Context-Dependent Data Envelopment Analysis-Measuring Attractiveness and Progress with Interval Data

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Abstract  Data envelopment analysis (DEA) is a method for recognizing the efficient frontier of decision making units (DMUs). This paper presents a Context-dependent DEA which uses the interval inputs and outputs. Context-dependent approach with interval inputs and outputs can consider a set of DMUs against the special context. Each context shows an efficient frontier including DMUs in particular levels. The Context-dependent DEA with interval inputs and outputs can measure (i) the attractiveness when DMUs showing weaker performance are selected as an appraisal context, and (ii) the interval progress when DMUs showing better performance are selected as the appraisal context.

Keywords  Interval Inputs and Outputs, Context-Dependent Data Envelopment Analysis, Attractiveness, Progress, Value Judgment.

1 Introduction

Data envelopment analysis (DEA), presented by Charnes, Cooper and Rhodes (CCR) [1], is a mathematical programming method used for measuring the relative efficiency of decision making units (DMUs) with several outputs and inputs.

Among DMUs from a given set, DEA recognizes efficient DMUs. All we know is that adding or omitting an inefficient DMU or a set of inefficient DMUs does not change the efficiencies of the DMUs and the efficient frontier. The inefficiency scores alter only if the efficient frontier is changed, i.e., the performance of DMUs is only influenced by the identified efficient frontier. On the other hand, the context often influences the consumer choice, e.g., a circle shows big when small circles surround it, and small when bigger ones surround it. A product also may show attractive against some products with less attractiveness, and unattractive when we compare it with more attractive products [2]. Considering this result of the DEA framework, someone could ask “what is the relative attractiveness of a special DMU when compared to others?” mentioned in [3], someone accepts that existence or non-existence of a third option, say DMU_{z} (or a group of DMUs), influences the relative attractiveness of DMU_{x} which are compared to DMU_{y}. Relative attractiveness depends on the evaluation context made from different choices (or DMUs).

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Indeed a set of DMUs can be shared among different levels of efficient frontiers. If we omit DMUs which are efficient in their best and worst conditions, afterwards a new second-level efficient frontier will be made for the remaining (inefficient) DMUs. With omitting this new second-level efficient frontier, a third-level efficient frontier is made, and this process continues, until no DMU is remained. Each efficient frontier provides an assessment context that measures the relative attractiveness, e.g., the second-level efficient frontier is considered as the assessment context for measuring the relative attractiveness of the DMUs located on the first-level (main) efficient frontier. On the other part, the performance of DMUs on the third-level efficient frontier can be measured concerning the first-or second-level efficient frontier. With continuing this procedure, the context-dependent DEA is gained that can measure the relative attractiveness and the relative progress when DMUs having worse performance and better performance are chosen as the assessment context, respectively. The existence or non-existence (or the shape) of the assessment context (efficient frontier) influence the relative attractiveness or progress of DMUs on a different level of efficient frontier. When DMUs in a particular level are observed that have equal performance, the attractiveness value or the progress value lets distinguish the “equal performance” based on the same particular assessment context (or third option). In this paper we have interval inputs and interval outputs so the attractiveness measure and progress measure aren’t the same in [4], context-dependent data envelopment presented by Seiford, et al because they just consider in their crisp condition. For interval inputs and outputs sake also the evaluation contexts change, i.e., we have the best condition and worst condition in efficient frontier so we measured in two states; (i) the attractiveness of DMU considered when DMUs having worse performance are chosen as the evaluation context (ii) the progress of DMU considered when DMUs having better performance are chosen as the evaluation context. Note that different input/output values affect the assessment of a DMU’s performance. Thus, the incorporation of value judgment in measuring the relative attractiveness and progress, is very important as well. We use the result of a context-dependent DEA with value judgment presented by Zhu [5] in this paper and incorporate it with a context-dependent DEA. The rest of the paper is as follows: The next section represents the context-dependent DEA with interval inputs and outputs. Then we merge the value judgment into the context-dependent DEA. After that, we use this method in the numerical example in one of the Iran’s commercial bank. We provide our conclusions in the last section.

2 Context-dependent DEA

Let DMU \( j \) \((j = 1, 2, \ldots, n)\) are decision making units that produces \( y_j = (y_{j1}, \ldots, y_{jr}) \) by using \( x_j = (x_{j1}, \ldots, x_{jm}) \) that \( x_j^L \leq x_j \leq x_j^U \). Assume that \( j^j = \{DMU_j; j = 1, \ldots, n\} \) is all \( n \) DMUs’ set. Now we define \( J^{++} = J^j - E_i^{++} \) and \( E_i^{++} = \{DMU_j; \phi^+ = 1, \phi^* = 1\} \). \( E_i^{++} \) reaches from two models below.
\[
\begin{align*}
\phi^+(l, k) &= \max_{\lambda_j, \phi^- (l, k)} \phi^-(l, k) \\
\text{s.t.} & \quad \lambda_k x^L_k + \sum_{j \in F(J)} \lambda_j x^L_j \leq x^U_k, \\
\phi^*(l, k) &= \max_{\lambda_j, \phi^- (l, k)} \phi^+(l, k) \\
\text{s.t.} & \quad \lambda_k x^U_k + \sum_{j \in F(J)} \lambda_j x^L_j \leq x^L_k,
\end{align*}
\]

\[
\begin{align*}
\bar{\lambda}_k y^L_k + \sum_{j \in F(J)} \lambda_j y^U_j \geq \phi^- (l, k) y^L_k, \\
\bar{\lambda}_k y^U_k + \sum_{j \in F(J)} \lambda_j y^L_j \geq \phi^+ (l, k) y^U_k, \\
\lambda_j \geq 0, \quad j \in F(J^l). \tag{1} \quad \lambda_j \geq 0, \quad j \in F(J^l). \tag{2}
\end{align*}
\]

which \((x_k, y_k)\) is the input and output vector of DMU\(_k\), and \(j \in F(J^l)\) means DMU\(_j \in J^l\). The below algorithm helps us recognize these efficient frontiers due to models (1) and (2). The efficient frontiers can be easily obtained by using the DEA Excel Solver provided in [6].

**Step 1:** Set \(l = 1\). Consider the all DMUs in \(J^1\) by models (1) and (2) to reach the first-level efficient DMUs, set \(E_1^{++}\) (the first-level efficient frontier).

**Step 2:** Assume that \(J^{l+1} = J^l - E_1^{++}\) to omit the efficient DMUs from future DEA runs. If \(J^{l+1} = \emptyset\) then stop.

**Step 3:** assess the new subset, \(J^{l+1}\) by models (1) and (2) to obtain a new set of efficient DMUs, \(E_1^{++}\) (the new efficient frontier).

**Step 4:** Let \(l = l + 1\). Go to step 2.

**Theorem 1.** \(\phi^- (l, k) \leq \bar{\phi}^- (l, k) \leq \phi^- (l, k)\) that \(\phi^- (l, k)\) and \(\phi^+ (l, k)\) are obtained from (1) and (2) in order and \(\bar{\phi}^- (l, k)\) can be obtained by following model:

\[
\begin{align*}
\bar{\phi}^-(l, k) &= \max_{\lambda_j, \phi^- (l, k)} \bar{\phi}(l, k) \\
\text{s.t.} & \quad \sum_{j \in F(J^l)} \lambda_j \bar{x}_j \leq \bar{x}_k, \\
& \quad \lambda_j \bar{y}_j \geq \bar{\phi}(l, k) \bar{y}_k, \\
& \quad \lambda_j \geq 0, \quad j \in F(J^l). \tag{3}
\end{align*}
\]

**Proof.** First we show that \(\bar{\phi}^-(l, k) \leq \phi^- (l, k)\). For this purpose we choose the optimal solution \((\lambda_j^*, \bar{\phi}^-(l, k))\) of model (3); at the same time with we have:
\[
\begin{align*}
\sum_{j \in F(p)} \lambda_j^* \bar{x}_j &\leq \bar{x}_k \iff \sum_{j \in F(p)} \lambda_j^* x_{j}^L \leq \sum_{j \in F(p)} \lambda_j^* \bar{x}_j \leq \bar{x}_k (1 - \lambda_k^*) \leq x_k^U (1 - \lambda_k^*) \\
\sum_{j \in F(p)} \lambda_j^* \bar{y}_j &\geq \check{\phi}(l,k) \bar{y}_k \iff \sum_{j \in F(p)} \lambda_j^* y_{j}^U \geq \sum_{j \in F(p)} \lambda_j^* \bar{y}_j \geq \bar{y}_k (\check{\phi}^*(l,k) - \lambda_k^*) \geq y_k^U (\check{\phi}^*(l,k) - \lambda_k^*)
\end{align*}
\]

\[
\begin{align*}
\lambda_o^* x_{io}^U + &\sum_{j \in F} \lambda_j^* x_{j}^L \leq x_{io}^U , \quad i = 1, ..., m, \\
\lambda_o^* y_{io}^L + &\sum_{j \in F} \lambda_j^* y_{j}^U \geq \check{\phi}^*(l,k) y_{io}^L , \quad r = 1, ..., s,
\end{align*}
\]

So \((\lambda_j^*, \check{\phi}^*(l,k))\) is a feasible solution for model (1) and since the objective function of the model (1) has been maximized the theorem is proved, \(\check{\phi}^*(l,k) \leq \check{\phi}^- (l,k)\).

In the same way we can prove that; \(\check{\phi}^- (l,k) \leq \check{\phi}^* (l,k)\).

Showing that these sets \(E_{l^+}^\ast\) of DMUs have following properties is easy:

1. \(J^* = \cup E_{l^+}^*\) and \(E_{l^+}^* \cap E_{l'}^*\) for \(l \neq l'\).
2. The DMUs in \(E_{l^+}^*\) are Dominated through some of the DMUs in \(E_{l^+}^*\) if \(l \leq l'\).
3. DMUs in the set \(E_{l^+}^*\) is efficient with due attention to DMUs in set \(l < l'\).

Now context-dependent DEA with \(E_{l^+}^*\) can measure attractiveness of DMUs. Evaluate a specific \(DMU_{q^\prime}\) from a specific level \(E_{l^+}^*\) \((l=1, ..., L)\). Attractiveness is obtained from models below:

\[
\begin{align*}
\Omega_{q^\prime}^*(d) &= \max_{\lambda_j^*, \Omega_{q^\prime}^\ast (d)} \Omega_{q^\prime}^\ast (d), \quad d = 1, ..., L - l_o \\
\text{s.t.} \quad &\sum_{j \in F(E_{q^\prime}^+)} \lambda_j^* x_{j}^L \leq x_{q^\prime}^U , \\
&\sum_{j \in F(E_{q^\prime}^+)} \lambda_j^* y_{j}^U \geq \Omega_{q^\prime}^\ast (d) y_{q^\prime}^L , \\
&\lambda_j^* \geq 0, \quad j \in F(E_{l^+}^+) .
\end{align*}
\]

\(\Omega_{q^\prime}^+(d) = \max_{\lambda_j, \Omega_{q^\prime}^+ (d)} \Omega_{q^\prime}^+ (d), \quad d = 1, ..., L - l_o \)

\[
\begin{align*}
\text{s.t.} \quad &\sum_{j \in F(E_{q^\prime}^+)} \lambda_j^* x_{j}^L \leq x_{q^\prime}^L , \\
&\sum_{j \in F(E_{q^\prime}^+)} \lambda_j^* y_{j}^L \geq \Omega_{q^\prime}^+ (d) y_{q^\prime}^L , \\
&\lambda_j^* \geq 0, \quad j \in F(E_{l^+}^+) .
\end{align*}
\]

\(\Omega_{q^\prime}^+ (d)\) can be obtained by following model:
\[
\tilde{\Omega}_q^*(d) = \max_{\lambda \in \mathcal{F}(E_{i_0}^{++})} \tilde{Q}_q(d), \quad d = 1, \ldots, L - l_o
\]
\[
\text{s.t.} \quad \sum_{j \in \mathcal{F}(E_{i_0}^{++})} \lambda_j \tilde{x}_j \leq \tilde{x}_q, \quad i = 1, \ldots, m,
\]
\[
\sum_{j \in \mathcal{F}(E_{i_0}^{++})} \lambda_j \tilde{y}_j \geq \tilde{Q}_q(d) \tilde{y}_q^*, \quad r = 1, \ldots, s,
\]
\[
\lambda_j \geq 0, \quad j \in \mathcal{F}(E_{i_0}^{++}).
\]

where DMU \( q \) is from a particular level \( E_{i_0}^{++} \); \( l_o \in \{1, \ldots, L - 1\} \). We have:

\[
\begin{align*}
A_q^+(d) &> 1 \\
A_q^+(d) &> 1 \quad \text{for each } d = 1, \ldots, L - l_o \\
A_q^-(d) &> 1
\end{align*}
\]

Definition 1.

\( A_q^+(d) = \frac{1}{\Omega_q^+(d)} \) is called the (output-oriented) d-degree attractiveness of DMU \( q \) from a specific level \( E_{i_0}^{++} \). When DMU \( q \) is in the best condition, and DMUs in the evaluation context \( l_o + d \); \( d = 1, \ldots, L - l_o \) are in their worst condition.

\( A_q^-(d) = \frac{1}{\Omega_q^-(d)} \) is called the (output-oriented) d-degree attractiveness of DMU \( q \) from a specific level \( E_{i_0}^{++} \). When DMU \( q \) is in its own worst condition and DMUs in the evaluation context \( l_o + d \); \( d = 1, \ldots, L - l_o \) are in their own best condition.

Theorem 2. \( \Omega_q^+(d) \leq \tilde{\Omega}_q^*(d) \leq \Omega_q^-(d) \) that \( \Omega_q^+, \tilde{\Omega}_q^*(d) \) and \( \Omega_q^-, \Omega_q^-(d) \) are obtained from (5) , (6) and (4).

Proof. First we show that \( \tilde{\Omega}_q^*(d) \leq \Omega_q^+(d) \). For this purpose we choose the optimal solution of the model (2), \( (\lambda^*, \tilde{\Omega}_q^*(d)) \); at the same time with we have:
\[
\begin{aligned}
\min \quad & \sum_{j} \lambda_j^* x_j^L \\
\text{s.t.} \quad & \sum_{j} \lambda_j^* x_j^U \leq x_q^U \\
& \sum_{j} \lambda_j^* y_j^L \geq P_q^*(g)y_q^L \\
& \lambda_j^* \geq 0, \ j \in F(E_{l_{m+d}}^*)
\end{aligned}
\]

\[
\begin{aligned}
\min \quad & \sum_{j} \lambda_j^* x_j^L \\
\text{s.t.} \quad & \sum_{j} \lambda_j^* x_j^U \leq x_q^U \\
& \sum_{j} \lambda_j^* y_j^L \geq P_q^*(g)y_q^L \\
& \lambda_j^* \geq 0, \ j \in F(E_{l_{m+d}}^*)
\end{aligned}
\]

So \( (\lambda^*, \tilde{\Omega}_q^*(d)) \) is a feasible solution for model (4), and since the objective function of the model (4) has been maximized the theorem is proved, \( \tilde{\Omega}_q^*(d) \leq \Omega_q^-(d) \).

In the same way we can prove that; \( \Omega_q^+(d) \leq \tilde{\Omega}_q^*(d) \).

Let each DMU in the first-level efficient frontier represents a choice. We usually compare a specific DMU in \( E_{l_{m+d}}^+ \) with other DMUs that are in the same level as well as with relevant choices that be used as evaluation contexts, i.e., the relevant choices are those DMUs in the second-or third-level efficient frontier, etc. with given the alternatives (evaluation contexts), we are enabled by using models (4), (5), (6) to select the best option or the most attractive one. In models (4), (5), (6), each efficient frontier of \( E_{l_{m+d}}^+ \) is an assessment context for measuring the relative attractiveness of DMUs in \( E_{l_{m+d}}^+ \).

\[
\begin{aligned}
\max \quad & A_q^+(d) \\
\text{s.t.} \quad & \sum_{j} \lambda_j^* x_j^L \leq x_q^U \\
& \sum_{j} \lambda_j^* y_j^L \geq P_q^*(g)y_q^L \\
& \lambda_j^* \geq 0, \ j \in F(E_{l_{m+d}}^*)
\end{aligned}
\]

Note that \( A_q^+(d) > 1 \). The larger the value of \( A_q^+(d) \), the more attractive the DMU \( q \) is, because this DMU \( q \) makes itself more distinctive and different from the assessment context \( E_{l_{m+d}}^+ \). This property enables us to rank the DMUs in \( E_{l_{m+d}}^+ \) based upon their attractiveness scores and recognize the best one.

The progress measure for a specific DMU \( q \ in E_{l_{m+d}}^+ \) is obtained in the following context-dependent DEA;

\[
P_q^*(g) = \max_{\lambda_j^*, P_q^*(g)} P_q^*(g), \ g = 1, \ldots, l_n - 1
\]

s.t.

\[
\begin{aligned}
& \sum_{j} \lambda_j^* x_j^L \leq x_q^U \\
& \sum_{j} \lambda_j^* y_j^L \geq P_q^*(g)y_q^L \\
& \lambda_j^* \geq 0, \ j \in F(E_{l_{m+d}}^*)
\end{aligned}
\]

\( \tilde{P}_q^* \) can be obtained by following model:
\[ \tilde{P}_q^*(g) = \max_{\lambda_j, \tilde{P}_q(g)} \tilde{P}_q(g), \quad g = 1, \ldots, l_0 - 1 \]

\[ \begin{align*}
\text{s.t.} \quad & \sum_{j \in \mathcal{F}(E_{i_q}^g)} \lambda_j \tilde{x}_j \leq \tilde{x}_q, \\
& \sum_{j \in \mathcal{F}(E_{i_q}^g)} \lambda_j \tilde{y}_j \geq \tilde{P}_q(g) \tilde{y}_q, \\
& \lambda_j \geq 0, \quad j \in \mathcal{F}(E_{i_q}^{++}).
\end{align*} \]  

(9)

**Theorem 3.** \( P_q^+ (g) \leq \tilde{P}_q^*(g) \leq P_q^- (g) \) that \( P_q^+ (g), \tilde{P}_q^*(g) \) and \( P_q^- (g) \) are obtained from (7), (9), and (8).

**Proof.** First we show that \( \tilde{P}_q^*(g) \leq P_q^- (g) \). For this purpose, we choose the optimal solution of the of model (9); At the same time with we have:

\[ \begin{align*}
\left\{ \begin{array}{l}
\sum_{j \in \mathcal{F}(E_{i_q}^g)} \lambda_j^* \tilde{x}_j \leq \tilde{x}_q \Leftrightarrow \sum_{j \in \mathcal{F}(E_{i_q}^g)} \lambda_j^* x_j^L \leq \sum_{j \in \mathcal{F}(E_{i_q}^g)} \lambda_j^* \tilde{x}_j \leq \tilde{x}_q \leq x_q^U \\
\sum_{j \in \mathcal{F}(E_{i_q}^g)} \lambda_j^* \tilde{y}_j \geq \tilde{P}_q^*(g) \tilde{y}_q \Leftrightarrow \sum_{j \in \mathcal{F}(E_{i_q}^g)} \lambda_j^* y_j^U \geq \sum_{j \in \mathcal{F}(E_{i_q}^g)} \lambda_j^* \tilde{y}_j \geq \tilde{P}_q^*(g) \tilde{y}_j \geq \tilde{P}_q^*(g) y_q^L \\
\sum_{j \in \mathcal{F}(E_{i_q}^g)} \lambda_j^* x_j^L \leq x_q^U \\
\sum_{j \in \mathcal{F}(E_{i_q}^g)} \lambda_j^* y_j^U \geq \tilde{P}_q^*(g) y_q^L
\end{array} \right. 
\end{align*} \]

So \( (\lambda^*, \tilde{P}_q^*(g)) \) is a feasible solution for model (7) and since the objective function of the model (7) has been maximized the theorem is proved, \( \tilde{P}_q^*(g) \leq P_q^- (g) \).

In the same way we can prove that; \( P_q^+ (g) \leq \tilde{P}_q^*(g) \).

\[ \begin{align*}
P_q^+ (g + 1) & > P_q^+ (g) \\
\tilde{P}_q^*(g + 1) & > \tilde{P}_q^*(g) \\
P_q^-(g + 1) & > P_q^- (g)
\end{align*} \]

We have; \( P_q^+ (g) \leq P_q^- (g) \).

**Definition 2.** \( P_q^+ (g) \) is called the (output-oriented) g-degree progress of DMU\( _q \) from a specific level \( E_{i_q}^{++} \). When DMU\( _q \) is in the best condition and DMUs in the evaluation context \( l_0 - g \); \( g = 1, \ldots, l_0 - 1 \) are in their worst condition.
\( \hat{P}_q^*(g) \) is called the (output-oriented) g-degree progress of DMU\(_q\) from a specific level \( E_{l_o}^{++} \).

when

\[
\begin{align*}
    x_q^L &\leq \tilde{x}_q^L \leq x_q^U, \\
    y_q^L &\leq \tilde{y}_q^L \leq y_q^U,
\end{align*}
\]

is and \( \text{DMU}_j \cdot j \in F(E_{l_o-g}^{++}) \) in the evaluation context \( l_o - g \geq 1, \ldots, l_o - 1 \) are in

\[
\begin{align*}
    x_j^L &\leq \tilde{x}_j^L \leq x_j^U, \\
    y_j^L &\leq \tilde{y}_j^L \leq y_j^U.
\end{align*}
\]

\( \hat{P}_q^- (g) \) is called the (output-oriented) g-degree progress of DMU\(_q\) from a specific level \( E_{l_o}^{++} \).

When \( \text{DMU}_q \) is in its own worst condition and DMUs in the evaluation context \( l_o - g \geq 1, \ldots, l_o - 1 \) are in their own worst condition.

Each efficient frontier, \( E_{l_o-g}^{++} \), includes a possible purpose for a particular DMU in \( E_{l_o}^{++} \) to improve its performance. Here the progress is a level-by-level improvement. A larger

\[
\begin{align*}
    P_q^+ (g) & \quad \text{Shows more progress for DMU}_q. \quad \text{Thus, a smaller value of}
    \hat{P}_q^*(g) \quad \text{is preferred.}
\end{align*}
\]

3 Context-dependent DEA with value judgment

Both attractiveness and progress are measured radially with due attention to different levels of efficient frontiers, in the previous section. The measurement does not need a priori information on the importance of the attributes (input/output) that feature the performance of DMUs. However, different properties play different roles in the evaluation of a DMU’s overall performance. Therefore, we present value judgment into the context-dependent DEA.

3.1 Incorporating value judgment into attractiveness measure

In order to incorporate value judgment into our measures of attractiveness and progress, we first specify a set of weights related to the s outputs, \( u_r \) (\( r = 1, \ldots, s \)) such that \( \sum_{r=1}^{s} u_r = 1 \).

Based upon [5], we develop the following linear programming problem for \( \text{DMU}_q = (x_q, y_q) = (x_{q_1}, \ldots, x_{q_m}, y_{q_1}, \ldots, y_{q_n}) \) in \( E_{l_o}^{++} \), \( l_o \in \{1, \ldots, L-1\} \):
\[ \Phi^*_q(d) = \max \sum_{r=1}^{\ell} u_r \Phi^r_q(d), \ d = 1, \ldots, \ell - 1 \]
\[ \text{s.t.} \sum_{j \in F(E_{i+d}^+)} \lambda^r_{ij} x^L_{ij} \leq x^U_{iq}, \ i = 1, \ldots, m, \]
\[ \sum_{j \in F(E_{i+d}^+)} \lambda^r_{ij} y^U_{ij} \geq \Phi^r_q(d) y^L_{rq}, \ r = 1, \ldots, s, \]
\[ \Phi^r_q(d) \leq 1, \ r = 1, \ldots, s, \]
\[ \lambda^r_j \geq 0, \ j \in F(E_{i+d}^+). \]

\[ (10) \]

\[ \Phi^*_q(d) = \max \sum_{r=1}^{\ell} u_r \Phi^r_q(d), \ d = 1, \ldots, \ell - 1 \]
\[ \text{s.t.} \sum_{j \in F(E_{i+d}^+)} \lambda^r_{ij} x^L_{ij} \leq x^U_{iq}, \ i = 1, \ldots, m, \]
\[ \sum_{j \in F(E_{i+d}^+)} \lambda^r_{ij} y^U_{ij} \geq \Phi^r_q(d) y^L_{rq}, \ r = 1, \ldots, s, \]
\[ \Phi^r_q(d) \leq 1, \ r = 1, \ldots, s, \]
\[ \lambda^r_j \geq 0, \ j \in F(E_{i+d}^+). \]

\[ (11) \]

\[ \Phi^*_q(d) \leq \Phi^*_q(d) \leq \Phi^*_q(d) \]
that \[ \Phi^*_q(d), \Phi^*_q(d) \text{ and } \Phi^*_q(d) \]
are obtained from \( (10), (12) \text{ and } (11) \) in order.

**Theorem 4.** \[ \Phi^*_q(d) \leq \Phi^*_q(d) \leq \Phi^*_q(d) \]
that \[ \Phi^*_q(d), \Phi^*_q(d) \text{ and } \Phi^*_q(d) \]
are obtained from \( (10), (12) \text{ and } (11) \) in order.

**Proof.** First we show that \[ \Phi^*_q(d) \leq \Phi^*_q(d). \]
For this purpose we choose the optimal solution of model \( (12) \); At the same time with we have:
\[
\begin{aligned}
\sum_{j \in F(E_{l_o}^-)} \lambda_j^* \tilde{x}_{iq} & \leq \tilde{x}_{iq} \quad \iff \quad \sum_{j \in F(E_{l_o}^+)} \lambda_j^* x_{ij}^U \leq \sum_{j \in F(E_{l_o}^+)} \lambda_j^* \tilde{x}_{iq} \leq \tilde{x}_{iq} \leq x_{iq}^U, \\
\sum_{j \in F(E_{l_o}^+)} \lambda_j^* \tilde{y}_{jq} & \geq \tilde{\Phi}_q^r(d) \tilde{y}_{rq} \quad \iff \quad \sum_{j \in F(E_{l_o}^+)} \lambda_j^* y_{ij}^U \geq \sum_{j \in F(E_{l_o}^+)} \lambda_j^* \tilde{y}_{jq} \geq \tilde{\Phi}_q^r(d) y_{rq}^L \geq \tilde{\Phi}_q^r(d) y_{rq}^L, \\
\sum_{j \in F(E_{l_o}^+)} \lambda_j^* x_{ij}^L \leq x_{iq}^U, & \\
\sum_{j \in F(E_{l_o}^+)} \lambda_j^* y_{ij}^U \geq \tilde{\Phi}_q^r(d) y_{ij}^L, & \quad r = 1, \ldots, s, \\
\end{aligned}
\]

So \((\lambda^*, \tilde{\Phi}_q^r(d), \tilde{\Phi}_q^s(d))\) is a feasible solution for model (10), and since the objective function of the model (10) has been maximized the theorem is proved, \(\tilde{\Phi}_q^r(d) \leq \Phi_q(d)\).

In the same way we can prove that; \(\Phi_q^r(d) \leq \Phi_q^s(d)\).

**Definition 3.**
\[
\bar{A}_{q}^*(d) = \frac{1}{\Phi_q^r(d)}
\]
is called the (output-oriented) value judgment (VJ) attractiveness of DMU\(q\) from a specific level \(E_{l_o}^+\). When DMU\(q\) is in the best condition and DMUs in the evaluation context \(l_o + d; d = 1, \ldots, L - l_o\) are in their best condition.

\[
\tilde{A}_{q}^*(d) = \frac{1}{\Phi_q^r(d)}
\]
is called the (output-oriented) value judgment (VJ) \(d\)-degree attractiveness of DMU\(q\) from a specific level \(E_{l_o}^+\). When \(j \in F(E_{l_o}^+)\) in the evaluation context \(l_o + d; d = 1, \ldots, L - l_o\) are in
\[
\begin{aligned}
x_q^L \leq \tilde{x}_q \leq x_q^U \\
y_q^L \leq \tilde{y}_q \leq y_q^U
\end{aligned}
\]
and DMU\(j\) is
\[
\begin{aligned}
x_j^L \leq \tilde{x}_j \leq x_j^U \\
y_j^L \leq \tilde{y}_j \leq y_j^U
\end{aligned}
\]

\[
\bar{A}_{q}^*(d) = \frac{1}{\Phi_q^r(d)}
\]
is called the (output-oriented) value judgment (VJ) \(d\)-degree attractiveness of DMU\(q\) from a specific level \(E_{l_o}^+\). When DMU\(q\) is in its own worst condition and DMUs in the evaluation context \(l_o + d; d = 1, \ldots, L - l_o\) are in their own best condition.

\[
\begin{aligned}
\bar{A}_{q}^*(d) > 1 \\
\tilde{A}_{q}^*(d) > 1
\end{aligned}
\]

It is obvious that \(\tilde{A}_{q}^*(d) > 1\). The larger score \(\tilde{A}_{q}^*(d)\) is, the more attractive the DMU\(q\)

\[
\begin{aligned}
\bar{A}_{q}^*(d) > 1 \\
\tilde{A}_{q}^*(d) > 1
\end{aligned}
\]
shows under the weights \(u_o (u=1, \ldots, s)\). Now we can rank DMUs in the same level by their VJ attractiveness scores integrated with the preferences over outputs.

We can prioritize the DMUs with higher values of the \(r_o\) th output with increasing the value of the corresponding weight \(u_o\). These user-specified weights appear the relative degree
of desirability of the outputs. The constraints of $\Phi_q^+(d) < 1$ (r = 1,…, s) is for making sure

that $\text{DMU}_q$ make itself as distinctive as possible, $\text{DMU}_q$ is not allowed to decrease some of its
outputs to achieve higher levels of other better outputs which is preferred.

Obviously, different weight combinations cause different attractiveness scores.

$$
\begin{align*}
\bar{A}_q^-(d) & \quad \Phi_q^-(d) \\
\bar{A}_q^+(d) & \quad \Phi_q^+(d)
\end{align*}
$$

(or $\bar{A}_q^+(d)$) is an overall attractiveness of $\text{DMU}_q$ in terms of outputs when the
inputs don’t change their status quo. On the other hand, the attractiveness of $\text{DMU}_q$ in terms

of each output dimension can be measured through each individual optimal value of

$\left( \frac{1}{\Phi_q^+(d)} \right)$

(r = 1,…, s).

Note that $\bar{A}_q^-(d)$ is not equal to $\sum_{r=1}^s u_r A_q^{r-}(d)$ which $\bar{A}_q^-(d) = \frac{1}{\Phi_q^+(d)}$.

$$
\begin{align*}
\bar{A}_q^-(d) & \quad \sum_{r=1}^s u_r A_q^{r-}(d) \\
\bar{A}_q^+(d) & \quad \sum_{r=1}^s u_r A_q^{r+}(d)
\end{align*}
$$

Definition 4. For $\text{DMU}_q \in E_{I_o^+}$; $I_o \in \{1,…,L\}$, the optimal value $\hat{A}_q^+(d) = \frac{1}{\Phi_q^+(d)}$ is called

the (output-oriented) VJ d-degree output-specific attractiveness measure.
Suppose \( \Phi_q(d) = y_{rq} - s_q^-(d) \) in (4) since \( \Phi_q(d) \leq 1 \),
\[
\begin{align*}
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d) \\
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d)
\end{align*}
\]
(4)
\[
\begin{align*}
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d) \\
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d)
\end{align*}
\]
(4)
\[
\begin{align*}
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d) \\
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d)
\end{align*}
\]
(4)
\[
\begin{align*}
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d) \\
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d)
\end{align*}
\]
(4)
\[
\begin{align*}
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d) \\
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d)
\end{align*}
\]
(4)
\[
\begin{align*}
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d) \\
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d)
\end{align*}
\]
(4)
\[
\begin{align*}
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d) \\
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d)
\end{align*}
\]
(4)
\[
\begin{align*}
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d) \\
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d)
\end{align*}
\]
(4)
\[
\begin{align*}
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d) \\
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d)
\end{align*}
\]
(4)
\[
\begin{align*}
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d) \\
\Phi_q(d) & \leq \Phi_q(d) \leq \Phi_q(d)
\end{align*}
\]
(4)
contexts (particular efficient frontiers $E_{i+}^{++}$). So, the output-specific attractiveness measure can identify the difference between $DMU_q \in E_{i+}^{++}$ and $E_{i+}^{++}$ in terms of a particular output. With the output-particular (or input-particular) attractiveness measures. On the other hand, if $\Phi_q^{+}(d) = 1$
\[ \Phi_q^{+}(d) = 1, \] then other DMUs in $E_{i+}^{++}$, or their combinations can also produce the amount of the $r_o$th output of $DMU_q$, say, $DMU_q$ does not show better performance with due attention to this specific output dimension. Therefore, $DMU_q$ should improve its performance on the $r_o$ output to identify itself in the future.

### 3.2 Incorporating value judgment into progress measure

Similar to the development in the previous section, we can define the output-oriented VJ progress measure:

\[
\tilde{P}_q^+(g) = \max \sum u_j p_j^+(g), \quad d = 1, \ldots, L - l_o
\]
\[ \text{s.t.} \quad \sum \lambda_j x_{ij}^r \leq x_{ij}^U, \quad i = 1, \ldots, m, \]
\[ \sum \lambda_j y_{ij}^U \geq p_j^+(g)y_{ij}^U, \quad r = 1, \ldots, s, \]
\[ P_j^+(g) \geq 1, \quad r = 1, \ldots, s, \]
\[ \lambda_j \geq 0, \quad j \in F(E_{i+}^{++}). \quad (16) \]

\[
\tilde{P}_q^+(g) = \max \sum u_j \tilde{p}_j^+(g), \quad g = 1, \ldots, L - l_o
\]
\[ \text{s.t.} \quad \sum \tilde{\lambda}_j \tilde{x}_{ij} \leq \tilde{x}_{iq}, \quad i = 1, \ldots, m, \]
\[ \sum \tilde{\lambda}_j \tilde{y}_{ij} \geq \tilde{P}_j^+(g)\tilde{y}_{ij}, \quad r = 1, \ldots, s, \]
\[ \tilde{P}_j^+(g) \leq 1, \quad r = 1, \ldots, s, \]
\[ \tilde{\lambda}_j \geq 0, \quad j \in F(E_{i+}^{++}). \quad (17) \]
**Theorem 6.** \[ F_q^* \leq P_q^* \leq \bar{P}_q^* \]

that \( \bar{P}_q^* \), \( P_q^* \) and \( P_q^* \) are obtained from (16), (18) and (17) in order.

**Proof.** First we show that \( \bar{P}_q^* \leq P_q^* \) for this purpose we choose the optimal solution of model (18); Along with we have:

\[
\begin{align*}
\sum_{j \in F(E_{w-q}^+)} \lambda_j^* x_{ij}^L & \leq \bar{x}_{ij}^L \Leftrightarrow \sum_{j \in F(E_{w-q}^+)} \lambda_j^* x_{ij}^L \leq \bar{x}_{ij}^L \leq x_{ij}^U, \quad i = 1, \ldots, m, \\
\sum_{j \in F(E_{w-q}^+)} \lambda_j^* y_{ij}^L & \leq \bar{P}_q^*(g) \bar{y}_{ij}^L \Leftrightarrow \sum_{j \in F(E_{w-q}^+)} \lambda_j^* y_{ij}^L \leq \bar{P}_q^*(g) \bar{y}_{ij}^L \leq \bar{P}_q^*(g) y_{ij}^L, \\
\sum_{j \in F(E_{w-q}^+)} \lambda_j^* y_{ij}^L & \leq x_{ij}^U, \quad i = 1, \ldots, m, \\
\sum_{j \in F(E_{w-q}^+)} \lambda_j^* y_{ij}^L & \leq \bar{P}_q^*(g) y_{ij}^L, \quad r = 1, \ldots, s,
\end{align*}
\]

So \((\lambda^*, \bar{P}_q^*)\) is a feasible solution for model (15) and since the objective function of the model (15) has been maximized so that theorem is proved, \( \bar{P}_q^* \leq \bar{P}_q^* \).

In the same way we can prove that; \( \bar{P}_q^* \leq \bar{P}_q^* \).

**Definition 6.** The optimal value \( P_q^*(g) \) is called the (output-oriented) VJ g-degree progress of \( DMU_q \) in a specific level \( E_{l+}^{++} \).

\[
\begin{align*}
P_q^*(g) \quad \text{and} \quad \bar{P}_q^*(g)
\end{align*}
\]

The larger the \( \bar{P}_q^*(g) \) is, the greater progress value is expected for \( DMU_q \). The user-specified weights show the relative degree of desirability of improvement on the individual output levels.
Suppose \( \tilde{P}_q^*(g) \) represent the optimal value of (16), (17), (18) for a specific \( g \in \{1,...,l_o-1\} \). By Zhu [4], we know that \( \sum_{j \in F(E_{i^+}^{l_o-g})} \lambda_j^r y_{iq}^U = P_q^{r^*}(g) y_{iq}^L \) holds in optimal solution for each \( r = 1,...,s \). Take into consideration the following linear programming problem:

\[
\begin{align*}
\text{Max} & \quad \sum_{r=1}^{m} S_i^-(g) , \ g = 1,...,l_o-1 \\
\text{s.t.} & \quad \sum_{j \in F(E_{i^+}^{l_o-g})} \lambda_j x_{iq}^L + S_i^-(g) = x_{iq}^U, \ i = 1,...,m, \\
& \quad \sum_{j \in F(E_{i^+}^{l_o-g})} \lambda_j y_{iq}^U = P_q^{r^*}(g) y_{iq}^L, \ r = 1,...,s, \\
& \quad S_i^-(g) \geq 0, \ i = 1,...,m, \\
& \quad \lambda_j \geq 0, \ j \in F(E_{i^+}^{l_o-g}). \quad (19)
\end{align*}
\]

\[
\begin{align*}
\text{Max} & \quad \sum_{r=1}^{m} \tilde{S}_i^-(g) , \ g = 1,...,l_o-1 \\
\text{s.t.} & \quad \sum_{j \in F(E_{i^+}^{l_o-g})} \lambda_j x_{iq}^L + \tilde{S}_i^-(g) = \tilde{x}_{iq}, \ i = 1,...,m, \\
& \quad \sum_{j \in F(E_{i^+}^{l_o-g})} \lambda_j y_{iq}^U = \tilde{P}_q^{r^*}(g) y_{iq}^L, \ r = 1,...,s, \\
& \quad \tilde{S}_i^-(g) \geq 0, \ r = 1,...,s, \\
& \quad \lambda_j \geq 0, \ j \in F(E_{i^+}^{l_o-g}). \quad (21)
\end{align*}
\]

Theorem 7. \( \text{Max} \sum_{i=1}^{m} S_i^-(g) \geq \text{Max} \sum_{i=1}^{m} \tilde{S}_i^-(g) \geq \text{Max} \sum_{i=1}^{m} S_i^-(g) \),

That \( \text{Max} \sum_{i=1}^{m} S_i^-(g) \), \( \text{Max} \sum_{i=1}^{m} \tilde{S}_i^-(g) \) and \( \text{Max} \sum_{i=1}^{m} S_i^-(g) \) are obtained from (19), (21) and (20).

Proof. Assume that \( (\lambda_j^r, \text{Max} \sum_{i=1}^{m} S_i^-(g)) \) is an optimal solution for model (19). Let ́s consider the first constraints of above models as;
\[
\sum_{j \in F(E_{i \rightarrow q}^{\L})} \lambda_j^i x_{ij}^U \geq \sum_{j \in F(E_{i \rightarrow q}^{\L})} \lambda_j^i \bar{x}_{ij} \geq \sum_{j \in F(E_{i \rightarrow q}^{\L})} \lambda_j^i x_{ij}^L \geq x_{iq}^U \geq \bar{x}_{iq} \geq x_{iq}^L, \]

and now it is easy to show that; \( \text{Max} \sum_{i=1}^{m} S_i^- (g) \geq \text{Max} \sum_{r=1}^{l_q} \bar{S}_r^-(g) \geq \text{Max} \sum_{i=1}^{m} S_i^- (g) \).

**Definition 7**. (Preferred global efficient target and preferred local efficient target) The following points:
\[
\begin{aligned}
\hat{x}_{iq}^- &= x_{iq}^U - S_i^- (g), i = 1, \ldots, m, \\
\hat{x}_{iq} &= \bar{x}_{iq} - S_i^- (g), i = 1, \ldots, m, \\
\hat{y}_r^- &= P_r^- (g) y_{rq}^L, r = 1, \ldots, s,
\end{aligned}
\]

are preferred global efficient targets for \( DMU_q \in E_{i \rightarrow q}^{\L}, l_q \in \{2, \ldots, L\} \), if \( g = l_q - 1 \); otherwise, if \( g < l_q - 1 \), they represent preferred local efficient targets,
\[
\begin{aligned}
P_q^*(g) \\
\hat{s}_i^+ (g)
\end{aligned}
\]
where \( P_q^*(g) \) are the optimal value in (16), (17) and (18), and \( \hat{s}_i^+ (g) \) present the optimal values in (19), (21) and (20).

### 4 Numerical example

Now we will consider the branches of one of the Iran’s commercial banks with 3 inputs and 5 outputs as our DMUs. The inputs are payable interest, personnel and non-performing loans and the outputs are the total sum of four main deposits, other deposits, loans granted, received interest and fee. These data were collected in 2005.

\( E_{1 \rightarrow q}^{\L} = \{17, 11, 10, 9, 8, 7, 6, 4, 1, 19\} \)

\( E_{2 \rightarrow q}^{\L} = \{18, 16, 15, 14, 5, 3, 2\} \)

\( E_{3 \rightarrow q}^{\L} = \{20, 13, 12\} \)

**Table 1** Inputs and outputs

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payable interest</td>
<td>The total sum of four main deposits</td>
</tr>
<tr>
<td>Personnel</td>
<td>Other deposits</td>
</tr>
<tr>
<td>Non-performing loans</td>
<td>Loans granted</td>
</tr>
<tr>
<td></td>
<td>Received interest</td>
</tr>
<tr>
<td></td>
<td>Fee</td>
</tr>
</tbody>
</table>
Table 2  Input − data for the 20 bank branches

<table>
<thead>
<tr>
<th>DMU_j</th>
<th>X_1^L</th>
<th>X_1^U</th>
<th>X_2^L</th>
<th>X_2^U</th>
<th>X_3^L</th>
<th>X_3^U</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5007.37</td>
<td>9613.37</td>
<td>36.29</td>
<td>36.86</td>
<td>87243</td>
<td>87243</td>
</tr>
<tr>
<td>2</td>
<td>2926.81</td>
<td>5961.55</td>
<td>18.8</td>
<td>2016</td>
<td>9945</td>
<td>12120</td>
</tr>
<tr>
<td>3</td>
<td>8732.7</td>
<td>17752.5</td>
<td>25.74</td>
<td>27.17</td>
<td>47575</td>
<td>50013</td>
</tr>
<tr>
<td>4</td>
<td>945.93</td>
<td>1966.39</td>
<td>20.81</td>
<td>22.54</td>
<td>19292</td>
<td>19753</td>
</tr>
<tr>
<td>5</td>
<td>8487.07</td>
<td>17521.66</td>
<td>14.16</td>
<td>14.8</td>
<td>3428</td>
<td>3911</td>
</tr>
<tr>
<td>6</td>
<td>13759.35</td>
<td>27359.36</td>
<td>19.46</td>
<td>19.46</td>
<td>13929</td>
<td>15657</td>
</tr>
<tr>
<td>7</td>
<td>587.69</td>
<td>1205.47</td>
<td>27.29</td>
<td>27.48</td>
<td>27827</td>
<td>29005</td>
</tr>
<tr>
<td>8</td>
<td>4646.39</td>
<td>9599.61</td>
<td>24.52</td>
<td>25.07</td>
<td>9070</td>
<td>9983</td>
</tr>
<tr>
<td>9</td>
<td>1554.29</td>
<td>3427.89</td>
<td>20.47</td>
<td>21.59</td>
<td>412036</td>
<td>413902</td>
</tr>
<tr>
<td>10</td>
<td>17528.31</td>
<td>36297.54</td>
<td>14.84</td>
<td>15.05</td>
<td>8638</td>
<td>10229</td>
</tr>
<tr>
<td>11</td>
<td>2444.34</td>
<td>4955.78</td>
<td>20.42</td>
<td>20.54</td>
<td>500</td>
<td>937</td>
</tr>
<tr>
<td>12</td>
<td>7303.27</td>
<td>14178.11</td>
<td>22.87</td>
<td>23.19</td>
<td>16148</td>
<td>21353</td>
</tr>
<tr>
<td>13</td>
<td>9852.15</td>
<td>19742.89</td>
<td>18.47</td>
<td>21.83</td>
<td>17163</td>
<td>17290</td>
</tr>
<tr>
<td>14</td>
<td>4540.75</td>
<td>9312.24</td>
<td>22.83</td>
<td>23.96</td>
<td>17918</td>
<td>17964</td>
</tr>
<tr>
<td>15</td>
<td>3039.58</td>
<td>6304.01</td>
<td>39.32</td>
<td>39.86</td>
<td>51582</td>
<td>55136</td>
</tr>
<tr>
<td>16</td>
<td>6585.81</td>
<td>13453.58</td>
<td>25.57</td>
<td>26.52</td>
<td>20975</td>
<td>23992</td>
</tr>
<tr>
<td>17</td>
<td>4209.18</td>
<td>8603.79</td>
<td>27.59</td>
<td>27.95</td>
<td>41960</td>
<td>43103</td>
</tr>
<tr>
<td>18</td>
<td>1015.52</td>
<td>2037.82</td>
<td>13.63</td>
<td>13.93</td>
<td>18641</td>
<td>19354</td>
</tr>
<tr>
<td>19</td>
<td>5800.38</td>
<td>11875.39</td>
<td>27.12</td>
<td>27.26</td>
<td>19500</td>
<td>19569</td>
</tr>
<tr>
<td>20</td>
<td>119948</td>
<td>120208</td>
<td>14943</td>
<td>17495</td>
<td>297674</td>
<td>27934.19</td>
</tr>
</tbody>
</table>

Table 3  Output − data for the 20 bank branches

|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
Table 4 The attractiveness scores in $E^1$ when $E^2$ and $E^3$ is chosen as the evaluation context

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Level</th>
<th>$A^+_q (1)$</th>
<th>$A^+_q (2)$</th>
<th>$A^+_r (1)$</th>
<th>$A^+_r (2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>8.474</td>
<td>2.032</td>
<td>18.867</td>
<td>6.25</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>17.857</td>
<td>3.021</td>
<td>55.555</td>
<td>16.129</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>3.267</td>
<td>1.557</td>
<td>6.289</td>
<td>2.949</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>12.987</td>
<td>1.680</td>
<td>58.823</td>
<td>12.345</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>22.222</td>
<td>5.714</td>
<td>50</td>
<td>22.222</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>12.195</td>
<td>2.298</td>
<td>55.555</td>
<td>17.857</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>24.390</td>
<td>9.433</td>
<td>34.482</td>
<td>22.222</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>166.666</td>
<td>40</td>
<td>333.333</td>
<td>17.857</td>
</tr>
<tr>
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<td>17.241</td>
<td>5.524</td>
<td>30.030</td>
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</tr>
<tr>
<td>19</td>
<td>1</td>
<td>13.333</td>
<td>3.861</td>
<td>45.454</td>
<td>13.888</td>
</tr>
</tbody>
</table>

Table 5 The attractiveness scores in $E^2$ when $E^1$ and $E^3$ is chosen as the evaluation context

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Level</th>
<th>$A^+_q (1)$</th>
<th>$A^+_q (1)$</th>
<th>$P^+_q (1)$</th>
<th>$P^+_q (1)$</th>
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<td>1.706</td>
<td>13.510</td>
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</tr>
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</tr>
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<td>0.768</td>
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</table>

Table 6 Output-specific attractiveness scores in $E^1$ (in the best condition) when $E^2$ is chosen as the evaluation context

<table>
<thead>
<tr>
<th>DMUs</th>
<th>The total sum of four main deposits</th>
<th>Other deposits</th>
<th>Loans granted</th>
<th>Received interest</th>
<th>Fee</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
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<td>1.298</td>
<td>4.405</td>
<td>4.608</td>
<td>3.300</td>
</tr>
<tr>
<td>8</td>
<td>1.941</td>
<td>62.5</td>
<td>4.716</td>
<td>4.081</td>
<td>14.705</td>
</tr>
<tr>
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</table>
Table 7 Output-specific attractiveness scores in $E_1$ (in the worst condition) when $E_2$ is chosen as the evaluation context

<table>
<thead>
<tr>
<th>DMUs</th>
<th>The total sum of four main deposits</th>
<th>Other deposits</th>
<th>Loans granted</th>
<th>Received interest</th>
<th>Fee</th>
</tr>
</thead>
<tbody>
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<tr>
<td>6</td>
<td>1.706</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
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<tr>
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<td>5.714</td>
<td>23.255</td>
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</tr>
<tr>
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</tbody>
</table>

5 Conclusions

To measure the attractiveness and progress of DMUs with due attention to a given evaluation context, context-dependent DEA is developed.

Different levels of efficient frontiers are used as evaluation contexts instead of the traditional first-level efficient frontier. In the basic form of DEA, adding or omitting inefficient DMUs does not change the efficiencies of the existing DMUs and the efficient frontier, but the context-dependent DEA, adding or omitting inefficient DMUs changes the performance of both efficient and inefficient DMUs, i.e., the context-dependent DEA performance depends on not only the efficient frontier, but also the inefficient DMUs. DEA is made by this change more powerful and allowed to locally and globally recognize better choices. Value judgment is integrated into the context-dependent DEA through a particular set of weights showing the preferences over different output (or input) measures. Specially, the attractiveness measure can be used to (i) recognize DMUs that have better performance and (ii) can rank DEA efficient DMUs.

In our application, we are able to differentiate the performance of choices. We have not considered the section 3.3, in [5], that has wrongly obtained the weights.

The input-oriented context-dependent DEA is considered as same as the output-oriented context dependent DEA, though in this case, what we obtain from the attractiveness model is the attractiveness value, and what we obtain from the progress model is the inverted form of the progress value.
References