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Extending the Super Efficiency Method to Rank the Non-Extreme Efficient Units

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Abstract This article will address the extension of super efficiency method to rank the non-extreme efficient decision making units. Many methodologies have introduced methods that can rank efficient units, amongst which, the super efficiency method due to its ability to provide meaningful geometrical as well as economic analyses has a significant place. But the common problem with all the super efficiency models is their inability to rank the non-extreme efficient units. The writing will first, introduce a method for finding a face with the smallest dimension including a non-extreme unit. In continuation, a new production possibility set is constructed by eliminating such surface. Removing such surface in done by eliminating decision making on the surface from production possibility set. Then, super efficiency of the non-extreme efficient unit can be measured under the new production possibility set. At conclusion, the proposed method will be clarified through an example and its results are compared with the existing methods.

Keywords: Data Envelopment Analysis (Dea), Non-Extreme Efficient Unit, Production Possibility Set (Pps), Ranking, Super Efficiency.

1 Introduction

Since publication of the CCR paper, by Charnes et al. [1], various topics such as financial management and performance evaluation, ranking, productivity, returns to scale, resource allocation and ..., have been analyzed, calculated and formulated by different DEA methods. The first step in evaluating the performance of DMUs with multiple input/output is to measure their efficiency which is calculated by using different models in DEA (CCR, BCC, SBM, …) under various technologies such as constant or variable returns to scale, free disposal hall and ..., assumptions. Based on the DEA properties and the fact that the DEA's production possibility set is generated using the observed units, usually there is more than one decision making unit on the frontier of production possibility set (approximate production function or efficient frontier). These units are called DEA efficient units and since their efficiencies are identical, there is a definite need for ways to compare them.

This comparison is known as ranking and there are two major ranking methods:

- ✓ The ones that rank all efficient units
- ✓ Those that only rank the extreme efficient units.

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Methods capable of ranking all efficient units are, in turn, divided in three groups namely: common set of weights, cross-evaluation and interval efficiency, each with significant limitations that make their ranking-ability ambiguous. For instance:

The existence of alternative optimal solutions and the inability to provide complete ranking for efficient units, make the common set of weights method problematic.

Cross-Evaluation suffers from existence of alternative optimal solutions, zero weights and the incompatibility of the cross-evaluation matrix ills.

In comparison to the other two methods, the internal Efficiency method has not been fully investigated yet. But the problem of overlapping frontiers of efficiency and inefficiency can be mentioned as hindrance to its ability to provide thorough ranking. Also, there is not a uniform process for generating inefficient frontiers in this method, meaning that different processes lead to different inefficient frontiers.

The super efficiency method for ranking of only efficient units, first introduced by Anderson and Petersen [2], does not have any of the above problems and its major advantage is its ability to provide valuable economic analysis capability. The basic idea of super efficiency is to remove the evaluation DMU from the observation and measure its performance based on the new production possibility set. Super efficiency value, a number more than one, indicates the importance of the under evaluation unit. For example, consider Anderson and Petersens' super efficiency [2] in the input-oriented model. Assume under evaluating unit is not an observed DMU, and we want to construct a DMU while its outputs are equal as that of evaluating DMU. The input values of such unit must been equal to super efficiency value of the evaluating unit × its inputs. Thus, imposed surplus input values to system are (super efficiency value of the evaluating units-1) × its inputs, i.e.; the more value of super efficiency, the more imposed surplus inputs to system. The more value of super efficiency, the more importance of evaluating unit. This is a significant analysis in economic. But the common problem with all the super efficiency models is their inability to rank the non-extreme efficient units.

Recently, Gholam Abri et al. [3] proposed a method by using representation theorem for ranking non-extreme efficient units in BCC model. Their basic idea is to calculate convex combination of the super efficiency values of reference extreme units of evaluating non-extreme efficient DMU as its super efficiency value. In this method, a face with the smallest dimension including non-extreme efficient unit is needed to identify its reference extreme DMUs. The suggested method to identify such surface by Gholam Abri et al. [3], as mentioned by them, is not applicable from the computational point of view. Another problem in their method is that non-extreme efficient unit may be have alternative representations of its reference extreme units. This was noted by Gholam Abri et al. [3]. They described this ambiguity by interval numbers, while the upper and lower bounds of super efficiency of non-extreme efficient units were obtained by solving two linear programming problems. Therefore, super efficiency of some non-extreme efficient units may be expressed as interval numbers. This has no consistency with the precise nature of data and super efficiency values of extreme efficient units. To overcome these difficulties and present a method for ranking non-extreme efficient units which has more consistency with super efficiency methodology, this paper at first proposes a mathematical programming problem to obtain a face with the smallest dimension including a non-extreme efficient unit. Then, the identified surface is removed from production possibility set. In this situation, measuring super efficiency value of the evaluating non-extreme efficient unit is conceivable.

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Accordingly, the present paper has been developed as follows: second section includes essential preliminaries. The main idea will be described in section three. In the next section an example is provided to illustrate the method. We review the results in the final section.

2 Prerequisites

Consider n DMUs, each unit with m input and s output. Also, suppose that DMU_j $(j=1,...,n)$ by using inputs $x_{ij}(i=1,\ldots,m)$ produces outputs $y_{rj}(r=1,\ldots,s)$.
The linear model of CCR (multiplier form of CCR in input oriented), introduced by Charnes et al. [1] to calculate the efficiency of DMU_o is as follows:

$$\begin{aligned}
&\text{Max} \sum_{r=1}^{s} u_r y_{ro}, \\
&s.t. \\
&\quad \sum_{i=1}^{m} v_i x_{io} = 1, \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j=1,...,n, \\
&\quad u_r \ge 0, \quad r=1,...,s, \\
&\quad v_i \ge 0, \quad i=1,...,m,
\end{aligned} \tag{1}$$

in which $u_r(r=1,\ldots,s)$ and $v_i(i=1,\ldots,m)$ are the output and input weights, respectively, are determined by solving the model. The CCR model is designed with the assumption of *constant* returns to scale.

By variable returns to scale assumption, the BCC model, introduced by Banker et al. [4], calculates the efficiency of the DMUs as follows:

$$\begin{aligned}
&\text{Max} \sum_{r=1}^{s} u_r y_{ro} + u_0, \\
&s.t. \\
&\quad \sum_{i=1}^{m} v_i x_{io} = 1, \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0, \quad j=1,...,n, \\
&\quad u_r \ge 0, \quad r=1,...,s, \\
&\quad v_i \ge 0, \quad i=1,...,m, \\
&\quad u_0 \quad \textit{free},
\end{aligned} \tag{2}$$

The dual of the above model, called envelopment form of BCC in input oriented, works as follows:

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$$
\begin{aligned}
& Min\ \theta, \\
& s.t. \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io},\ \ i=1,...,m, \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro},\ \ r=1,...,s, \\
& \quad \sum_{j=1}^{n} \lambda_j = 1, \\
& \quad \lambda_j \ge 0, \quad j=1,...,n,
\end{aligned} \tag{3}
$$

The above model is built on the BCC production possibility set which is depicted by T_v, and is as follows:

$$T_v = \left\{ (x_1,..,x_m, y_1,...,y_s) \middle| \sum_{j=1}^{n} \lambda_j x_{ij} \le x_i, i=1,..,m, \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_r, r=1,..,s, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0, j=1,...,n \right\}$$

Definition 1. DMU_o is called BCC Efficient when:

- optimal value in (2) equals to 1; and
- for at last one optimal solution, all multipliers $u_r(r=1,\ldots,s)$ and $v_i(i=1,\ldots,m)$ are positive.

Otherwise, it is BCC non-efficient.

Definition 2. Reference set of DMU_o, which is shown by E_o, is defined as:

$$E_o = \left\{ j \middle| \lambda_j^* \text{ is possitive in one of the optimal solutions of model (3)} \right\}$$

Definition 3. DMU_o is BCC extreme efficient if:

- it is BCC efficient; and
- only its reference be itself (that is: $|E_o| = 1$)

Definition 4. DMU_o is BCC non-extreme efficient if:

- it is BCC efficient; and
- has references to other than itself (that is: $|E_o| > 1$)

There does usually more than one efficient unit exist in a DEA models. One of efficient units' ranking methods popular with researchers is the super efficiency method introduced by Anderson and Petersen [2].

Multiplier and envelopment forms of Anderson and Petersens' model [2] are respectively as:

$$
\begin{aligned}
& \text{Max} \sum_{r=1}^{s} u_r y_{ro} + u_0, \\
& s.t. \\
& \quad \sum_{i=1}^{m} v_i x_{io} = 1,
\end{aligned}
$$

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$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0, \quad j = 1,\dots,n, \quad j \ne o,$$
$$u_r \ge 0, \quad r = 1,\dots,s,$$
$$v_i \ge 0, \quad i = 1,\dots,m,$$
$$u_0 \quad free, \tag{4}$$

$$Min \ \theta,$$
$$s.t.$$
$$\sum_{\substack{j=1 \\ j\ne o}}^{n} \lambda_j x_{ij} \le \theta x_{io}, \quad i = 1,\dots,m,$$
$$\sum_{\substack{j=1 \\ j\ne o}}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r = 1,\dots,s,$$
$$\sum_{\substack{j=1 \\ j\ne o}}^{n} \lambda_j = 1,$$
$$\lambda_j \ge 0, \quad j = 1,\dots,n, \quad j \ne o, \tag{5}$$

Super efficiency value of model (5) is more than unity for extreme efficient units, if model (5) is feasible. This score is number one for non-extreme efficient units and is equal to efficiency score of non-efficient DMUs. Model (5) has three defects which are as follows:

1- Infeasibility in some cases
2- Instability of the optimal value of model (5) in relation to change in inputs of evaluating unit
3- Inability to rank non-extreme efficient units

Several models have been proposed to solve the first two mentioned problems which SBM model [5] is one of the most successful model. Yet, there is the third problem in all of the super efficiency models.

3 Ranking of Non-Extreme Efficient Units

One of the super efficiency models' shortcomings, including Anderson and Petersen's [2], is their inability to rank the non-extreme efficient units. Usually, the number of non-extreme efficient units, when compared with the extreme ones, is insignificant. However, when there is more than one non-extreme efficient unit, their ranking by the super efficiency models is impossible. Consider the following explanation:

In the above graph, units A, C, E and G are extreme efficient, and B, D, F, L and M are non-extreme efficient.

The basic idea of super efficiency method is to reconstruct production possibility set regardless evaluating unit and then measuring its performance in new PPS (measuring super efficiency). As can be seen in Figure 1, removing each of the extreme efficient units A, C, E and G is changed PPS and so their performances. But, this is clear from Figure 1 which removing any of non-extreme efficient units B, D, F, L and M is not changed PPS. Therefore, their performances are not changed, and they are nevertheless efficient with efficiency score

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one. As a result, there is no differentiation among non-extreme efficient units and so they cannot be ranked.

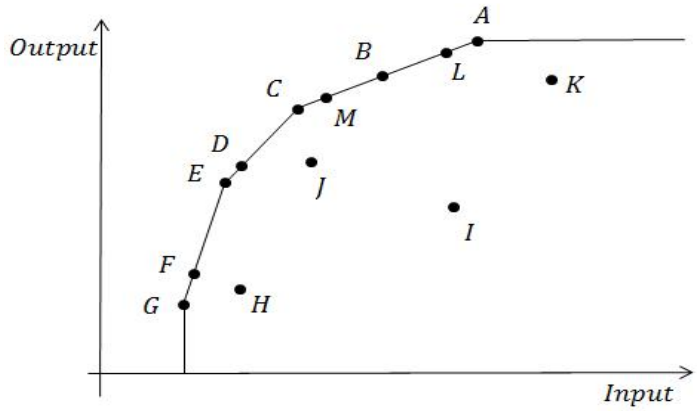

Fig. 1 Production possibility set and both efficient and non-efficient units

If we want to extend super efficiency idea to rank non-extreme efficient units, we must answer to this question that how removing a non-extreme efficient unit causes to change PPS? The proposal of this paper is to find a face with the smallest dimension including non-extreme efficient unit and then remove such surface from the PPS. To find and remove a face with the mentioned property, all of DMUs laying on the face must be distinguished and removed. This means that evaluation non-extreme unit and its references must be eliminated form PPS. To discuss, consider Figure 2. Line segment AC is the face with the smallest dimension including non-extreme efficient unit B. To remove unit B from PPS, we must simultaneously eliminate units A, B, C, M and L from the set of observations. In this situation, non-extreme efficient B is not a member of new PPS as shown in Figure 2.

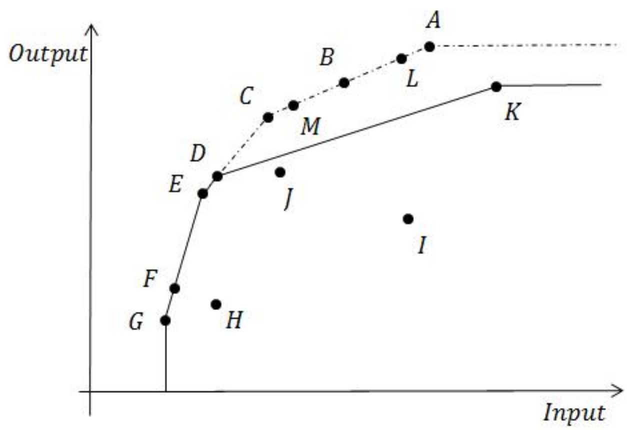

Fig. 2 Production possibility set after removing the face with the smallest dimension including B

A similar procedure can remove non-extreme efficient units D and F from PPS, which corresponding Figures are observed in the following Figures 3, 4.

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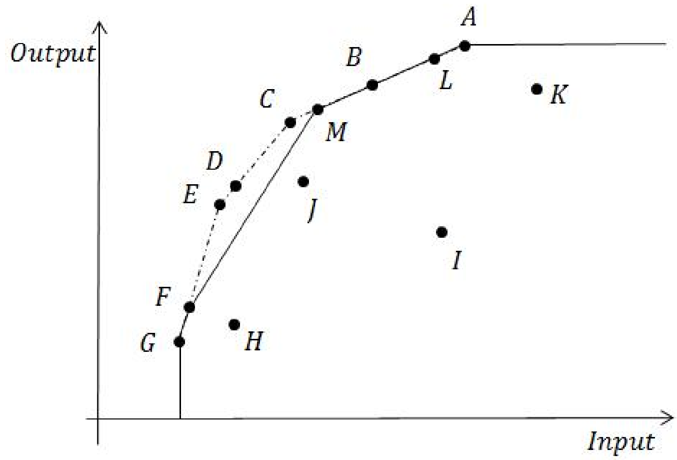

Fig. 3 Production possibility set after removing the face with the smallest dimension including D

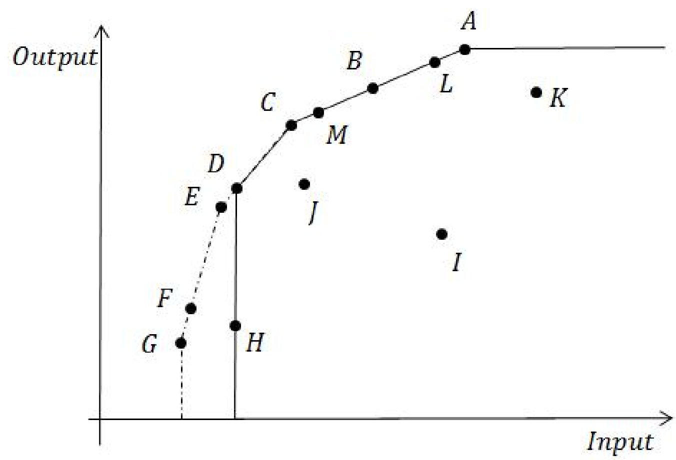

Fig. 4 Production possibility set after removing the face with the smallest dimension including F

Let DMU_o is a non-extreme efficient unit. Following model is proposed to distinguish reference set and the smallest face including DMU_o as:

$$
\begin{aligned}
&\text{Min} \sum_{j=1}^{n} k_j, \\
&s.t. \\
&\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 + d_j = 0, \quad j = 1,\ldots,n, \\
&\qquad \sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} + u_0 = 0, \\
&\qquad \left(1 - k_j\right) \le M d_j, \quad j = 1,\ldots,n, \\
&\qquad u_r \ge 0, \quad r = 1,\ldots,s, \\
&\qquad v_i \ge 0, \quad i = 1,\ldots,m,
\end{aligned}
$$

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$$d_j \geq 0, \quad j=1,...,n,$$
$$k_j \in \{0,1\}, \quad j=1,...,n,$$
$$u_0 \quad free, \tag{6}$$

Now, consider $u^* = (u_1^*,...,u_s^*)$, $v^* = (v_1^*,...,v_m^*)$, $d^* = (d_1^*,...,d_n^*)$, $k^* = (k_1^*,...,k_n^*)$ and u_0^* as optimal multipliers of model (6) with optimal value K^*.

Lemma 1. $H = \left\{ (x_1,..,x_m, y_1,...,y_s) \middle| \sum_{r=1}^{s} u_r^* y_r - \sum_{i=1}^{m} v_i^* x_i + u_0^* = 0 \right\}$ is a supporting hyperplane for production possibility set of T_v.

Proof. We have $\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} + u_0 = 0$ as a constraint of problem (6). Thus, $\sum_{r=1}^{s} u_r^* y_{ro} - \sum_{i=1}^{m} v_i^* x_{io} + u_0^* = 0$.

On the other hand, $\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} + u_0^* + d_j^* = 0$, $d_j^* \geq 0, (j=1,...,n)$. So, $\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} + u_0^* \leq 0$ $(j=1,...,n)$. As a result, $\sum_{r=1}^{s} u_r^* \sum_{j=1}^{n} \lambda_j y_{rj} - \sum_{i=1}^{m} v_i^* \sum_{j=1}^{n} \lambda_j x_{ij} + u_0^* \leq 0$, for $\sum_{j=1}^{n} \lambda_j = 1$, $\lambda_j \geq 0$ $(j=1,...,n)$. Thus, for each $(x_1,..,x_m, y_1,...,y_s) \in T_v$, we have $\sum_{r=1}^{s} u_r^* y_r - \sum_{i=1}^{m} v_i^* x_i + u_0^* \leq 0$. This completes the proof.

Lemma 2. If $d_t^* = 0$, then hyperplane $H = \left\{ (x_1,..,x_m, y_1,...,y_s) \middle| \sum_{r=1}^{s} u_r^* y_r - \sum_{i=1}^{m} v_i^* x_i + u_0^* = 0 \right\}$ contains DMU_t.

Proof. We have $\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} + u_0^* + d_j^* = 0$ $(j=1,...,n)$. Therefore, $\sum_{r=1}^{s} u_r^* y_{rt} - \sum_{i=1}^{m} v_i^* x_{it} + u_0^* = 0$. Thus, hyperplane $\sum_{r=1}^{s} u_r^* y_r - \sum_{i=1}^{m} v_i^* x_i + u_0^* = 0$ contains DMU_t.

Theorem 3. If $\sum_{r=1}^{s} \bar{u}_r y_r - \sum_{i=1}^{m} \bar{v}_i x_i + \bar{u}_0 = 0$ is a supporting hyperplane for PPS at DMU_o, a non-extreme efficient unit, then the number of units on such hyperplane is not less than K^*.

Proof. By a contra positive assumption, let the number of units laying on $\sum_{r=1}^{s} \bar{u}_r y_r - \sum_{i=1}^{m} \bar{v}_i x_i + \bar{u}_0 = 0$ is $\bar{K} < K^*$. Without losing generality, assume the hyperplane contains $DMU_1,...,DMU_{\bar{K}}$ and the rest of units are not on it. We define $\bar{d}_j = 0 - \left(\sum_{r=1}^{s} \bar{u}_r y_{rj} - \sum_{i=1}^{m} \bar{v}_i x_{ij} + \bar{u}_0 \right)$. So, $\bar{d}_j = 0$ $(j=1,...,\bar{K})$, $\bar{d}_j > 0$ $(j=\bar{K}+1,...,n)$. Based on the constraint $(1-k_j) \leq Md_j$ $(j=1,...,n)$ in model (6), we define $\bar{k}_j = 1$ $(j=1,...,\bar{K})$, and

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$\bar{k}_j = 0\ (j = \bar{K}+1,...,n)$. As a result, multipliers $\bar{u} = (\bar{u}_1,...,\bar{u}_s)$, $\bar{v} = (\bar{v}_1,...,\bar{v}_m)$, $\bar{d} = (\bar{d}_1,...,\bar{d}_n)$, $\bar{k} = (\bar{k}_1,...,\bar{k}_n)$ and $\bar{u}_0$ can construct a feasible solution for model (6) while objective function value of such solution is $\bar{K}$. Thus, $K^* \le \bar{K}$. This is a contradiction and so the proof is completed.

Lemma 4. If a point lays on a hyperplane and can be expressed as a convex combination of some other points, then the hyperplane contain these points. [6]

Theorem 5. DMU_t is a member of the reference set of DMU_o (a non-extreme efficient unit) if and only if $d_t^* = 0$ in optimal solution of model (6).

Proof. Assume DMU_t is a member of the reference set of DMU_o. Therefore, there is an optimal solution of problem (3) as$(\lambda_1^*,...,\lambda_n^*)$, which $\lambda_t^* > 0$. As a result, DMU_o is a convex combination of some of the observed DMUs which DMU_t is one of them. On the other hand, by lemma 1, $\sum_{r=1}^{s} u_r^* y_r - \sum_{i=1}^{m} v_i^* x_i + u_0^* = 0$ is a supporting hyperplane for PPS including DMU_o. By lemma 4, DMU_t lays on this hyperplane. Thus, $d_t^* = 0$.

Conversely, this is clear that $u^* = (u_1^*,...,u_s^*)$, $v^* = (v_1^*,...,v_m^*)$ and u_0^* are optimal multipliers of model (2) in evaluating DMU_o. We have $d_t^* = 0$, then $\sum_{r=1}^{s} u_r^* y_{rt} - \sum_{i=1}^{m} v_i^* x_{it} + u_0^* = 0$, in model (2). Based on the complementary slackness theorem, there is an optimal solution for model (3) as $(\lambda_1^*,...,\lambda_n^*)$ in which $\lambda_t^* > 0$. So, by definition 2, DMU_t is a member of the reference set of DMU_o.

Define $F_o = H \quad T_v$, intersection $\sum_{r=1}^{s} u_r^* y_r - \sum_{i=1}^{m} v_i^* x_i + u_0^* = 0$ and PPS. Thus, F_o is a face of PPS. By using theorem 5, the set of the efficient units laying on F_o is E_o. PPS by removing them from the observations is reconstructed as:

$$\bar{T}_v = \left\{ (x_1,..,x_m,y_1,...,y_s) \middle| \sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j x_{ij} \le x_i, i = 1,..,m, \sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j y_{rj} \ge y_r, r = 1,..,s, \sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j = 1, \lambda_j \ge 0,\ j \in \{1,...,n\} - E_o \right\}$$

Theorem 6. $(x_o, y_o) \notin \bar{T}_v$.

Proof. By a contra positive assumption, suppose $(x_o, y_o) \in \bar{T}_v$. So, we have:

$$\sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j x_{ij} \le x_i,\ \ i = 1,..,m,$$

$$\sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j y_{rj} \ge y_r,\ \ r = 1,..,s,$$

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$$\sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j = 1,$$

$$\lambda_j \geq 0, \; j \in \{1,...,n\} - E_o .$$

If the above relations are held in equality situation, then DMU_o is a convex combination of units which are not references of DMU_o. This is a contradiction. Otherwise, if one of the above inequalities is held strictly, then $\left(\sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j x_{1j},..., \sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j x_{mj}, \sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j y_{1j},..., \sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j y_{sj} \right) \in \overline{T_v}$ and is dominant to DMU_o. This is also a contradiction. Thus, we have $(x_o, y_o) \notin \overline{T_v}$.

Now, we present following model for ranking non-extreme efficient units, as an extension of model (5), as:

$$Min \; \theta,$$

$$\sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \; i = 1,..,m,$$

$$\sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j y_{rj} \geq y_{ro}, \; r = 1,..,s,$$

$$\sum_{\substack{j=1 \\ j \notin E_o}}^{n} \lambda_j = 1,$$

$$\lambda_j \geq 0, \; j = 1,...,n, j \notin E_o \tag{7}$$

4 Example

Here, we use the presented example by Gholam Abri et al. [3] to illustrate the method proposed in this paper, and to compare its results with Gholam Abri et al.'s method [3]. Thirteen units with one input and output are considered in Figure 5.

Obtained results report of applying super efficiency method of Anderson and Petersen [2], Gholam Abri et al.'s method [3], and the proposed method in this paper are provided in the following Table 1.

Based on the super efficiency of units in the second column of Table 1, DMUs are divided into three groups non-efficient DMUs including H, I, J, K, M, non-extreme efficient units containing C, D, F, L, and extreme efficient units A, B, E, G. This is clear that non-extreme efficient units cannot be ranked by their super efficiency scores. The third column of Table 1 shows super efficiency of non-extreme efficient units by using the proposed method of Gholam Abri et al. [3]. The obtained scores are convex combinations of super efficiency of extreme efficient units. Although, non-extreme efficient units can be ranked by these scores, the procedure for finding such convex combinations is computationally burdensome. This fact is also confirmed by Gholam Abri et al. [3].

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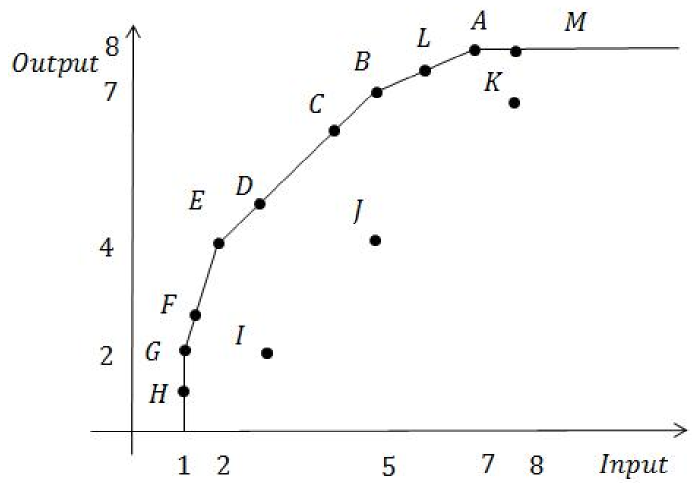

Fig. 5 Generated PPS by Gholam Abri et al. [3]

Table 1 The results of different methods

DMU	Anderson and Petersen [2]	Gholam Abri et al. [3]	Reference set	Proposed score
A	1.1428	-	-	-
B	1.0666	-	-	-
C	1	1.086	B, C, D, E	1.125
D	1	1.10553	B, C, D, E	1.1667
E	1.125	-	-	-
F	1	1.1875	G, E, F	1.3333
G	1.25	-	-	-
H	0.9999	-	-	-
I	0.333	-	-	-
J	0.3	-	-	-
K	0.625	-	-	-
L	1	1.1047	A, B, L	1.1667
M	0.875	-	-	-

Table 2 Comparing different ranking methods

DMUs	Anderson and Petersen [2]	Gholam Abri et al. [3]	Proposed method
A	2	2	2
B	4	4	4
C	5	6	8
D	5	7	7
E	3	3	3
F	5	5	5
G	1	1	1
H	9	9	9
I	12	12	12
J	13	13	13
K	11	11	11
L	5	8	6
M	10	10	10

Reference sets of non-extreme efficient units by using model (6) are reported in the fourth column of Table 1. We categorize non-extreme efficient units by using the number of their references. We then rank units in the related categories by using model (7). The least number

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of references in a category, the highest preference of category for ranking. We have two categories of non-extreme efficient units, DMUs F, L with three references and DMUs C, D with four references. We firstly rank units F, L and then units C, D which their super efficiency, by model (7), is presented in the last column of Table 1. Ranking summary of the units, based on the different mentioned methods, is provided in Table 2.

5 Conclusion

This paper has presented a method for ranking non-extreme efficient units. The method is to find a face with the smallest dimension including non-extreme efficient unit and then to remove the face from PPS. In this situation, measuring super efficiency of the evaluating non-extreme efficient unit is possible. One of the important advantages of the proposed method related to the suggested method by Gholam Abri et al. [3] is presenting a practical and simple method to distinguish the face with the smallest dimension including the evaluating unit. The model to distinguish face with the smallest dimension can be applied to determine reference set of non-efficient units. This paper propose also to use SBM super efficiency method instead of BCC method to overcome the problems in BCC super efficiency (infeasibility and instability). One can mix two suggested models in this paper to present a programming problem in order to remove the face and measure super efficiency, simultaneously.

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