Stochastic Fuzzy Discrimination Information Measure Cost Function in Image Processing

A. Fatemi[*]

Abstract A new cost function based on stochastic fuzzy discrimination information measure is introduced in this paper. Focusing on their significant parts, this cost function is used to find the optimal value of threshold for denoising image. It is, in fact, an extension of fuzzy entropy cost function by the present author. Multivariable normal distribution is used for creating focus on significant parts of an image. At the end, the results of this cost function are compared to previous ones by applying it to some images. By using multivariate normal distribution on the images as the cost function weight, the center of the image is more considered by the algorithm. Consequently, the best results will be produced by the new cost function.

Keywords: Fuzzy Sets, Stochastic, Entropy, Discrimination Information Measure, Image Processing.

1 Introduction

In fuzzy sets theory and image processing, many distances have been defined and used to compare sets or images. [1-5]

Comparing it with the ideal image, it would be possible to evaluate a contracted image. Beside the Euclidian distance, fuzzy entropy was used as a cost function for this comparison by a previous research [6]. A new index of fuzzy entropy in stochastic situation, one of which is a symmetric form of discrimination information measure, was also introduced [7]:

$$D_{SFS}(A,B) = I_{SFS}(A,B) + I_{SFS}(B,A)$$

$$I_{SFS}(A,B) = \sum p_i \left[\mu_A(x) \log \frac{\mu_A(x)}{\frac{1}{2}(\mu_A(x) + \mu_B(x))} + (1 - \mu_A(x)) \log \frac{1 - \mu_A(x)}{1 - \frac{1}{2}(\mu_A(x) + \mu_B(x))} \right] \qquad (1)$$

where A and B are fuzzy sets in the same reference set X; p is a stochastic plan on X.

This paper aims at using a new cost function including this symmetric form of discrimination information measure and total entropy with a multivariate normal plan. This

---

* Corresponding Author. (✉)
   E-mail: fatemi@iausdj.ac.ir (A. Fatemi)

A. Fatemi
   Islamic Azad University, Sanandaj Branch, Department of Mathematics, Kurdistan, Iran

plan is used to evaluate images and to compare the results with the non-stochastic version of this cost function.

2 Stochastic Fuzzy Entropy Cost Function in Image Processing

As it was aforementioned in the introduction, this section aims at using the following formula instead of the other cost function to choose the best threshold in image denoising:

$$C(A) = \alpha D_{SFS}(A,C) + (1-\alpha)E_T(A)$$

Were A and B are the fuzzy version of denoised image and the ideal image, and

$$E_T(A) = E_S(A) + H(P)$$

$$E_S(A) = \sum_{i=1}^{n} p_i e(x_i)$$

$$e(x_i) = 1 - \sqrt{(\mu_A(x_i) - v(x_i))^2} \tag{2}$$

$H(P)$ is Shannon entropy and $E_T(A)$ is total entropy for a multivariate normal distribution.

In Fig. 1, an image of human face as an ideal image C, and its constructed versions with different threshold from d=.01 to d=.19 are considered to recognize the noised pixel.

Ideal image C   d=.01   d=.02   d=.03   d=.04   d=.05   d=.06   d=.07   d=.08   d=.09

d=.10   d=.11   d=.12   d=.13   d=.14   d=.15   d=.16   d=.17   d=.18   d=.19
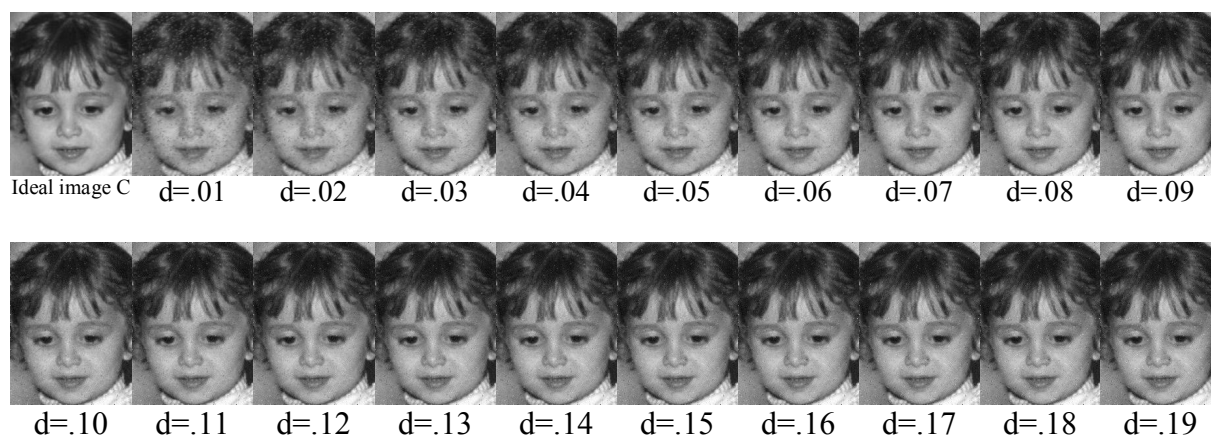
Fig. 1 ideal and constructed images

Since central image pixels include important information in human face image, multivariate normal distribution in new cost function can be used.
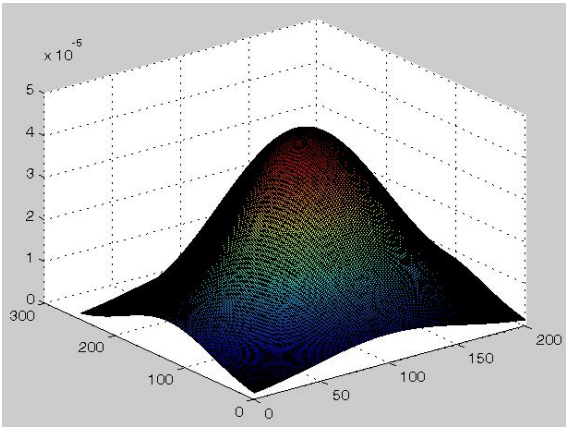
Fig. 2. multivariate normal distribution

The new stochastic cost function and the non-stochastic cost function for each of the above images are computed and compared:

Table 1 Comparison of new stochastic cost function with non-stochastic cost function

| Threshold for recognize noised pixels to construction	New stochastic cost function	Non stochastic cost function	Threshold for recognize noised pixels to construction	New stochastic cost function	Non stochastic cost function
d=0.01	00.00963	00.065957	d=0.11	00.00601	00.061394
d=0.02	00.008857	00.064958	d=0.12	00.005974	00.061348
d=0.03	00.007309	00.063022	d=0.13	00.005969	00.061351
d=0.04	00.007477	00.063181	d=0.14	00.005993	00.061383
d=0.05	00.007057	00.06266	d=0.15	00.006008	00.061406
d=0.06	00.006606	00.062102	d=0.16	00.006026	00.061427
d=0.07	00.006404	00.061837	d=0.17	00.006056	00.061474
d=0.08	00.006217	00.061612	d=0.18	00.006124	00.061537
d=0.09	00.006104	00.061481	d=0.19	00.006164	00.0616
d=0.10	00.006039	00.061407	Min.	00.005969	00.061348
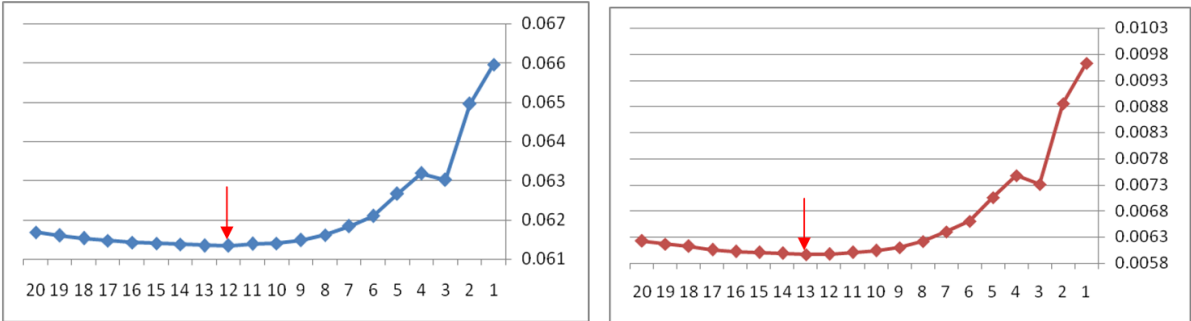
Diagram1 Comparison of new stochastic cost function with non stochastic cost function

3 Conclusion

Our results showed that for a human face image the new cost function proposed threshold d=0.13 and the non-stochastic cost function proposed threshold d=0.12. Looking at the constructed images, we see that the sharpness of constructed image decreases as construction algorithm threshold decreases. By using multivariate normal distribution on the images as the cost function weight, the center of the image is more considered by the algorithm.

Therefore, we are avoiding the miss of real information focused in the center of images.

The new cost function with threshold d=0.13 instead d=0.12, preserves the information at the center of image.

References

1. Cekanavicius, V., Murauskas, G., (2002). Statistics and its applications. Part 2, TEV, Vilnius, 272 (in Lithuanian).
2. Grudin, M. A., (1997). A compact multi-level model for the recognition of facial images. Ph.D. thesis, Liverpool John Moores University.
3. Phillips, P.J., Moon, H. J., Rizvi, S. A., Rauss, P. J., (2000). The FERET evaluation methodology for face recognition algorithms. PAMI 22 10, 1090–1104.
4. Phillips, P. J., O'Toole, A. J., Cheng, Y., Ross, B., Wild, H. A., (1999). Assessing algorithms as computational models for human face recognition. NIST TR.
5. Vytautas, P., (2004). Distance measures for PCA-based face recognition, Pattern Recognition Letters, 25 (6), 711-724.
6. Pasha, E., Farnoosh, R., Fatemi, A., (2006). Fuzzy Entropy as Cost Function in Image Processing, Proceedings of the 2nd IMT-GT conference.
7. Fatemi, A., (2011). Entropy of Stochastic Intuitionistic Fuzzy Sets, Journal of Applied Sciences 11 (4), 748-751.