International Journal of Applied Operational Research
Vol. 2, No. 2, pp. 25-36, July 2012

Journal homepage: www.ijorlu.ir

Cost Analysis of Fuzzy Queuing Systems

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Received: January 30, 2012 ; **Accepted:** June 11, 2012

Abstract Regarding the fact that getting a suitable combination of the human resources and service stations is one of the important issues in the most service and manufacturing environments, In this paper, we have studied the two models of planning queuing systems and its effect on the cost of the each system by using two fuzzy queuing models of M/M/1 and M/E2/1. In the first section, we have compared two different fuzzy queuing models based on the costs of each model and fuzzy ranking methods are used to select optimal model due to the resulted complexity. This paper results in a new approach for comparing different queuing models in the fuzzy environment (regarding the obtained data from the real conditions) that it can be more effective than deterministic queuing models. Also a sensitivity analysis is carried out to help the decision maker in selecting the optimal model.

Keywords Cost Analysis, Fuzzy Queuing Systems, Fuzzy Ranking Methods

1 Literature Review

The optimal allocation of the human resources is one of the most important factors for decreasing the total cost in many existed systems of the real world such as manufacturing, producing, servicing, industrial, and etc. It can be said that generally the managers are looking forward to have a suitable combination of the human resources, the operator's skills to control the service stations. To achieve this goal, we can survey the different methods of the human resource allocation to each service station as well as the cost of each one. The queuing theory helps us to find the cost of different combinations of the human resources and service stations. The staff's allocation to the service stations and also its effects on the systems can be surveyed with different models of the queuing theory, and they can analyzed to find the best situation to reduce the production cost regarding the performance measures of the queuing system.

In this paper, the two different combinations of the human resources and service stations are analyzed in order to determine which assignment of operators leads to a minimum cost model. The first model is made of two series service stations of M/M/1 settled in a queue in which two service stations present a service to customers which mean of the servicing time in each stations is equal. The second model is Erlang model includes two stages and has a server which is able to do both services. These models are studied in the works of Hillier and Lieberman in [1] and Taha in [2].

In the queuing theory, it is usually assumed that the time between the two consecutive arrivals and the servicing time follow a special probability distribution. However, in the real

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world, this type of information is obtained using qualitative data and expressed by words like quick, medium, and slow rather than the probability distribution. It can be said that fuzzy queuing models are more realistic and practical than the classical ones.

In this paper, we also have considered the queuing system in a fuzzy environment to get a better analysis. The fuzzy queuing model is used by the experts like Parade [3], Li and Lee [4], Buckley [5], Negi and Lee [6], Kaoetal [7], Buckley et al. [8], Chen [9, 10] and Ke and lin[11]. The papers [4] and [8] can be referred to illustrate some samples which considered the time between the two consecutive arrivals and the servicing time based on the possibility theory in the fuzzy theory and used it to determine the optimized numbers of the servers. Kao et al. [7] also has used parametric planning method using α-cut approach to analyze the fuzzy queue system.

Chen [9] has presented a model using the fuzzy membership function and considered the fuzzy arrival rate and fuzzy cost regarding the fuzzy parametric planning method. Chen [10] has designed a non-linear numerical mixed integer planning with the binary variables using the possibility theory for the group arrival model with different numbers in each group in which the arrival time and the service time both are fuzzy numbers. In this paper, to survey the costs of different assignment models and to choose the best model in the uncertain situation, we have assumed that the time between the two consecutive arrivals and servicing time are fuzzy numbers and based on Zadeh's theory in [12] and similar to the models used in the articles [13] and [14], the cost measure for each model is evaluated.

Also the fuzzy ranking methods of Decooman [15], Liou and Wang [16] and Nakamura [17] are used for decision making in an uncertain environment. According to the researches done in this field, results obtained by Nakamura [17] are more precise than the other models therefore, in this paper, Nakamura method [17] has been used for fuzzy ranking.

The rest of the article follows as: in the second chapter, we deal with the presented models, their operational measures, and also the models' cost. In the third chapter, it is explained how to convert the queuing system arrival data into the fuzzy numbers and the method to change these numbers into the fuzzy numbers will be explained. It is also explained that each fuzzy queuing model can be considered as a classical model of queuing theory and finally we will discuss the decision making method by explaining the ranking method of fuzzy number. In the fourth chapter, a practical example is illustrated and Nakamura [17] method is used to find the model with the minimum cost due to the overlap of the fuzzy results. At the end, a sensitivity analysis of the both models' costs is performed in the fifth chapter. We conclude the paper in section six.

2 Problem definition

In this paper, we have studied two different models of planning a queuing system and each of them is explained completely and each one's cost is evaluated.

2.1 First model

In this model, two service stations are situated in series as shown in Fig. (1). the servicing time in the system is in fact the sum of servicing time in both stations.

It is assumed that the time between arrivals to the system follows an exponential distribution with parameter λ and the service time in every station follows an exponential distribution with parameter μ_1. The number of the servers in each station is one and the mean service time is $\frac{1}{\mu_1}$ in each station. In a series system, all customers are demanding for the both services. Moreover, the order of getting services is also the same for the all customers. Each customer waits in the queue of first station after entering to the system. After getting service in the first station, he joins to the second station and at last after getting service in the second station, he gets out of the system.

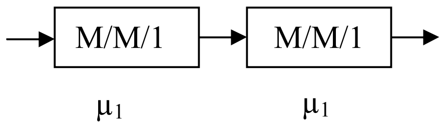

Fig. 1 First model including two working stations

In this system, it is assumed that the system capacity in all stations is unlimited. In this case, every server can be considered as a M/M/1 queuing model where the number of customers' arrival in each station follows a Poisson distribution with parameter λ. Regarding the point that each service stations can be considered as independent stations, the computation of operational performance measure is done easily. In the other words, if L_i is the expected mean of the number of the customers in the service station i of queuing system, then based on M/M/1 model, it can be calculated as following:

$$L_i = \frac{\lambda_i}{\mu_i - \lambda_i} \tag{1}$$

Regarding the point that probability distribution of number of arrivals and probability distribution of service time for both servers are the same, so it is concluded that:

$$L_1 = L_2 = \frac{\lambda}{\mu_1 - \lambda} \tag{2}$$

and the expected mean of the number of the customer in the whole system in long term, L is calculated as follow:

$$L = L_1 + L_2 = 2L_1 = 2L_2 \tag{3}$$

2.2 Second Model

In this model, the customers arrive based on the Poisson distribution with rate λ. It is assumed that two service stations have been replaced with a single service station that can perform two services consecutively. Since this single service station presents two services to each customer thus it is assumed that the total servicing time follows an Erlang distribution (two services in one stage). If the service time follows an Erlang distribution with 2 degrees of freedom with the mean $\frac{1}{\mu_2}$, it can be assumed that in this system, the service station includes two consecutive stages in which the service time in each stage follows an exponential distribution with the parameter $2\mu_2$ (Fig. 2).

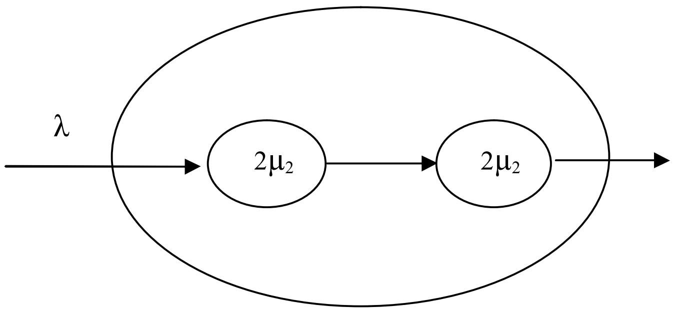

Fig. 2 The service model in Erlang service time

At first, the customer passes the first stage which its service time has an exponential distribution with the mean $\frac{1}{2\mu_2}$. Then, he enters to the second stage which its service time has an exponential distribution with the mean $\frac{1}{2\mu_2}$ (the expected mean of servicing time). The customer leaves the system after entering both stages. Because the two services are not separable practically (They are only divided into two stages mathematically) therefore there will be only one customer in one of the service stages and the other stage is without any customer.

If $L^{'}$ is the expected mean of the number of the customers who get the service in this system, based on $M/E_2/1$ queuing model, it can be calculated as follow:

$$L^{'} = \frac{1+r}{2r} \times \frac{\lambda^2}{\mu_2(\mu_2 - \lambda)} + \frac{\lambda}{\mu_2} \tag{4}$$

Since r = 2 in this model, $L^{'}$ will be as follow:

$$L^{'} = \frac{3}{4} \times \frac{\lambda^2}{\mu_2(\mu_2 - \lambda)} + \frac{\lambda}{\mu_2} \tag{5}$$

2.3 Two Models Costs Function

After introducing the model and its parameters, it is needed to optimize the queuing system. In the other words, it is needed to search for a system that the sum of its costs is minimum.

2.3.1 The Cost Function of the First Model

The cost of first model includes the expected mean of the cost of idle servers $(C_1(m - L + L_q))$, the expected mean of the cost of the servers who present service $(C_2(L - L_q))$, the expected mean of the cost of the customers' time consuming in queue in a unit of time $(C_5 L_q)$, and the total cost of the customers' time consuming who are getting the service $(C_6(L - L_q))$ in which we have:

C_1: Cost of keeping an idle server in a unit of time.
C_2: Operational costs related to a server who presents a service.
C_5: Cost of the customers' time consuming in queue in a unit of time.
C_6: Cost of the customers' time consuming while getting service in a unit of time.
L : Expected mean of the number of the customers in a system in long term.
L_q : Expected mean of the number of the customers in queue in long term.
m : Number of the servers.

The function of the total cost as the sum of the mentioned costs is calculated as follow:

$$C = C_1(m - L + L_q) + C_2(L - L_q) + C_5 L_q + C_6(L - L_q) \tag{6}$$

Regarding the point that $C_1 = C_2$ in this model, the sum of $C_1(m - L + L_q)$ and $C_2(L - L_q)$ is written in summary, $C_1 m$ and also considering the point that $C_5 = C_6$, the sum of $C_5 L_q$ and $C_6(L - L_q)$ is shortly written as $C_6 L_q$ therefore the cost function of the first system is as follow:

$$C(m) = C_1 m + C_6 L \quad , \quad m = 2 \tag{7}$$

2.3.2 The cost functions of the second model

The costs of this model similar to the first model include four types of costs that their sum as a cost function is explained as follow:

$$C(m) = C'_1 m' + C'_6 L' \quad , \quad m = 1 \tag{8}$$

C'_1 : Cost of server in a unit of time.

C'_6: Cost of the customers' time consuming in a unit of time.
L' : Expected mean of the number of the customers in a system in long term.

3 The fuzzy Model

The system data which are gathered from the real world problems always has a sort of ambiguity. The reason is that in most of the systems in the real world, there is some information such as the arrival rate of the customers into the system or the rate of servicing can be expressed with the linguistic data, and it cannot be suitably expressed with the probability distributions. The "fuzzy set" is a tool to consider these ambiguities. On the other hand, to determine the parameters of the model in the real world, generally, the experts' ideas or sampling data can be used. It can be claimed that there is a sort of ambiguity in the both mentioned methods. In the first case (experts' idea), the ambiguity is due to the lack of preciseness and enough specialty. In the second case (sampling data), the ambiguity is due to the lack of enough sampling data.

Because of this ambiguity, in this paper, it is preferred using the fuzzy parameters instead of the certain ones for the suggested queue model. To estimate a parameter as a fuzzy number and to change the input data (for instance the customers' arrival rate (λ) and the servicing rate (μ)) into the fuzzy numbers, the method of Buckley and Qu method [18] is used. The brief description of this method is as follows: suppose θ is the unknown parameter of the model and its value must be estimated from the sample of $X_1, X_2, \ldots, X_n$. First its (1-β)100% confidence interval $[\theta_1(\beta), \theta_2(\beta)]$ is determined as follows,

$$[\theta_1(\beta), \theta_2(\beta)] = [\overline{X} - Z_{\beta/2}\frac{\sigma}{\sqrt{n}}, \overline{X} + Z_{\beta/2}\frac{\sigma}{\sqrt{n}}]$$

where $\overline{X}, \sigma$ are respectively the average and the standard deviation of X_i's and $Z_{\beta/2}$ is a standard normal variable where the value of cumulative normal distribution function at this point is $1 - \beta/2$. In the fuzzy estimation concept, the different confidence intervals for all different values of β's $(0 < \beta < 1)$ are generated and the fuzzy number is concluded by considering these different intervals.

According to Zadeh development theory [12] and by the use of the possibility theory concepts and fuzzy Morkov chain [19], li and liu [3] proposed that every fuzzy queuing model could be considered as a classical queuing theory model by considering following changes, the probability distribution function of the time between two consecutive arrivals which is assumed to follow the exponential distribution with the parameter λ, in the fuzzy environment, the arrival rate is considered to be $\tilde{\lambda}$ which is an approximation of the mean of its possibility distribution and also the servicing rate which is assumed to be the parameter μ, in the fuzzy environment, it is considered to be the fuzzy number $\tilde{\mu}$ which $\tilde{\mu}$ is the mean of its possibility distribution in the fuzzy environment.

Accordingly, all the functions (models) can be defined for the fuzzy parameters of $\tilde{\lambda}$ and $\tilde{\mu}$ by

$$\mu_{f(\tilde{\lambda},\tilde{\mu})}(z) = \sup_{\lambda,\mu\in R} \{\mu_{\tilde{\lambda}}(\lambda) \wedge \mu_{\tilde{\mu}}(\mu) \,|\, z = f(\lambda, \mu)\} \tag{9}$$

Because of the complexity of defining the membership functions in this method, in this paper, the method of Buckley and Qu [18] is used that is based on the concept of α-cut as follows.

$$L_1(\alpha) = \min\{ L \,|(\lambda, \mu, \alpha)\} \tag{10}$$

$$L_2(\alpha) = \max\{ L \,|(\lambda, \mu, \alpha)\} \tag{11}$$

$$L(\alpha) = [L_1(\alpha), L_2(\alpha)] \tag{12}$$

$\widetilde{L}$ is determined by above equations and based on the fuzzy value of $\widetilde{L}$, the fuzzy cost of each model is determined

Since the superiority of each model cannot be studied intuitively, to decide in an uncertain environment and to compare the different decisions in the fuzzy environment, ranking methods such as Decooman [15], Liou and Wang [16] and Nakamura [17] have been proposed. Regarding the activities done in this field, the obtained results of Nakamura method [17] are more precise than the other models, so for ranking the fuzzy costs, this method is used in this paper.

3.1 Nakamura method

This method defines a fuzzy preference rate, $\mu_N(\widetilde{A},\widetilde{B})$ for the fuzzy alternative pairs, $\widetilde{A}$ and $\widetilde{B}$ with the following membership function:

$$\mu_N(\widetilde{A},\widetilde{B}) = \begin{cases} \dfrac{(1-\beta)T_1 + \beta T_3}{T_\beta} & if \;\; T_\beta \neq 0 \\ \dfrac{1}{2} & if \;\; T_\beta = 0 \end{cases} \tag{13}$$

where,

$$T_\beta = (1-\beta)(T_1 + T_2) + \beta(T_3 + T_4) \tag{14}$$

$$T_1 = \int_{\{\alpha \,|\, \underline{A}_\alpha > \underline{B}_\alpha\}} [\underline{A}_\alpha - \underline{B}_\alpha] d\alpha \tag{15}$$

$$T_2 = \int_{\{\alpha \,|\, \underline{A}_\alpha < \underline{B}_\alpha\}} [\underline{B}_\alpha - \underline{A}_\alpha] d\alpha \tag{16}$$

$$T_3 = \int_{\{\alpha \,|\, \overline{A}_\alpha > \overline{B}_\alpha\}} [\overline{A}_\alpha - \overline{B}_\alpha] d\alpha \tag{17}$$

$$T_4 = \int_{\{\alpha \,|\, \overline{A}_\alpha < \overline{B}_\alpha\}} [\overline{B}_\alpha - \overline{A}_\alpha] d\alpha \tag{18}$$

where $\underline{A}_\alpha$ and $\overline{A}_\alpha$ are respectively the lower and upper bound of α-cut of fuzzy number A. Proposed method of Nakamura needs more calculations in comparison to the other models, but it is more precise [17]. β is a criterion to evaluate risk. If the decision maker is interested in risk, β will be considered more than 0.5 and if he is not interested in risk, β will be less than 0.5 and in the case of neutrality, it will be equal to 0.5.

If $\mu_N(\widetilde{A},\widetilde{B})>\frac{1}{2}$, $\widetilde{A}$ would be preferred to $\widetilde{B}$ and If $\mu_N(\widetilde{A},\widetilde{B})<\frac{1}{2}$, $\widetilde{B}$ would be preferred to $\widetilde{A}$ and if $\mu_N(\widetilde{A},\widetilde{B})=\frac{1}{2}$, there is no preference between $\widetilde{A}$ and $\widetilde{B}$.

4 Numerical example

Regarding the point that nowadays the most important problem of factories is to determine a suitable combination of the human resources, the staff's abilities and the service stations, In this paper, a queuing optimization model is considered to select between the two mentioned models and we wants to decide about employing the required human resources to control the service stations in order to get optimized solution according to the cost function. In the first model, the two service stations are set in series and each system is managed by an operator independently. Accordingly, the first stage output must enter the second service station and go out of it after receiving the second service. To estimate each model parameter, 30 samples are gathered from the real world and their fuzzy value is determined using the mentioned methods in the third chapter of this article.

The obtained diagrams for each parameter of this model are as follow,

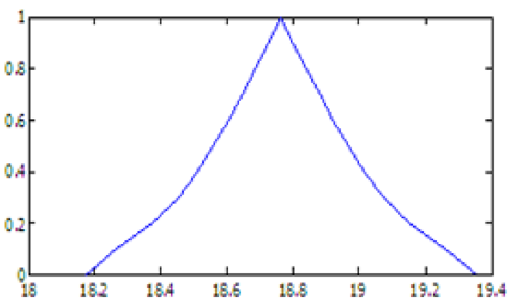

Fig. 3 The fuzzy number of the servicing rate for the first model (μ_1)

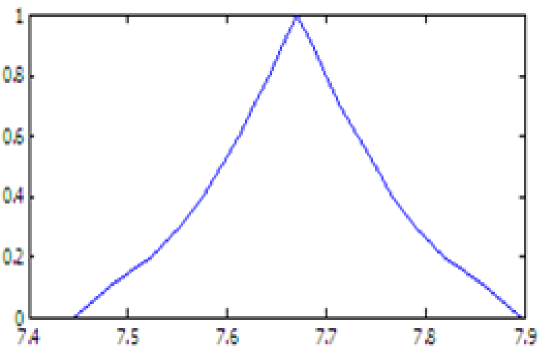

Fig. 4 The fuzzy number of the arrival rate for the first model (λ)

In the first model, the number of arrivals into the system follows a Poisson distribution with the parameter λ and the time of servicing follows an exponential distribution with the parameter μ_1 as shown in Fig. 3 and Fig. 4. The operators have the ability to work with one of the service stations. Fuzzy cost of each operator is C_1 = (15000, 17000, 2000). The customers' time consuming cost in the system is C_6 that is considered by the fuzzy number (19000, 24000, 27000).

In the second model (M / E_2 / 1), it is considered that the number of the arrivals into the system follows a Poisson distribution with the parameter λ (parameter λ is considered the same in both models). Since in this model, service stations present two services successively to the customers and time of each service follows an exponential distribution thus it is assumed that the servicing time to the customers follows an Erlang distribution with parameters 2 and $2\mu_2$ which its mean value is $\frac{1}{\mu_2}$.The parameter μ_2 is shown in Fig. 5. In this model, operator has the ability to work with both service stations and because of his ability, the operator cost is more than the first one that is $C_1^{'}$= (25000, 32000, 35000). The cost of time consuming in the system for the customer has been considered equal to the first model.

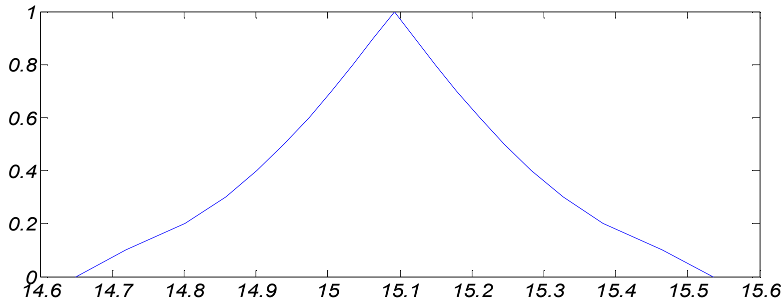

Fig. 5 The fuzzy number of the servicing rate for the second model (μ_2)

After estimating λ, μ_2 and μ_1, we obtained fuzzy value of L and L' for each of the models as follow,

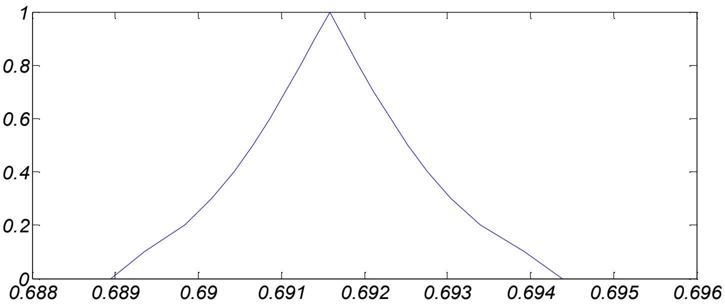

Fig. 6 The obtained fuzzy number for the expected mean of the number of customers in the first model (L)

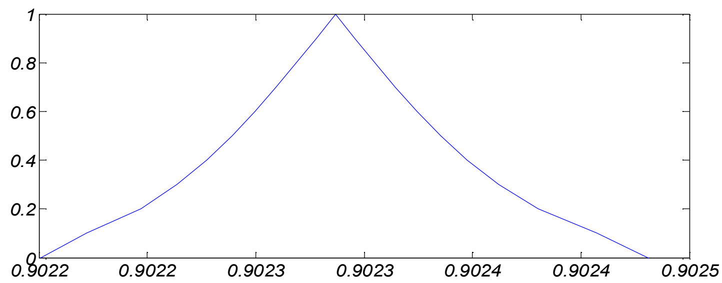

Fig. 7 obtained fuzzy number for the expected mean of the number of the customers in the second model (L')

Now, by getting all the parameters and according to the formulas 7 and 8, the cost function of first model is shown by fuzzy number A and the cost function of second model is shown by fuzzy number B in Fig. 8.

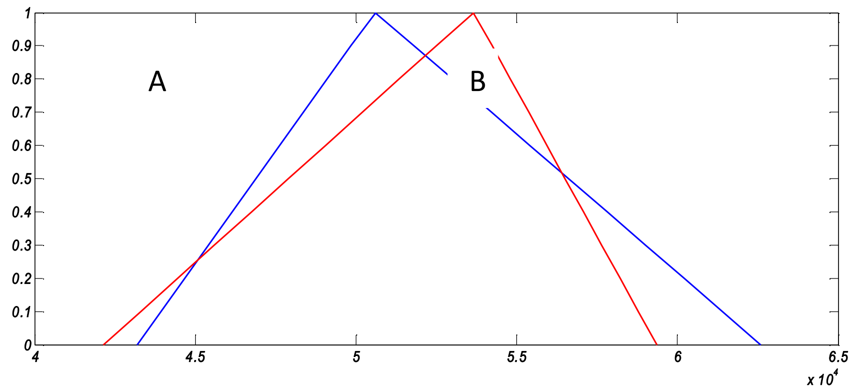

Fig. 8 The costs functions of the first and the second model.

By comparing the total cost for the both models, it is observed that the result of cost analysis leads to the fuzzy values for costs of each model that has overlapped with each other. To decide about the case which provides the minimum cost, the decision making techniques in uncertain environments have been applied. To compare these costs, we use the fuzzy ranking method of Nakamura [17] which has been introduced in the last section. Therefore, regarding the above diagrams and calculation of the area of the surface below the curve, there will be a preference fuzzy rate $\mu_N(\widetilde{A},\widetilde{B})$ for the fuzzy alternatives $\widetilde{A}$ and $\widetilde{B}$ that is calculated using MATLAB software as follows:

$$T_1 = \int_{\{\alpha \mid \underline{A}_\alpha > \underline{B}_\alpha\}} [\underline{A}_\alpha - \underline{B}_\alpha] d_\alpha = 946.8983$$

$$T_2 = \int_{\{\alpha \mid \underline{A}_\alpha < \underline{B}_\alpha\}} [\underline{B}_\alpha - \underline{A}_\alpha] d_\alpha = 131.8184$$

$$T_3 = \int_{\{\alpha \mid \overline{A}_\alpha > \overline{B}_\alpha\}} [\overline{A}_\alpha - \overline{B}_\alpha] d_\alpha = 838.2469$$

$$T_2 = \int_{\{\alpha \mid \overline{A}_\alpha < \overline{B}_\alpha\}} [\overline{B}_\alpha - \overline{A}_\alpha] d_\alpha = 544.2251$$

$$T_\beta = (1-\beta)(T_1 + T_2) + \beta(T_3 + T_4) = 1230.594$$

$$\mu_N(\widetilde{A},\widetilde{B}) = \frac{(1-\beta)T_1 + \beta T_3}{T_\beta} = \frac{\frac{1}{2}(946.8983 + 838.2469)}{1230.594} = 0.73$$

Regarding the point that decision maker is neutral thus β is considered to be 0.5 and since $\mu_N(\widetilde{A},\widetilde{B}) > \frac{1}{2}$ therefore A is preferred to B.

5 Sensitivity Analysis

To analyze the sensitivity of the models, a sensitivity analysis is performed based on the different value for the costs $C_1, C_1^{'}$ and C_6 and their results are shown in the Table 1.

Table 1 The sensitivity analysis of the two models

rows	C_1	C_6	$C_1^{'}$	$\mu_N(\widetilde{A},\widetilde{B})$	Selected model
1	(15000, 17000, 22000)	(19000, 24000, 27000)	(25000,32000, 35000)	0.73	B
2	(15000, 17000, 22000)	(24700, 31200, 35100)	(25000,32000, 35000)	0.53	B
3	(15000, 17000, 22000)	(24700, 31200, 35100)	(28750,36800, 40250)	0.509237	B
4	(19500, 22100, 28600)	(24700, 31200, 35100)	(28750,36800, 40250)	0.519237	B

In the first row of Table 1, the value of costs which are assumed in the numerical example is mentioned. In the second row, we analyzed the cases of increasing 10, 20 and 30 percent of the customers' time consuming cost (C_6) in both directions and it is concluded that the fuzzy rate preference is almost the same for the both models when we increase the cost of the customers' time consuming approximately 30 percent. In the third row, the different conditions of positive and negative shifts in the operator cost presented in the second model are analyzed and the neutral condition is resulted when we increase the operator cost approximately 15 percent, in the fourth row, the cost in these two models will be equal when an increase of approximately 30 percent occurs in the operator cost of the first model.

Regarding the obtained fuzzy numbers, selecting both models won't have any difference while getting the fuzzy preference rate of 0.5 considering the optimized models in the case of small shifts in each of the mentioned parameters.

6 Conclusions

The fuzzy queuing model has more applicability in the real environments than the exact systems although there are a few papers in the field of using these models and also their comparisons. Some papers, done in this field, have applied only a fuzzy queuing model and there is not any comparison between the different fuzzy queuing models. In this paper, we compared two practical systems to study the different conditions of the operator's allocation in the queuing systems in real environment. To analyze the conditions more precise and more practical, it is assumed that the rate of arrivals and the servicing rate are fuzzy data also it is considered that the system costs are fuzzy numbers according to the experts experience to express the uncertain condition in the system completely. To study the model validity, the mentioned models are analyzed in different scenarios of the costs by the sensitivity analysis on the models' fuzzy costs. Regarding the conditions of the production system, the achieved results can help the decision maker in choosing the number of staff and their qualification, according to the costs of the system.

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