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Obtaining a possible allocation in the bankruptcy model using the Shapley value

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Abstract Data envelopment analysis (DEA) is an effective tool for supporting decision-makers to assess bankruptcy, uncertainty concepts including intervals, and game theory. The bankruptcy problem with the qualitative parameters is an economic problem under uncertainty. Accordingly, we combine the concepts of the DEA game theory and uncertain models as interval linear programming (ILP), which can be applied to all areas of studies such as bankruptcy assessment. An optimal allocation is achieved based on the players of the different coalitions and Shapley values by considering a kind of the interval games. Indeed, the Shapely value is one idea which player i ' s is share equal to i 's expected marginal contribution if the players join the coalition each time. Also, the Shapley value is one reasonable allocation which we used in this paper based on game theory**.** Finally, we solve a numerical example, using the Shapely value concept.

Keyword: DEA, Bankruptcy, Interval Programming, Shapley Value.

1 Introduction

Bankruptcy is one of the most important concepts of economics. Data envelopment analysis (DEA) has been usually considered to assess efficiency of decision-making units (DMUs) and is the most distinguished methods for the bankruptcy assessment. The additive DEA method of bankruptcy assessment, developed by Premachandra et al. [1], that used a set of financial ratios as input and output variables and studied the reduction of outputs and increase in inputs in DMUs. Actually, bankruptcy should use as a process designed to remove those firms that are inefficient. Therefore, in bankruptcy assessment models, bankruptcy is based on financial ratios. Altman's work on prediction of bankruptcy uses financial ratio [2] and in the next researches, Premachandra et al. use DEA for bankruptcy assessment [1]. Game theory is the study of mathematical models as a game for recognition economic behaviors. In fact, game theory has many applications in the fields of mathematics like uncertainty. In real-world, from different types of methods, that to deal with uncertainty is game theory and interval programming. Interval methods can be given by using qualitative concepts in an interval form for obtaining the optimal solution in the feasible regions studied with better evaluation compared to other methods. Some researchers have studied ILP problems [3-8]. In the proposed methods for solving ILP, the model is converted to two sub-models: one sub-model

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gives the best value of the objective function that it is optimistic value, and the other obtains the worst value that it is pessimistic value.

This paper presents a new model for bankruptcy assessment as interval and prediction using interval values. In this model, we use the DEA game theory and the proposed model for bankruptcy assessment [9]. Actually, the proposed model for bankruptcy assessment was one model in DEA game theory that we extend it as an interval DEA game model. In fact, in this paper, we use a combination of the DEA game theory and interval programming and specify bankruptcy interval as the optimistic and pessimistic bankruptcy intervals. Finally, we apply the Shapley concept for allocating estate and then compare results obtained by solving the model. The Shapely value is one idea: player i ' s share is equal to i ' s expected marginal contribution if the players join the coalition one at a time, in a uniformly random order. Indeed, The Shapley value is one reasonable allocation which we used in this paper. Moreover, some researchers used the Shapely value concept for proposing allocation schemes based on cooperative game theory for reducing costs [10].

In fact, the values obtained from the model rather than different coalitions of the players is the game's final results (games payoff) that could be the Shapley value in this model, and on the other word, the amount allocated to each player during the games rather than its remaining estate and whether or not to pay the debt. Also, we consider some uncertain indicators as an interval indicator [11].

We study bankruptcy models of the directional distance function and modified directional distance model in DEA in Section 2. Section 3 contains basic definitions and in Sections 4, ILP model is provided. In section 5, a bankruptcy assessment model using combine interval models and DEA game theory has been introduced and a numerical example in section 6 and final results has presented.

2 Directional distance function of DEA

The DEA method is a linear programming methodology to measure the efficiency of DMUs. Suppose there is a set of 'n' DMUs with 'm' inputs and 's' outputs, $\{y_{rj}\}$ be the value of the r-th output from the DMU_j and x_{ij} be the value of the i-th input in the DMU_j. All data are supposed to be non-negative and is given as follows:

$$x_{ij} \geq 0, x_j \neq 0, j = 1,...,n$$

$$y_{rj} \geq 0, y_j \neq 0, j = 1,...,n$$

The proposed models in DEA can be used as a directional distance function. According to [12], direction distance model has been considered in the measurement of efficiency.

$$\begin{aligned} &\theta^* = \min \theta \\ &s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \ i = 1,...,m \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \ r = 1,...,s \\ &\lambda_j \geq 0, \ j = 1,...,n \end{aligned} \tag{1}$$

In Model (1), x_{io} and y_{ro} show the input and output studied DMU. This model has a feasible solution. Now, DEA models can be used as a directional distance function for measurement of efficiency. Now we use the directional distance function for bankruptcy assessment.

Model (2) is the variable returns to scale (VRS) DEA model for the directional distance function that was expanded by Chambers. In this method, the directional distance function can be computed for each input and output vector associated with a production possibility set [13] that β can be unrestricted:

$$\begin{aligned} \max \quad & \beta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \beta g_{y_{ro}} \\ & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \beta g_{x_{io}} \\ & \lambda_j \geq 0,\ j = 1,2,\ldots,n \\ & g_{y_{ro}}, g_{x_{io}} \geq 0 \\ & g_x = \max_j \left\{ x_{ij} \right\} - x_{io} \quad i = 1,\ldots,m \\ & g_y = y_{ro} - \min \left\{ y_{rj} \right\} \quad r = 1,\ldots \end{aligned} \tag{2}$$

Actually, the technical inefficiency is measured by β in the DMUs. In Model (2), g_x and g_y are direction vectors. Now, we study bankruptcy assessment using the modified directional distance function of DEA as follows:

A modified DEA is given by considering the worst relative efficiency. This model is studied as an output-oriented one. Firstly, the worst position of DMU (undesirable DMU) is introduced, such that the input and output of this DMU are as follows [14]:

$$\begin{aligned} x_i^{\max} &= \max_j (x_{ij}) \quad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n \\ y_r^{\min} &= \min_j (y_{rj}) \quad r = 1,2,\ldots,s;\ j = 1,2,\ldots,n \end{aligned}$$

Where n, m, and s are the number of DMUs, inputs and outputs, respectively. x_{io} and y_{ro} show inputs and outputs of the unit under assessment. For bankruptcy assessment, we change the output of the DMU in the direction of the output vector. In this case, the production possibility set is defined following:

$$T_{CRS}^{BR} = \left\{ (x, y) : x \leq \sum_{j=1}^{n} \lambda_j x_{ij},\ y \geq \sum_{j=1}^{n} \lambda_j y_{rj};\ \lambda_j \geq 0 \right\}.$$

So, oriented-output bankruptcy model is presented as follows:

$$
\begin{aligned}
& \max \ \beta^{BR} \\
& s.t. \ \sum_{j=1}^{n} \lambda_j^{BR} y_{rj} \le y_{ro} - \beta^{BR} g_y; \ r = 1,2,\ldots,s \\
& \sum_{j=1}^{n} \lambda_j^{BR} x_{ij} \ge x_{io} \quad i = 1,2,\ldots,m \\
& g_y, \beta^{BR} \ge 0 \\
& g_y = y_{ro} - \min_j \{y_{rj}\}. \ r = 1,2,\ldots,s
\end{aligned}
\tag{3}
$$

The worst relative efficiency can be computed by Model (3) in the direction of undesirable DMU and the amount of bankruptcy is calculated by a distance measure between the considered point and the worst point according to undesirable point.

3 Basic definitions

Definition 3.1. Let $N = \{1,\ldots,n\}$ be a set of creditors. It can be said that a bankruptcy problem is a pair (E,d), where $d = (d_1,d_2,\ldots d_n) \in R^n$ such that $d_i \ge 0$ for all $0 \le i \le n$ and $0 \le E \le \sum_{j \in N} d_j$.

We suppose a bankruptcy problem with creditors and we interpret d_i as the amount that the creditors demand, where $E \in R$ is the amount of the estates that may be returned (repaid).

Note 3.1. A solution for the problem of bankruptcy (E,d) or, in brief, an allocation, is an n-tuple. Also, $E = \sum_{j \in N} x_j$ where x_j represents the amount allocated to j-th creditor, which can be calculated by the Shapely value method.

In fact, when we want to choose a state that leads to the most efficient one, we use the allocation which is one of the most important and highly used topics in optimization have been used in all directions. The use of optimization models allows us to evaluate the different aspects of the allocation of the objective function. Here, it can be said that the allocation is a function of assigning a unique allocation to each bankruptcy problem [15].

Definition 3.2. By an n-person cooperative game in characteristic function form, we mean a pair (N,v), where $N = \{1,\ldots,n\}$ is a set of players and $v : 2^n \to R$ where 2^n denotes number of the subsets of N and $v(\varnothing) = 0$.

We usually refer to subsets S of N as coalitions and to the number $v(S)$ as the worth S. The allocation may be interpreted of estates to each player as maximum profit or minimum cost. Now, we consider a fixed set of players by game (N,v), where c is a characteristic function [15].

Definition 3.3. The bankruptcy game (E,D) is corresponding to the bankruptcy problem as follows:

$$
v_{E,d}(S) = \max \left\{ E - \sum_{j \in N \setminus S} d_j, 0 \right\}
$$

In this case, $v(S)=0$ shows the bankruptcy. If $E-\sum_{j\in N\backslash S} d_j$ is negative, then the player is bankrupt and nonzero value would indicate non-bankruptcy. Also, if the obtained value for $E-\sum_{j\in N\backslash S} d_j$ is positive, then it is interpreted as a non-bankruptcy [16-18].

Now, using the concepts in the above section, we present a model for bankruptcy with concepts of DEA game theory [6].

$$
\begin{aligned}
&\max E_j-\sum_{j\in N} d_j\\
&\quad \sum_{j=1}^{n}\lambda_j y_{rj}\le y_{ro}-(E_j-\sum_{j\in N} d_j)g_y\\
&\quad \sum_{j=1}^{n}\lambda_j x_{ij}\ge x_{io}\quad i=1,\dots,m\\
&\quad \sum_{j=1}^{n}\lambda_j =1\\
&\quad E_j-\sum_{j\in N} d_j\ge 0\\
&\quad g_y=y_{ro}-\min\{y_{rj}\}.\quad r=1,\dots,s
\end{aligned}
\tag{4}
$$

E_j is the initial value of the total estates of j-th organization and $N=\{1,\dots,n\}$ indicates the total number of the organizations, and x_{ij} and y_{rj} are input and output of j-th organization, x_{io} and y_{ro} represent the input and output under review organization and as the amount that the i-th creditor demands, and g_y is a production direction vector. Really, Model (4) describes bankruptcy assessment by using game theory that presents a different interpretation and further consideration of the aspects of economic bankruptcy.

We mentioned that $N=\{1,\dots,n\}$ represents the total number of the players and the objective function represents the net estates or the remaining estates after payment of claims. In this model, a firm is interpreted as a set of players. This model is based on the total estates and demands. The value obtained from solving the model is interpreted as the net assets or the remaining asset value after payment of the claims of j-th player. Also, positive values obtained for objective function are represented as non-bankruptcy. For more interpretation, see [9].

4 Interval linear programming

Many problems in real-world are uncertain. To deal with these problems, interval linear programming (ILP) is proposed. Several methods have been proposed for solving ILP models [2, 4, 19, 5, 6, 7, 8]. One method proposed by Tong [6] entitled the best and the worst method (BWC) obtains the upper and lower bounds of the objective function.

An interval number $[X^-,X^+]$ is shown as $\overset{\pm}{X}$ where $X^-\le X^+$. If $X^-=X^+$, then $\overset{\pm}{X}$ is degenerate. If A^- and A^+ are two matrices in $\mathbb{R}^{m\times n}$ such that $A^-\le A^+$. Then the set of matrices

$\overset{+}{A^{-}} = [A^{-}, A^{+}] = \left\{ A \middle| A^{-} \le A \le A^{+} \right\}$

And the matrices A^{-} and A^{+} are called its bounds. Center and radius matrices are defined as: $\Delta_{A^{\pm}} = \frac{1}{2}\left(A^{+} - A^{-}\right)$ and $A^{c} = \frac{1}{2}\left(A^{-} + A^{+}\right)$. A special case of an interval matrix is an interval vector $\mathbf{x}^{\pm} = \left\{ x | x^{-} \le x \le x^{+} \right\}$ where $x^{-}, x^{+} \in \mathbb{R}^{n}$. Interval arithmetic has been studied in [20].

Consider the following ILP model:

$$\begin{aligned} & Max \quad z^{\pm} = \sum_{j=1}^{n} c_j^{\pm} x_j^{\pm} \\ & s.t. \quad \sum_{j=1}^{n} a_{ij}^{\pm} x_j^{\pm} \le b_i^{\pm}, \quad i = 1, 2, \ldots, m, \\ & \qquad x_j^{\pm} \ge 0, \quad j = 1, 2, \ldots, n. \end{aligned} \tag{5}$$

According to [18], the best and worst values of the objective function of Model (5) are the solution of two the best and the worst models and are obtained as follows:

$$\begin{aligned} & Max \quad z^{+} = \sum_{j=1}^{n} c_j^{+} x_j \\ & s.t. \quad \sum_{j=1}^{n} a_{ij}^{-} x_j \le b_i^{+}, \quad i = 1, 2, \ldots, m, \\ & \qquad x_j^{\pm} \ge 0, \quad j = 1, 2, \ldots, n. \end{aligned} \tag{6}$$

$$\begin{aligned} & \mathrm{Max} \quad z^{-} = \sum_{j=1}^{n} c_j^{-} x_j \\ & s.t. \quad \sum_{j=1}^{n} a_{ij}^{+} x_j \le b_i^{-}, \quad i = 1, 2, \ldots, m, \\ & \qquad x_j^{\pm} \ge 0, \quad j = 1, 2, \ldots, n. \end{aligned} \tag{7}$$

Models (6) and (7) have the largest and smallest feasible space, respectively.

5 A bankruptcy assessment model using combine interval models and DEA game theory

In this section, we introduce a new model of bankruptcy assessment using ILP and DEA game theory. In Model (4), by converting the outputs to interval, we have an interval game model as follows:

$$
\begin{aligned}
& \max\ E_j - \sum_j d_j \\
& s.t.\ \sum_{j=1}^{n} \lambda_j \left[\underline{y}_j , \overline{y}_j \right] \le \left[\underline{y}_o , \overline{y}_o \right] - \left(E_j - \sum_j d_j \right) g_y \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} \ge x_{io} \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad g_y = \overline{y}_{ro} - \min\{ \underline{y}_{rj} \} \\
& \quad E_j - \sum_j d_j \ge 0 \\
& \quad \lambda_j \ge 0
\end{aligned}
\tag{8}
$$

Now, according to Models (6) and (7), the optimistic and pessimistic models are as follows:

Pessimistic model:

$$
\begin{aligned}
& \max\ E_j - \sum_j d_j \\
& s.t.\ \sum_{j=1}^{n} \lambda_j \left[\underline{y}_j \right] \le \left[\overline{y}_o \right] - \left(E_j - \sum_j d_j \right) g_y \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} \ge x_{io} \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad g_y = \overline{y}_{ro} - \min\{ \underline{y}_{rj} \} \\
& \quad E_j - \sum_j d_j \ge 0 \\
& \quad \lambda_j \ge 0
\end{aligned}
\tag{9}
$$

Optimistic model:

$$
\begin{aligned}
& \max\ E_j - \sum_j d_j \\
& s.t.\ \sum_{j=1}^{n} \lambda_j \left[\overline{y}_j \right] \le \left[\underline{y}_o \right] - \left(E_j - \sum_j d_j \right) g_y \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} \ge x_{io} \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad g_y = \overline{y}_{ro} - \min\{ \underline{y}_{rj} \} \\
& \quad E_j - \sum_j d_j \ge 0 \\
& \quad \lambda_j \ge 0
\end{aligned}
\tag{10}
$$

The solutions obtained from solving Models (9) and (10) are the bankruptcy bounds.

6 Numerical example

Consider the information system consisting of 8 companies in the form of the play. Indicators of the estate, debt, and sale as crisp values, and performance evaluation as qualitative value converted to interval are listed in Table 1 [6]. Now for using interval analysis in this system, it is assumed that performance evaluation is an interval as $[0,100]$, 0 is lower bound and 100 is upper bound (for instance, very weak performance is (0, 10), weak performance is (10, 30) and average performance is (30, 50), and etc.) Directional vector is positive. Using the estates, debts, sales and performance assessment given in Table 1, the interval bankruptcy of the eight companies in the form of a game with 8 players and two strategies (recipient of their demands and payment of the debts) is calculated by Model (8).

Table 1 Values of the indicators of the players

	Estate	Debt	Sale	Performance	Bankruptcy interval
$DMU1$	123	46	11	Weak	[No answer,0.96]
$DMU2$	234	78	3	Very weak	[No answer,1]
$DMU3$	176	68	164	Good	[0.02, 0.59]
$DMU4$	28	17	165	Very good	[0.23, 0.68]
$DMU5$	530	71	171	Average	[0,0.4]
$DMU6$	69	69	76	Weak	[No answer, 0.66]
$DMU7$	109	85	37	Good	[0,0.28]
$DMU8$	135	72	60	Good	[0.37,0.88]

6.1 Results and Analysis

By using the definition of bankruptcy game theory, we discuss the amount of the estate available after paying our demands and analysis model as follows. In this example, the estate of the companies is more than the debts (or finally equal). We consider these companies in a competitive game. The performance of the players in the game is measured with each other, and then the bankruptcy is calculated. In fact, in this competitive game, some players may have better performance and, due to the bankruptcy of opponent player, remove him/her from the competition. Now, we consider using the values of estate and liabilities, all coalitions between players that have been obtained. In this case, it is clear that in the game with 8 players, how can a player form different coalitions with 7 other players and allocate its estate and debt. In fact, by calculating the amount remaining after payment of debts, how can the player continues his/her activities in the field of the economy with the other players? The amounts remaining after payment of debts in all the coalitions are listed in Table 2. For further explanation, consider the first player who has a total debt of 46. Now, 7 other players want this debt from Player 1 as $d = (2,4,5,6,8,10,11)$. Afterwards, we survey the amount of available estate after payment (or even without payment) of debts, by calculating the bankruptcy game under a competitive game and Shapley values [21]; [22]. How does a player's desire with an estate amount more than debt?

In fact, the performance of the players compared to bankruptcy (or non-bankruptcy) is measured relative to each other, and their performances are measured in comparison with each other whether or not to go bankrupt. Since this game is a competitive game under a social activity, some players may be weak in performance due to weakness in each of the indicators.

The values of the estates, total debts and demands vector for each player are shown in Table 2.

We make up different coalitions of a player to the other players (S) and then calculate the value of $v_{E,d}(S)$, for example; the coalitions of players 1 and 6 are given in Table 3, respectively. For example, single coalitions mean that the first player must allocate its estate rather than to the total debt of other players except Player 1 in the coalition. The values in Table 3 show that since the estate of the player is more than debt, then the player will be able to pay its debts, but, due to poor performance, this player will become bankrupt. Consider the last coalition of Player 1 that all players are present. The amount of $v_{E,d}(S)$ is 123, which means that the players can refuse to pay their debts!

Table 2 values of estate, total debts and demands vector for each player

Player	E	$\sum_{j\in N\setminus S} d_j$	d
1	123	46	$(2,4,5,6,8,10,11)$
2	234	78	$(8,6,12,15,20,5,3)$
3	176	68	$(4,6,12,15,10,18,3)$
4	28	17	$(2,2,1,4,4,3,1)$
5	530	71	$(8,16,12,7,14,5,9)$
6	69	69	$(6,8,12,15,20,5,3)$
7	109	85	$(4,6,12,15,24,5,19)$
8	135	72	$(1,6,12,19,22,5,3)$

Now, the value that each player had to do in comparison to its coalition during the game defines the Shapley value, so a player can pay its demands to any coalition with any number of players or even with respect to prediction of the bankruptcy model, and not pay its debts.

Based on the above models and calculations, any player during its demand, rather than the players, will be the receiver of the demands. Naturally, other players may have slightly or even more debt compared to other players, they can—after the payment of their debts—administrate matters relating to the production and development dealing with economic problems. Now, it seems that the Shapley value in this model is the amount allocated to each player during the game rather than its estate and debt. It is necessary to mention that any allocation in this game is defined in accordance with the range of the output (performance assessment). The calculated bankruptcy shows a kind of interval game concept. The results can be different rather than games and other players, and produce some other Shapley value. A flowchart and figure are given below. It can be said, in the Shapely value, player i ' s share is equal to i ' s expected marginal contribution if the players join the coalition.

A flowchart and figure are given below.

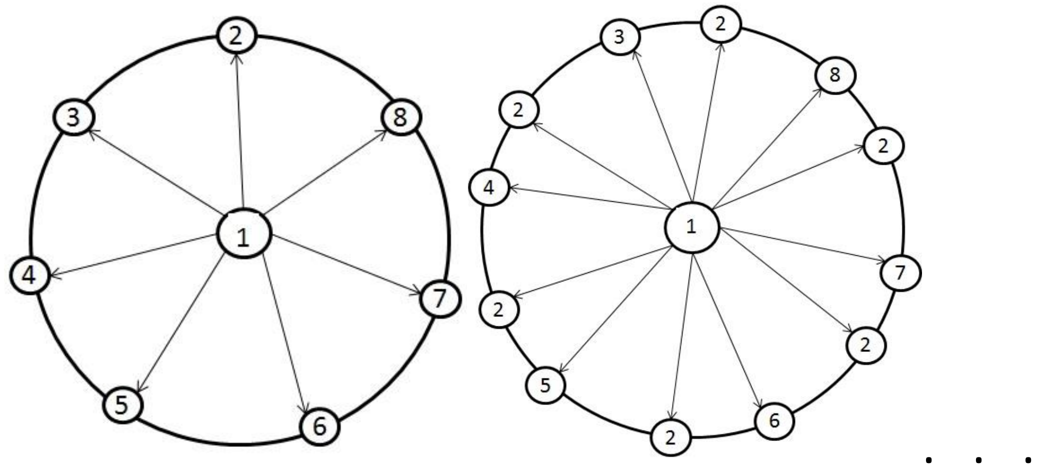

Fig. 1 An overview of the game coal

Table 3 The performance of player 1 in comparison to the other player's coalitions

S	$v_{E,d}(\mathrm{S})$		S	$v_{E,d}(\mathrm{S})$	
	Player 1	Player 2		Player 1	Player 2
$\{1\}$	79	8	$\{1,2,3,4\}$	94	41
.	.	.	.	.	.
.	.	.	.	.	.
	.	.	.	.	.
$\{7\}$	88	3	$\{4,5,6,7\}$	112	43
$\{1,2\}$	83	14	$\{1,2,3,4,5\}$	102	61
.	.	.	.	.	.
.	.	.	.	.	.
	.	.	.	.	.
$\{6,7\}$	98	8	$\{3,4,5,6,7\}$	117	55
$\{1,2,3\}$	88	26	$\{1,2,3,4,5,6,7\}$	112	66
.	.	.	.	.	.
.	.	.	.	.	.
	.	.	.	.	.
$\{5,6,7\}$	106	28	$\{2,3,4,5,6,7\}$	121	61
			$\{1,2,3,4,5,6,7\}$	123	69

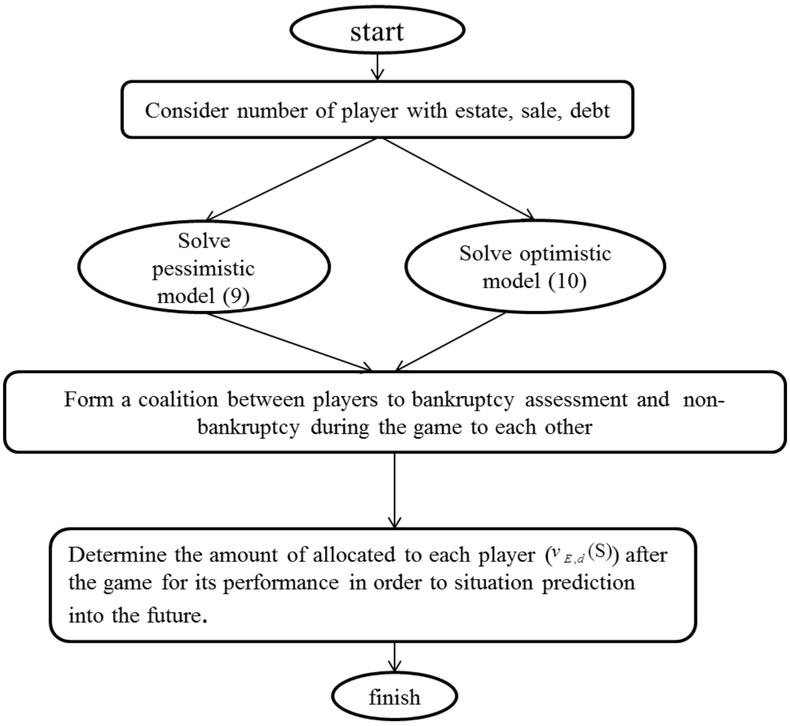

Fig. 2 The program flowchart of the interval games using the shapely value

7 Conclusion

In this paper, bankruptcy models have been studied by considering game theory and interval linear programming. An applied example has been analyzed by the proposed model in the field of problems in economics. The investigation demonstrated how the model, by studying a group of peer companies as a competitive game, can be considered with or without bankruptcy in the form of a model. Since, in this game, both factors and indexes are discussed, then this model forecasts bankruptcy and eliminates some players from the game, even when the estate is more than its debt. If a player fails in its activities, the player is removed from the rest of the game (competition) by its rival players.

In fact, interval programming allows us to use a combination of the qualitative indicators and game theory. Hence, this model specifies bankruptcy interval as the optimistic and pessimistic bankruptcy intervals. So, we used the Shapley concept that has demonstrated a bankrupt player during the game, with estate more than debt or at most equal to its debt, to allocate its estate to the other players or can even refuse to pay their debts! It is necessary to mention that in this game, each player can pay their demands or get some demands from the other players. Seemingly, this is a game with two strategies (as a recipient, or as paying), as players can pay their debts or not by considering the Shapley value, rather than different coalitions of the player. This model can create an applicable understanding of the game theory and ILP in the world of economics and mathematics.

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