this work for further model formulation:

3.2 Assumptions

1. Some projects have been done by some subcontractors with several skills. Each contractor can join a coalition for performing activities with high quality and descending cost.
2. The client can specify budget of the project by an estimation of project utilities.
3. Each contractor can join a coalition for performing activities with better quality and lower cost.
4. Project game is super-additive. It means that if contractor 1 (Figure 1) obtains implementation characteristics of the project ($y_{11}, y_{12}, ..., y_{1n}$) by its technical characteristics ($x_{11}, x_{12}, ..., x_{1m}$) and also contractor 2 obtains implementation characteristics of the project ($y_{21}, y_{22}, ..., y_{2n}$) by its technical characteristics ($x_{21}, x_{22}, ..., x_{2m}$), the amount of implementation characteristics of the project in coalition 2 ($y_{c1}, y_{c2}, ..., y_{cn}$) will increase certainly and it will be more or equal to the sum of the implementation characteristics of the project for subcontractors 1 and 2.

4 Model formulation

For the given values of variables $x_{ki}$ and considering the values of $y_i$, the project profit function is formulated for each contractor. In the next step, for the calculated values of variables $x_{ci}$ (for each coalition $c_L$) and their project utilities, the project profit function is formulated for them.

4.1 Formulation of the project profit function

Based on the previous section, the project profits function can be formulated as follows.
Table 2 Definition of input, output and payment in the formulation of the project profits function

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual</td>
<td>Technical characteristics of the subcontractors</td>
<td>Implementation characteristics of the project by subcontractors</td>
</tr>
<tr>
<td>$x_{11}, x_{12}, \ldots, x_{1n}$</td>
<td>$y_{11}, y_{12}, \ldots, y_{1n}$</td>
<td>$V (1) = f(y_{11}, y_{12}, \ldots, y_{1n})$</td>
</tr>
<tr>
<td>$x_{21}, x_{22}, \ldots, x_{2n}$</td>
<td>$y_{21}, y_{22}, \ldots, y_{2n}$</td>
<td>$V (2) = f(y_{21}, y_{22}, \ldots, y_{2n})$</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>$x_{k1}, x_{k2}, \ldots, x_{kn}$</td>
<td>$y_{k1}, y_{k2}, \ldots, y_{kn}$</td>
<td>$V (k) = f(y_{k1}, y_{k2}, \ldots, y_{kn})$</td>
</tr>
</tbody>
</table>

Coalitional

| $x_{c1}, x_{c2}, \ldots, x_{cm}$ | $y_{c1}, y_{c2}, \ldots, y_{cn}$ | $V (c_1) = f(y_{c1}, y_{c2}, \ldots, y_{cn})$ |
| $x_{c1}, x_{c2}, \ldots, x_{cm}$ | $y_{c1}, y_{c2}, \ldots, y_{cn}$ | $V (c_2) = f(y_{c1}, y_{c2}, \ldots, y_{cn})$ |
| \ldots | \ldots | \ldots |
| $x_{c1}, x_{c2}, \ldots, x_{cm}$ | $y_{c1}, y_{c2}, \ldots, y_{cn}$ | $V (c_m) = f(y_{c1}, y_{c2}, \ldots, y_{cn})$ |

As shown in Table 2, technical characteristics of each contractor like human resources, finance (credit), expertise, financial resources, equipment and etc. should be identified at first. Each contractor affects the implementation characteristics of the project with regards to its technical characteristics. $Z$ denotes the payment of the client to subcontractors which is a function of implementation characteristics of the project (i.e. $y_{i1}, y_{i2}, \ldots, y_{in}$ or $y_{c1}, y_{c2}, \ldots, y_{cm}$)

Fig.2 Details of the input, output and payment in formulation of the project profits function

4.2 Cooperative game theory

A game contains a number of players, a set of strategies for each player, and a payoff that describes the outcome of the amount that each player wins or loses. Game theory can be divided into two parts: non-cooperative and cooperative [10]. In the non-cooperative game,
players see only their own strategic objectives and try to maximize their profits, but in the cooperative game, players cooperate to get more profits and fairly allocate cooperative gains. Cooperative game has two sections: Transferable Utility - game in which the profits can be transferred and Non-Transferable Utility - game in which the profits cannot be transferred. Cooperative game theory attempts to answer some questions such as which coalitions can be formed? How can the coalitional gains be allocated in order to keep a sustainable agreement? For allocating profits, some methods such as the Core and the Shapley value [13] are suggested. The TUGlab package (Transferable Utility Games laboratory) is a MATLAB program that can serve as a helpful complement to allocate profit of the project [24].

4.2.1 Core

The core is the set of allocations so that each coalition receives at least the rewards associated with that coalition. The core may be empty.

Let \( S \subset N \) be a coalition and let \( \bar{x} \in X \). The excess of coalition \( S \subset N \) for imputation \( \bar{x} \in X \) is defined by:

\[
e(S,x) = v(S) - \sum_{i \in S} x_i
\]

(1)

It is the amount by which the rewards allocated to the coalition \( S \) differs from the benefits associated with \( S \) [10]. The core of the game is:

\[
C(0) = \left\{ \bar{x} \in X \mid e(S,\bar{x}) \leq 0, \forall S \subset N \right\} = \left\{ \bar{x} \in X \mid v(S) \leq \sum_{i \in S} x_i, \forall S \subset N \right\}
\]

(2)

4.2.2 Shapley value

Fair allocation determines the amount that each member adds to a coalition. Players who add nothing should receive nothing, and players who are indispensable should be allocated a lot. The Shapley allocation is each player’s expected contribution to any possible sequencing of players joining the grand coalition.

An allocation \( \bar{x} = (x_1, x_2, \ldots, x_n) \) is called the Shapley value if:

\[
x_i = \sum_{S \in \Pi^i} \left[ v(S) - v(S - i) \right] \frac{(|S| - 1)!(|N| - |S|)!}{|N|!}, \quad i = 1, 2, \ldots, n
\]

(3)

Where \( \Pi^i \) is the set of all coalitions \( S \subset N \) containing \( i \) as a member (i.e., \( i \in S \)), \(|S| = \) number of members in \( S \), and \(|N| = n \) [10].

4.2.3 Equal profit method (EPM)

This method is based on the equal profit method [25] that provides a stable allocation for the players in the grand coalition. This method minimizes the maximum differences in the mutual relative utility of the players. It is called the Equal Profit Method (EPM):
\[
\min z \\
\text{s.t.} \\
z \geq \frac{y_i}{v(i)} - \frac{y_j}{v(j)}, \forall (i, j) \in P, \\
\sum_{i \in C} y_i \geq v(C), \text{ for all } C \subset P, C \neq P, \\
\sum_{i \in P} y_i = v(P).
\]

The first constraint set measures the difference between the relative utility of two players. The variable \( z \) represents the largest difference that should be minimized in the objective function.

### 4.3 Methodology

In the first step, technical characteristics of the subcontractors should be collected or the probabilistic coalition in the project should be defined. In the next step, implementation characteristics of the project are estimated and the contract rules are legislated based on implementation characteristics. In the fourth step, the project cost should be defined for each coalition which is formed. Finally, by using cooperative game theory methods in the TUGlab package, the profit of grand coalition is assigned to subcontractors.

The methodology of the previous sections is described briefly as follows:

- **Step 1**: Collecting the technical characteristics of the contractors (Identify matrix \( X \))
- **Step 1**: Defining the probabilistic coalitions
- **Step 2**: Estimating implementation characteristics of the project
- **Step 3**: Legislating contract based on implementation characteristics of the project
- **Step 4**: Defining cost of the project for each coalitions
- **Step 5**: Applying cooperative game theory methods for allocation of the payment

**Fig. 3** Methodology of the presented model formulation
5 Numerical example

Emam-Ali is a highway that connects the east-north of Tehran to its south. This highway starts from Darabad and passes the Avini highway and continues to Tondgooyan highway as the Haram-Ta-Haram project. Also, many junctions make accessibilities to the streets that connect the east of the Tehran to its west. It is like a communication channel and has a key role in facilitating of Tehran traffic.

Emam-Ali highway project started in 2011 and opened in 2013. It is one of the longest highways with 35 kilometers length, 56 bridges in 25 junctions and 27 bridges with 2 layers. This highway had to pass from old buildings in 7, 8, 13, 14 and 15 regions of Tehran municipality which was a main problem for municipality managers. This problem was solved and 7000 apartments (or 4000 houses) were bought and ruined for construction of Emam-Ali highway.

For facilitating, acceleration and increasing precision in performance, this project was divided into 6 phases. In this paper, three phases of this project which were devolved to three subcontractors are considered.

Contractor A: this phase started from Azadegan highway to Khavaran Highway (2.2 kilometers).

Contractor B: this phase started from Khavaran highway to Mahallati square (1 kilometers).

Contractor C: this phase started from Mahallati square to Piroozi Street (2.3 kilometers).

![Fig. 4 Phases of Emam Ali highway project that are considered (Emam Ali Hwy – [26])](image-url)
Table 3 Specifications of the subcontractors in Emam-Ali highway project.

<table>
<thead>
<tr>
<th>$X_{ki}$</th>
<th>Human resources</th>
<th>Equipment</th>
<th>Knowledge</th>
<th>Finance (million $)</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2000</td>
<td>800</td>
<td>High</td>
<td>90</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>900</td>
<td>450</td>
<td>Medium</td>
<td>50</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>1200</td>
<td>570</td>
<td>Medium</td>
<td>75</td>
<td>1</td>
</tr>
<tr>
<td>AB</td>
<td>2900</td>
<td>1250</td>
<td>Higher</td>
<td>140</td>
<td>1</td>
</tr>
<tr>
<td>BC</td>
<td>2100</td>
<td>1020</td>
<td>High</td>
<td>125</td>
<td>1</td>
</tr>
<tr>
<td>AC</td>
<td>3200</td>
<td>1370</td>
<td>Higher</td>
<td>165</td>
<td>1</td>
</tr>
<tr>
<td>ABC</td>
<td>4100</td>
<td>1820</td>
<td>Highest</td>
<td>215</td>
<td>1</td>
</tr>
</tbody>
</table>

Civil deputy of Tehran municipality defined the implementation characteristics and profit function of this project as follows:

$$z = f(y_1, y_2) = ay_1 - by_2 + c$$  \hspace{1cm} (5-1)

“a”, “b” are constant coefficients in this function and “c” is the amount of basic contract which is constant too. Of course, because of multiple changings in the project time and materials cost, some long term projects have supplementary in the amount of the contract, for example %25 amount of the basic contract.

As was defined in the notations section, $y_1$ and $y_2$ are implementation characteristics of the project for each subcontractor (or their coalition) which is described in this case as follows:

$y_1$: project quality (10: high, 5: medium, 1: low).

$y_2$: project time (10: expedition, 5: on time, 1: delay).

Based on the specifications of the subcontractors in Table 5 and the profit function of this project, payments of each subcontractor and the subcontractors’ coalitions are obtained as follows:

Table 4 Payments of each subcontractors and their coalitions in Emam-Ali highway project.

<table>
<thead>
<tr>
<th>$X_{ki}$</th>
<th>Payment (million $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>25</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
</tr>
<tr>
<td>AB</td>
<td>42</td>
</tr>
<tr>
<td>BC</td>
<td>45</td>
</tr>
<tr>
<td>AC</td>
<td>30</td>
</tr>
<tr>
<td>ABC</td>
<td>65</td>
</tr>
</tbody>
</table>

Finally, by using the cooperative game theory methods in the TUGlab package, the obtained payment is allocated to subcontractors in the grand coalition:

Table 5 Allocation of the coalition profit, according different methods

<table>
<thead>
<tr>
<th>Methods</th>
<th>Shapley</th>
<th>$\tau$ value</th>
<th>Core-center</th>
<th>EPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contractor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>30.33</td>
<td>30.357</td>
<td>30.375</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>15.33</td>
<td>15.357</td>
<td>15.375</td>
<td>22</td>
</tr>
<tr>
<td>Sum</td>
<td>65</td>
<td>65</td>
<td>65</td>
<td>65</td>
</tr>
</tbody>
</table>
The black area in Figure 5 represents the core which is not empty. Also, in a 3-person cooperative game, the game is convex if its core “touches” all three sides of the imputations triangle [24], so the game is convex.

![Figure 5](image)

**Fig. 5** Core for the grand coalition in Emam-Ali highway project.

### 6 Conclusion

Decrease of costs and time and increasing the project quality are very important for client and subcontractors. Therefore, subcontractors join together and form coalitions for improving the project profit. In this paper, it is tried to model the problem and consider technical characteristics of each contractor for calculation of improved profit. Finally, by using the cooperative game theory methods such as the Core, Shapley, EPM and $\tau$-value, the improved profit is allocated to the subcontractors. Results show that considerable cost savings under the grand coalition creates strong incentive for the cooperation. So, it is found that the potential of extra utilities of coalitions, and supper additive property are contingent. Therefore, the synergy of cooperation rises with the size of the coalition and maximizes in the grand coalition.

For more research, it can be focused on the uncertainty (or probability) in costs problems, technical characteristics and etc. Also, it can be assumed that the subcontractors don’t have any mutual information about the technical characteristics of other subcontractors and extend the proposed model for asymmetric games.

### References