

# Production planning considering undesirable outputs-A DEA based approach

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**Abstract** While the conventional DEA based production plans aim to minimize all the inputs consumption and maximize all the outputs production, there are many real world production systems may also generate undesirable by-products. One methodological difficulty associated with the previous DEA-based production planning models is how to incorporate undesirable factors in the planning models, while the simultaneous increase of desirable outputs and decrease of undesirable outputs could be considered. Based on the assumptions that demand changes can be forecasted for the next production season and inputs changes are under control, this paper propose a new DEA based model in a centralized decision making environment, which shares inputs/outputs changes among all of the units in such a way that individual efficiencies do not decrease. The proposed model ensures the improvement of individual units' performances, in addition to the overall efficiency. An empirical example is used to illustrate the proposed model.

**Keywords:** Data Envelopment Analysis (DEA), Production Planning, Undesirable Output, Decision Making Unit (DMU).

## 1 Introduction

Data envelopment analysis proposed by Charnes et al. [1], is an effective and widely used approach for measuring the relative efficiencies of a set of similar units, usually referred to as decision-making units (DMUs). Because of its development (such as the BBC model [2], the additive model [3], the FDH model [4] and the SBM model [5]) and widespread applications, DEA has attracted much attention from academics and practitioners. In the past few years, DEA has become increasingly popular in efficiency analysis and led to many new developments in concepts and methodologies (see Cooper et al. [6]).

The application of DEA as a non parametric quantitative tool is not restricted to assess the relative efficiency of peer units. Many extensions of the original work of Charnes et al. [1] have been proposed and used successfully in a wide range of applications. During recent

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years, more and more attentions have been paid to applying DEA to production planning problem as one of the most important practices for achieving the organizational objectives. Golany and Tamir [7] presented a DEA based resource allocation model which simultaneously determines input and output targets based on maximizing total output. Fare et al. [8] used DEA for modeling the possibility of reallocation of a fixed input. Cook and Kress [9] proposed a DEA based approach to assign a fixed or common cost to the various DMUs in an equitable way. Jahanshahloo et al. [10] proposed an equitable approach for assigning a fixed or common cost to all DMUs without solving linear programming problems. Hosseinzadeh Lotfi et al. [11] imposed Jahanshahloo et al. [10] approach on DMUs with fuzzy inputs and outputs in the case that fixed costs are imprecise. Korhonen and Syrjanen [12] developed a combined DEA-MOLP approach to a resource allocation problem in which a central unit controls the resources of a set of units. Lozano and Villa [13] considered a centralized DMU who owns or supervises all the operating units to maximize the efficiency of individual units. Lozano et al. [14] used DEA based Centralized target setting for regional recycling operations. Cook and Zhu [15] extended Cook and Kress [9] approach and provided a practical equitable approach to the cost allocation problem under the condition of variable returns to scale. Amirteimoori and Kordrostami [16] presented a DEA-based method for allocating fixed cost, allocating fixed input and setting fixed target to DMUs. Asmild et al. [17] reconsidered one of the centralized resource allocation BCC models proposed by Lozano and villa [13] and Lozano et al. [14] and suggest modifying it to only consider adjustments of previously inefficient units. Amirteimoori and Mohaghegh-Tabar [18] presented a DEA-based method for allocating fixed resources or costs across a set of DMUs. Du et al. [19] developed two DEA-based production planning approaches to find the most preferred production plans. Hosseinzadeh-Lotfi and Moghtaderi [20] developed Du et al.'s [19] model and presented new inputs and outputs plans based on the prediction of outputs changes in the next production season. Amirteimoori and Kordrostami [21] introduced a DEA approach based on the Du et al. [19] to making future production plans in a centralized decision making environment when demand changes can be forecasted in the next production season. Amirteimoori and Kordrostami [22] presented an optimal production planning model in a centralized decision-making environment which takes the size of operational units into consideration and the production level for each unit becomes proportional to the ability of the units.

As a nonparametric approach, DEA assumes that producing more outputs relative to fewer inputs is a criterion of efficiency. However, as mentioned in the seminal work of Koopmans [23], the production process may also generate undesirable outputs. If inefficiency exists in the production, the undesirable outputs should be reduced to improve the inefficiencies i.e., the undesirable and desirable outputs should be treated differently when we evaluate the production performance (see, Seiford and Zhu [4]). As an extension of the previous studies, this paper develops a DEA-based model to determine new production plans for all the individual units under a centralized decision-making environment, considering both the desirable and undesirable outputs.

We have organized this article into five major sections. Second section reviews the previous studies on efficiency measurement in presence of undesirable outputs. In the third section, we extend the work of Amirteimoori and kordrostami [22] for production planning with bad outputs, when demand change for the next production period is uncertain. Section four, illustrates the proposed model by a data set from a chain of poultry farms, in Guilan province, Iran. The last section develops discussion and conclusion.

## 2 Incorporating undesirable outputs in DEA models

Research on undesirable outputs has also been popularly pursued by DEA. It was first proposed by Fare et al. [25] and has been largely extended in the few past years. A number of studies have been carried out to deal with this type of outputs. For example, Scheel [26] used a data transformation approach to make undesirable factors desirable so that the resulting model preserves linearity. Using the classification invariance property, Seiford and Zhu [24] used the standard DEA model to improve the performance via increasing the desirable outputs and decreasing the undesirable outputs. Fare and Grosskopf [27] considered Seiford and Zhu [24] and suggested an alternative approach based on the directional distance function to increase good outputs and decrease undesirable outputs. Korhonen and Luptacik [28] used DEA to measure the eco-efficiency of 24 coal-fired power plants in presence of bad outputs. Jahanshahloo et al. [29] presented an approach to treat both undesirable inputs and outputs simultaneously in non-radial DEA models. Kordrostami and Amirteimoori [30] considered the efficiency evaluation of a set of interdependent decision making sub-units (DMSU) which make up a larger DMU with desirable and undesirable factors. Amirteimoori et al. [31] developed a DEA model which could be used to improve the relative performance via increasing undesirable inputs and decreasing undesirable outputs. Liang et al. [32] proposed an effective approach to deal with undesirable outputs and simultaneously reduces the dimensionality of data set.

Most recently, Lozano et al. [33] proposed a directional distance approach to deal with network DEA problems with undesirable outputs and applied their model to the problem of modeling and benchmarking airport operations in Spain. Akther et al. [34] studied the performance of 21 banks in Bangladesh and used a two stage network approach to maximize desirable outputs and minimize bad outputs. Li et al. [35] proposed some resource allocation models as a MOLP which considers the input reduction, desirable output reduction and undesirable output reduction. Wu et al. [36] proposed some new DEA models, which consider both economic and environmental factors in the allocation of a given resource. Three scenarios of the given resource in the next period and two objective functions are formulated for the three scenarios: maximizing the total desirable outputs and minimizing the total undesirable outputs. Hwang et al. [37] developed a new DEA model for performance evaluation where the simultaneous increase of desirable outputs and decrease of undesirable outputs are considered with a focus on identifying inefficiency as a result of higher levels of undesirable performance. Wang et al. [38] utilized improved DEA models to measure the energy and environmental efficiency of 29 administrative regions of China. Guo and Wu [39] presented an extended DEA model considering undesirable outputs using restrictions to realize a unique ranking of DMUs through the new “Maximal Balance Index” based on the optimal shadow prices. Most recently, Li et al. [40] in their paper used the Super-SBM model under undesirable outputs to measure regional environmental efficiency in China and then explored influential factors of China’s environmental efficiency by means of the Tobit regression model.

## 3 Production planning model

Production in large organizations with a centralized decision-making environment involves the participation of more than one individual unit, each contributing a part of the total production. Several DEA-based studies concerned such a centralized decision-making

environment and quite a few of them dealt with production planning problem. Recently, several researchers considered production planning concept in organizations with a set of  $n$  homogenous DMUs which act under supervision of a central decision-making unit and use same set of inputs to produce the same set of outputs [19]. The central DMU regularly faced by problem of arranging new input and output plans for all individual units in the next production season in order to maximize individual unit efficiency and entire organization performance, simultaneously. Considering desirable outputs as products of many production systems, the planning model must consider undesirable outputs. Although, few researchers have provided DEA models which consider bad outputs in efficiency analysis, we do not find any research which takes into account production planning in presence of bad (undesirable) factors.

Suppose there are a set of  $n$  DMU and unit  $j$  is denoted by  $DMU_j$  ( $j = 1, 2, \dots, n$ ), Each DMU consumes varying amounts of  $m$  different inputs  $x_{ij}$  ( $i = 1, 2, \dots, m$ ) to produce different outputs. Let us assume the production system produces  $s$  desirable outputs  $y_{rj}$  ( $r = 1, 2, \dots, s$ ) and  $k$  undesirable outputs  $w_{kj}$  ( $k = 1, 2, \dots, K$ ). The CCR efficiency of each DMU can be measured by the following multiplier model (1):

$$\begin{aligned}
 & \text{Max} \quad \sum_{r=1}^s u_r y_{ro} - \sum_{k=1}^K \mu_k w_{ko} \\
 & \text{s.t.} \\
 & \sum_{i=1}^m v_i x_{io} = 1 \quad , \quad (i = 1, \dots, m) \\
 & \sum_{r=1}^s u_r y_{rj} - \sum_{k=1}^K \mu_k w_{kj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad , \quad (r = 1, \dots, s) ; (k = 1, \dots, K) \\
 & v_i, u_r, \mu_k \geq \varepsilon \quad , \quad (j = 1, \dots, n)
 \end{aligned} \tag{1}$$

Suppose the demand change for output  $r$  ( $r = 1, 2, \dots, s$ ) in the next production season can be forecasted as  $D_r$ . To meet the supply and demand changes, the central unit will determine the most favorable input-output plans for all DMUs. Assume all  $D_r$  ( $r = 1, \dots, s$ ) can be either positive or negative, corresponding to an increase or a decrease in the demand for outputs  $r$ . The amount of change in input  $i$  ( $i = 1, \dots, m$ ) and undesirable output  $k$  ( $k = 1, 2, \dots, K$ ), also considered as  $C_i$  and  $G_k$ , respectively. We introduce the variables  $d_{rj}$  to represent the demand change of output  $r$  for  $DMU_j$  in the next production season,  $c_{ij}$  to represent the change in input  $i$  for  $DMU_j$  and  $g_{kj}$  to represent the change of undesirable output  $k$  for  $DMU_j$ . Therefore,  $\bar{x}_{ij} = x_{ij} + c_{ij}$ ,  $\bar{y}_{rj} = y_{rj} + d_{rj}$  and  $\bar{w}_{kj} = w_{kj} + g_{kj}$  represent the amount of total  $i$ -th input, total  $r$ -th desirable output and total  $k$ -th undesirable output of  $DMU_j$  in the next production season, respectively. Obviously,  $\sum_{j=1}^n c_{ij} = C_i$ ,  $\sum_{j=1}^n d_{rj} = D_r$  and  $\sum_{j=1}^n g_{kj} = G_k$ .

In the proposed approach, we believe that the inputs and outputs in the next season should be changed, such that each  $DMU_j$  has an efficiency score greater than or equal to its efficiency ( $e_j$ ) in the current season. Hence, we must have the following:

$$\begin{aligned}
& \frac{\sum_{r=1}^s u_r (y_{rj} + d_{rj}) - \sum_{k=1}^K \mu_k (w_{kj} + g_{kj})}{\sum_{i=1}^m v_i (x_{ij} + c_{ij})} \geq e_j \\
& s.t. \\
& \sum_{j=1}^n c_{ij} = C_i \quad , \quad (i = 1, \dots, m) \\
& \sum_{j=1}^n d_{rj} = D_r \quad , \quad (r = 1, \dots, s) \\
& \sum_{j=1}^n g_{kj} = G_k \quad , \quad (k = 1, \dots, K) \\
& v_i, u_r, \mu_k \geq \varepsilon \quad , \quad (j = 1, \dots, n) \\
& d_{rj} \geq 0 \quad \text{when} \quad D_r \geq 0 \\
& d_{rj} \leq 0 \quad \text{when} \quad D_r \leq 0 \\
& c_{ij} \geq 0 \quad \text{when} \quad C_i \geq 0 \\
& c_{ij} \leq 0 \quad \text{when} \quad C_i \leq 0 \\
& g_{kj} \geq 0 \quad \text{when} \quad G_k \geq 0 \\
& g_{kj} \leq 0 \quad \text{when} \quad G_k \leq 0
\end{aligned} \tag{2}$$

Obviously, the above model is nonlinear. If we have the change of variable  $\bar{d}_{rj} = u_r d_{rj}$ ,  $\bar{c}_{ij} = v_i c_{ij}$  and  $\bar{g}_{kj} = \mu_k g_{kj}$ , the following linear model will derive:

$$\begin{aligned}
& \frac{\left( \sum_{r=1}^s u_r y_{rj} + \sum_{r=1}^s \bar{d}_{rj} \right) - \left( \sum_{k=1}^K \mu_k w_{kj} + \sum_{k=1}^K \bar{g}_{kj} \right)}{\left( \sum_{i=1}^m v_i x_{ij} + \sum_{i=1}^m \bar{c}_{ij} \right)} \geq e_j \\
& s.t. \\
& \sum_{j=1}^n \bar{c}_{ij} = v_i C_i \quad , \quad (i = 1, \dots, m) \\
& \sum_{j=1}^n \bar{d}_{rj} = u_r D_r \quad , \quad (r = 1, \dots, s) \\
& \sum_{j=1}^n \bar{g}_{kj} = \mu_k G_k \quad , \quad (k = 1, \dots, K)
\end{aligned} \tag{3}$$

$$v_i, u_r, \mu_k \geq \varepsilon, \quad (j=1, \dots, n)$$

$$\bar{d}_{rj} \geq 0 \quad \text{when} \quad D_r \geq 0$$

$$\bar{d}_{rj} \leq 0 \quad \text{when} \quad D_r \leq 0$$

$$\bar{c}_{ij} \geq 0 \quad \text{when} \quad C_i \geq 0$$

$$\bar{c}_{ij} \leq 0 \quad \text{when} \quad C_i \leq 0$$

$$\bar{g}_{kj} \geq 0 \quad \text{when} \quad G_k \geq 0$$

$$\bar{g}_{kj} \leq 0 \quad \text{when} \quad G_k \leq 0$$

Suppose the change of r-th output assigned to DMU<sub>j</sub> referred to as  $\beta_j D_r$  and the input consumption for i-th input of DMU<sub>j</sub> referred to as  $\alpha_j C_i$ . In addition,  $\beta_j G_i$  is predicted as the change of k-th undesirable outputs of DMU<sub>j</sub>. Rationally,  $\alpha_j$  and  $\beta_j$  should be selected proportionately to the size of DMU<sub>j</sub>. In order to develop a feasible production planning model, first we determine the potential of each DMU in term of the magnitude size of the input and output:

**Definition 1-** The magnitude size of DMU<sub>o</sub> on the input side, denoted by  $MSI_o$ , is defined as the optimal objective value of the following linear programming model:

$$\begin{aligned} \text{Max } MSI_o &= \sum_{i=1}^m v_i x_{io} \\ \text{s.t.} \\ \sum_{i=1}^m v_i x_{ij} &\leq 1 \quad (i=1, \dots, m) \\ v_i &\geq \varepsilon, \quad (j=1, \dots, n) \end{aligned} \quad (4)$$

$MSI_o$  reflects the magnitude of DMU<sub>o</sub> in size and DMU<sub>o</sub> is said to be greater than DMU<sub>k</sub> in the input side if and only if  $MSI_o > MSI_k$ .

**Definition 2-** The magnitude size of DMU<sub>o</sub> on the output side, denoted by  $MSO_o$ , is defined as the optimal objective value of the following linear programming model:

$$\begin{aligned} \text{Max } MSO_o &= \sum_{r=1}^s u_r y_{ro} + \sum_{k=1}^K \mu_k w_{ko} \\ \text{s.t.} \\ \sum_{r=1}^s u_r y_{rj} + \sum_{k=1}^K \mu_k w_{kj} &\leq 1, \quad (r=1, \dots, s); (k=1, \dots, K) \\ u_r, \mu_k &\geq \varepsilon, \quad (j=1, \dots, n) \end{aligned} \quad (5)$$

Similarly,  $DMU_o$  is said to be greater than DMU<sub>k</sub> in the output side if and only if  $MSO_o > MSO_k$ . For each DMU<sub>o</sub> we let  $\alpha_o = MSI_o / \sum_{j=1}^n MSI_j$  and  $\beta_o = MSO_o / \sum_{j=1}^n MSO_j$  with  $\sum_{j=1}^n \alpha_o = \sum_{j=1}^n \beta_o = 1$ .

Note that the difficulty with these values with respect to  $\bar{c}_{ij}$ ,  $\bar{d}_{rj}$  and  $\bar{g}_{kj}$  is that there is no guarantee that they satisfy the model. Therefore, a rational objective is to introduce goal achievement variables for efficiency level and inputs and outputs levels. We define  $\bar{c}_{ij} - \alpha_j v_i C_i = a_{ij}^+ - a_{ij}^-$ ,  $\bar{d}_{rj} - \beta_j u_r D_r = b_{rj}^+ - b_{rj}^-$  and  $\bar{g}_{kj} - \beta_j \mu_k G_k = f_{kj}^+ - f_{kj}^-$ , in which  $a_{ij}^+$ ,  $a_{ij}^-$ ,  $b_{rj}^+$ ,  $b_{rj}^-$ ,  $f_{kj}^+$ ,  $f_{kj}^-$ ,  $s_j^+$  and  $s_j^-$  are The non negative deviation variables.

Based upon the result of models 3, 4 and 5 for all DMUs, we have the following planning model [22]:

$$\begin{aligned}
 \text{Min} \quad & \sum_{j=1}^n [s_j^+ + s_j^-] + \varepsilon_1 \sum_{r=1}^s \sum_{j=1}^n [b_{rj}^+ + b_{rj}^-] + \varepsilon_2 \sum_{i=1}^m \sum_{j=1}^n [a_{ij}^+ + a_{ij}^-] + \varepsilon_3 \sum_{k=1}^K \sum_{j=1}^n [f_{kj}^+ + f_{kj}^-] \\
 \text{s.t.} \quad & \left[ \sum_{r=1}^s u_r y_{rj} + \sum_{r=1}^s \bar{d}_{rj} \right] - \left[ \sum_{k=1}^K \mu_k w_{kj} + \sum_{k=1}^K \bar{g}_{kj} \right] - e_j \left[ \sum_{i=1}^m v_i x_{ij} + \sum_{i=1}^m \bar{c}_{ij} \right] \geq 0 \\
 & \left[ \sum_{r=1}^s u_r y_{rj} + \sum_{r=1}^s \bar{d}_{rj} \right] - \left[ \sum_{k=1}^K \mu_k w_{kj} + \sum_{k=1}^K \bar{g}_{kj} \right] - \left[ \sum_{i=1}^m v_i x_{ij} + \sum_{i=1}^m \bar{c}_{ij} \right] = s_j^+ + s_j^- \\
 & \bar{c}_{ij} - \alpha_j C_i v_i = a_{ij}^+ + a_{ij}^-, \quad (i=1, \dots, m) \\
 & \bar{d}_{rj} - \beta_j D_r u_r = b_{rj}^+ + b_{rj}^-, \quad (r=1, \dots, s) \\
 & \bar{g}_{kj} - \beta_j G_k \mu_k = f_{kj}^+ + f_{kj}^-, \quad (k=1, \dots, K) \\
 & \sum_{j=1}^n \bar{c}_{ij} = v_i C_i, \quad (i=1, \dots, m) \\
 & \sum_{j=1}^n \bar{d}_{rj} = u_r D_r, \quad (r=1, \dots, s) \\
 & \sum_{j=1}^n \bar{g}_{kj} = \mu_k G_k, \quad (k=1, \dots, K) \\
 & v_i, u_r, \mu_k, a_{ij}^+, a_{ij}^-, b_{rj}^+, b_{rj}^-, f_{kj}^+, f_{kj}^-, s_j^+, s_j^-, \varepsilon \geq 0, \quad (j=1, \dots, n) \\
 & \bar{d}_{rj} \geq 0 \quad \text{when} \quad D_r \geq 0 \\
 & \bar{d}_{rj} \leq 0 \quad \text{when} \quad D_r \leq 0 \\
 & \bar{c}_{ij} \geq 0 \quad \text{when} \quad C_i \geq 0 \\
 & \bar{c}_{ij} \leq 0 \quad \text{when} \quad C_i \leq 0 \\
 & \bar{g}_{kj} \geq 0 \quad \text{when} \quad G_k \geq 0 \\
 & \bar{g}_{kj} \leq 0 \quad \text{when} \quad G_k \leq 0
 \end{aligned} \tag{6}$$

The first inequality of model (6) guarantees that each DMU preserves its efficiency level and the second (goal) constraint insures that the new efficiency scores shift toward one. Note also that  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\varepsilon_3$  are considered as user-defined values to reflect the importance of the goal objectives ( $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 1$ ). We should point out that based upon the proposed production plan, the inputs and outputs changes in the next season should be allocated to all DMUs, such

that any efficiency score does not reduce. The new PPS describes a reliable reference for future decisions.

#### 4 Empirical Case study

The aim of this paper is to develop a DEA based production planning model in order to allocate resources and set targets for each of the units in a poultry chain. It has been assumed that central DMU has authority for sharing resources and presenting new production plans for next production season, when in addition to desirable outputs (Produced Meat and Feed Conservation Ratio), the process produces some undesirable outputs (Mortality and Condemn). In this section, we apply our DEA based model to a set of 13 poultry farms which are located in Guilan Province, Iran. These farms belong to the Green Hen poultry chain, with a central decision-making team which supervise all poultry's operation and make future production plans for them. The numerical production data of all 13 farms and their efficiency scores are presented in Table 1.

**Table 1.** Data for poultry chain

| DMU | Input                   |                        |                                   | Output             |                                |                             | Original efficiency |
|-----|-------------------------|------------------------|-----------------------------------|--------------------|--------------------------------|-----------------------------|---------------------|
|     | New Born Chicks (stock) | Feed Cost (1000 Rials) | Operational Expenses (1000 Rials) | Produced Meat (kg) | Feed Conversion Ratio (Number) | Mortality & Condemn (Stock) |                     |
| 1   | 12700                   | 587000                 | 155290                            | 28582.2            | 1.98                           | 640                         | 0.9778              |
| 2   | 14670                   | 663500                 | 174060                            | 32387.2            | 1.93                           | 710                         | 0.9842              |
| 3   | 13300                   | 590340                 | 169370                            | 28506.3            | 2.00                           | 1569                        | 0.9603              |
| 4   | 15000                   | 701440                 | 193240                            | 34075.0            | 1.95                           | 500                         | 1.0000              |
| 5   | 12000                   | 562620                 | 157730                            | 26256.5            | 1.98                           | 1014                        | 0.9354              |
| 6   | 14000                   | 614790                 | 177340                            | 29828.0            | 1.97                           | 1361                        | 0.9646              |
| 7   | 13000                   | 637380                 | 172570                            | 30158.7            | 2.03                           | 790                         | 0.9873              |
| 8   | 14900                   | 707620                 | 190780                            | 33414.6            | 2.04                           | 1035                        | 0.9578              |
| 9   | 13500                   | 650320                 | 169070                            | 30439.0            | 1.94                           | 764                         | 0.9636              |
| 10  | 12800                   | 577220                 | 166170                            | 28223.5            | 2.03                           | 790                         | 0.9724              |
| 11  | 19800                   | 921770                 | 225390                            | 44581.2            | 2.01                           | 1378                        | 1.0000              |
| 12  | 11000                   | 511640                 | 138220                            | 25683.4            | 2.00                           | 474                         | 1.0000              |
| 13  | 12600                   | 589600                 | 159630                            | 28405.3            | 1.88                           | 665                         | 0.9648              |

The inputs are “New Born Chicks”, “Feed Cost” and “Operational Expenses”. “New Born Chicks” refers to the newly hatched chick and “Feed Cost” refers to the diet cost for chicks and chickens, while “Operational Expenses” refers to the expenses such as labor expenses, rent, energy (gas, gasoline, power, and water), hygiene and safety cost. There are also three outputs, two of them are desirable (“Produced Meat” and “Feed Conversion Ratio”) and one is undesirable (“Mortality and Condemn”). “Produced Meat” refers to the total weight of all matured chickens, “Feed Conversion Ratio” refers to the amount of body weight gained for every kilogram of feed consumed, while “Mortality and Condemn” refers to the died or omitted chicks along the production season.

In order to develop our production plan, first, we take an epsilon model to calculate  $\varepsilon$  for models 1, 2 and 3. The computed amounts are 0.000000151, 0.000000086 and 0.0000218, respectively. By applying model 1, the efficiency scores of all DMUs are computed. The eighth column in table 1 shows these efficiency scores. Next, we used models 2 and 3 to compute the magnitude size of DMU<sub>j</sub> on the output and input sides. Then, based on the



defined equations, the values of  $\alpha$  and  $\beta$  were calculated. Table 2 summarizes the result as following:

Suppose that central DMU forecasted the demand change for produced meat ( $D_1$ ) and feed conversion ratio ( $D_2$ ) for the next production season as  $D_1 = 30000$ ,  $D_2 = 0$ , while it expects an increase in mortality and condemn ( $G_1$ ) equal to 450. Note,  $r_2$  is a non-controllable variable and its value determine by the system. The central DMU also determined the possible change of the inputs (new born chicks, feed cost and operational expenses) as  $C_1 = 10000$ ,  $C_2 = 600000$  and  $C_3 = 160000$ , respectively. The concern of the central DMU is to share total inputs and outputs changes among all units in such a way that total efficiency score improve. To this purpose, the new planning model (6) is used in this section. The new input and output targets of all poultries planned by model (6) are presented in Table 3 (columns 2 to 7), with the new CCR efficiencies in the eighth column:

**Table 2** the computed values for magnitude size of inputs and outputs

| DMU | MSI    | MSO    | $\alpha$ | $\beta$ |
|-----|--------|--------|----------|---------|
| 1   | 0.6470 | 0.6358 | 0.0703   | 0.0709  |
| 2   | 0.7303 | 0.7202 | 0.0793   | 0.0803  |
| 3   | 0.6624 | 0.6544 | 0.0720   | 0.0730  |
| 4   | 0.7795 | 0.7523 | 0.0847   | 0.0839  |
| 5   | 0.6276 | 0.5934 | 0.0682   | 0.0662  |
| 6   | 0.6908 | 0.6786 | 0.0750   | 0.0757  |
| 7   | 0.7052 | 0.6734 | 0.0766   | 0.0751  |
| 8   | 0.7826 | 0.7496 | 0.0850   | 0.0836  |
| 9   | 0.7137 | 0.6789 | 0.0775   | 0.0757  |
| 10  | 0.6480 | 0.6313 | 0.0704   | 0.0704  |
| 11  | 1.0000 | 1.0000 | 0.1086   | 0.1115  |
| 12  | 0.5663 | 0.5692 | 0.0615   | 0.0635  |
| 13  | 0.6528 | 0.6325 | 0.0709   | 0.0705  |

**Table 3** New plan and efficiency score for poultry chain

| DMU | Input                   |                        |                               | Output             |                                |                             | New efficiency |
|-----|-------------------------|------------------------|-------------------------------|--------------------|--------------------------------|-----------------------------|----------------|
|     | New Born Chicks (stock) | Feed Cost (1000 Rials) | Operational Cost (1000 Rials) | Produced Meat (kg) | Feed Conversion Ratio (Number) | Mortality & Condemn (Stock) |                |
| 1   | 13403                   | 629180                 | 166538                        | 30709.2            | 1.98                           | 672                         | 1.0000         |
| 2   | 15463                   | 711080                 | 186748                        | 34796.2            | 1.93                           | 746                         | 0.9902         |
| 3   | 14020                   | 633540                 | 180890                        | 30696.3            | 2.00                           | 1602                        | 0.9702         |
| 4   | 15000                   | 752248                 | 206792                        | 36592.0            | 1.95                           | 538                         | 1.0000         |
| 5   | 12682                   | 603540                 | 168642                        | 28242.5            | 1.98                           | 1044                        | 1.0000         |
| 6   | 14750                   | 659790                 | 189340                        | 32099.0            | 1.97                           | 1395                        | 0.9716         |
| 7   | 13766                   | 683340                 | 184826                        | 32411.7            | 2.03                           | 824                         | 1.0000         |
| 8   | 15750                   | 758620                 | 204380                        | 35922.6            | 2.04                           | 1073                        | 0.9670         |
| 9   | 14275                   | 696820                 | 181470                        | 32710.0            | 1.94                           | 798                         | 0.9828         |
| 10  | 13504                   | 619472                 | 177434                        | 30335.5            | 2.03                           | 822                         | 0.9865         |
| 11  | 20886                   | 986930                 | 242766                        | 47926.2            | 2.01                           | 1428                        | 1.0000         |
| 12  | 12462                   | 548540                 | 148060                        | 27579.4            | 2.00                           | 503                         | 1.0000         |
| 13  | 13309                   | 632140                 | 170974                        | 30520.3            | 1.88                           | 697                         | 0.9833         |

As shown in Table 3, the results indicate that the new efficiency scores of all units lie between the original efficiency score resulted from CCR model (1) and 1.0000. The results also show that based on the new plan, all of the efficiency scores are improved, while six out of the thirteen poultries are DEA efficient with three of them also efficient before applying the new

plan. As can be seen in Table 3, based on the new planning model the DMUs 1, 5 and 7 known as three newly DEA efficient units. Note, the new plan takes into account the potentiality (magnitude size) of units when developing the inputs/outputs arrangements, which ensures the results feasibility.

## 5 Conclusions

In recent years, many mathematical models have been developed to make a contribution to the problems of production planning in term of fixed cost allocation, resource allocation and target setting. The current paper developed a DEA-based approach for production planning in a centralized decision making environment, considering undesirable factors. Using a data set of 13 poultry farms the corresponding efficiency scores of new production plan have been computed and compared with those of the original plan. As shown in paper, the proposed model improved the efficiencies of all units significantly. The proposed approach, also allows the modeler to set priorities on objectives. It is widely applicable and fit well with real world examples.

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