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A new source of inefficiency due to the inappropriate choice of price in data envelopment analysis

R. Fallahnejad, E. Rezaei Hezaveh*

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Abstract Data envelopment analysis (DEA) is a useful tool for identifying well-performing (efficient) decision-making units (DMUs). In DEA, those units that are not placed on the efficiency frontier are considered to be inefficient units. Identifying inefficiency sources can help turn the units into more efficient ones. Therefore, studying inefficiencies is of utmost importance. The present paper aims to propose a cost production possibility set in a non-competitive environment (where prices can vary from one DMU to another). We compare the three PPSs so that we can introduce a new inefficiency source for DMUs based on the inappropriate choice of price by evaluating DMUs and comparing them with the existing cost production-possibility set frontier. And as a result, optimizing these price vectors can remove or, at least, reduce inefficiencies and create more efficient units.

Keywords: Data Envelopment Analysis, Price Efficiency, Inefficiency, Non-Competitive Environment**,** Unequal Prices.

1 Introduction

Data envelopment analysis (DEA) was proposed in [1] as a powerful tool for measuring relative efficiency of a set of decision-making units (DMUs).

If the data regarding input and output values of the decision-making units are available, we can discuss their technical efficiency measures. Moreover, if we also have the price data for input and output, it is also possible to analyze and evaluate the performance from the perspective of price and cost efficiencies.

Generally, DMUs can be assessed in competitive and non-competitive spaces. The prices of all DMUs are identical (or very similar) in competitive spaces.

In non-competitive spaces, prices can vary slightly or considerably in one or more indices. Prices can even vary in all indices and each input or output can have their own separate prices.

Various efficiencies have been investigated in previous studies on DEA. The concept of cost efficiency was first introduced by [2]. Later, [3] introduced linear programming models for the assessment of cost efficiency.

* **Corresponding Author. (✉)**
E-mail: Elham.rezaei@ms.khoiau.ac.ir , elham.rezaei50@gmail.com (E. Rezaei Hezaveh)

R. Fallahnejad
Department of Mathematics, Khorramabad Branch, Islamic Azad University, Khorramabad, Iran

E. Rezaei Hezaveh
Department of Mathematics, Khorramabad Branch, Islamic Azad University, Khorramabad, Iran

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When DMUs have identical inputs and outputs, and the prices of one DMU is multiple times higher than those of other DMUs, their cost efficiency will be the same and this is considered a flaw for CE. [4] and [5] discovered this flaw and tried to overcome it by posing the discussion of negotiation on various prices. [6] suggested a new production possibility set to solve this problem. Later, [7] presented a cost efficiency analysis and its application in comparing Japanese and American Electric Tools.

Various measures were taken to identify inefficiency sources, lost opportunities and cost efficiency in both competitive and non-competitive spaces.

For example, [8] studied technical and relative allocation inefficiency to the production and random production frontier. [9] investigated the specifications of technical and allocation inefficiency in random production and profit margin.

Later, he modeled allocation inefficiency in a transfer cost function and presented the related cost equations in 1997. [10] measured inefficiency in DEA and estimated the possible frontier.

Later, [11] estimated the shadow price of pollutants through production inefficiency and non-parametric distance function approach. [12] studied allocation and cost inefficiency in Spain State Hospital. [13] assessed allocation inefficiency through Basic System Approach. Later, [14] studied the index of output losses and input allocation inefficiency. [15] determined the sources of inefficiency in heterogeneous data using neural data in DAE. [16] introduced a format in DEA with a preferred structure for estimating overall inefficiency.

Later, [17] presented an analysis of profit instability in DEA through an overweight model. [6] and [7] proposed an interesting method for calculating efficiency where input prices can vary from one DMU to another.

They created new points by multiplying the price vector of each decision-making unit input by their input vectors and then created a production-possibility set based on these points.

After that, they calculated the efficiency of the corresponding units in the new space and considered radial efficiency as price efficiency.

The present paper aims to create a cost production-possibility set (similar to [7]) based on the modified units and observed prices and obtain unit price efficiencies in this production-possibility set.

In the next stage, we multiply each DMU by the price vectors of other DMUs (now, we have n^2 points).

We, then, use the points to create a new production-possibility set and measure the price efficiency of the DMUs in this new production-possibility set. Finally, we compare the DMUs of our new cost production-possibility set with a similar production-possibility set frontier in [7] and calculate their price efficiency in relation to that frontier.

Then, we use the price efficiencies obtained in previous stages and the price efficiency obtained in this stage to discover a new inefficiency source caused by the inappropriate choice of price.

The paper's structure is as follows: In section two, we introduce some basic concepts and definitions found in the literature regarding technical, price, and cost efficiencies.

In section three, after a brief discussion on the previous method proposed by [6] and [7], we propose our new method. Section four will provide an applied example, and finally section five will present a final conclusion.

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2 Backgrounds

Assume that there are n DMUs that use m " x_{ij} inputs" for production of s " y_{rj} output". The production possibility set is defined as the set of all Xs and Ys in which the output vector Y can be produced by the input vector X. Accepting basics of the inclusion of observations, convexity, feasibility, and constant returns to scale in DEA, the production possibility set is converted as follows:

$$P = \{(x, y) \mid x \geq X\lambda, y \leq Y\lambda, \lambda \geq 0\} \tag{1}$$

In the case that $1\lambda = 1$ constraint is added to the above-mentioned production possibility set; a production possibility set is generated with variable returns to scale. Based on the existence of units on the frontier of this production possibility set, evaluating units are classified into efficient and inefficient groups. Units on the efficient frontier are technically efficient and have an efficiency score of 1; otherwise, they are technically inefficient with score of efficiency less than 1. The CCR model for the input orientation of the envelopment form for the evaluated unit of DMU_o is as follows:

$$\begin{aligned} & \mathrm{E_o} = \min\theta \\ & \text{s.t} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io} \quad i = (1,\dots,m) \\ & \qquad \sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{ro} \quad r = (1,\dots,s) \\ & \qquad \lambda_j \geq 0 \quad j = (1,\dots,n) \end{aligned} \tag{2}$$

θ_o^* is the amount of radial efficiency corresponding to DMU_o. A point with coordinates of $(x_o^*, y_o^*) = (\sum_{j=1}^{n} \lambda_j^* x_j, \sum_{j=1}^{n} \lambda_j^* y_j)$ is the input-oriented projection of DMU_o on the strong efficiency frontier.

Assume that we are in a space where information about the price data for inputs and outputs of evaluated units is as c_{ij} $\forall i,j$ and p_{rj} $\forall r,j$. In general, since the market is not entirely competitive, prices vary from one unit to another. Suppose that we want to estimate the cost efficiency. In the traditional DEA, we first use the following model to find a point of frontier of the production possibility set with at least the same output value of evaluated unit of DMU_o and the least cost with the unit price vector o:

$$\begin{aligned} \min \quad & C_0 = c_o x \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_j \leq x \\ & \sum_{j=1}^{n} \lambda_j y_j \geq y_{ro} \\ & x, \lambda \geq 0 \end{aligned} \tag{3}$$

The cost efficiency of unit O is then defined as $\frac{c_o x^*}{c_o x_o}$ wherein that x^* is obtained by solving model (3).

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3 Cost efficiency and its factors in a non-competitive space

3.1 Proposed method by Tone et al [7]

The proposed method by [7] is briefly mentioned in this section.

Assume that there is n DMUs ($DMU_j, j=1,...,n$) each of which have m inputs $\left(x_{ij}\right), i=1,...,m$ for production of s outputs $\left(y_{rj}\right), r=1,...,s$. Furthermore, assume that $c=(c_1,...,c_m)$ is the price of inputs, and $p=\left(p_1,...,p_s\right)$ is the price of outputs. According to [6], if we use the initial model of cost efficiency by [2] for evaluation of DMUs in PPS (1), which have equal input and output, but different input prices, then the cost efficiency can be equal indicating the weakness of that method. Therefore, [6] provides the PPS (2) as follows:

$$P_c=\left\{\left(\bar{x},y\right)\middle|\bar{x}\geq\bar{X}\lambda, y\leq Y\lambda, \lambda\geq 0\right\} \tag{4}$$

Where, $\bar{X}=\left(\bar{x}_1,...,\bar{x}_n\right)$ is the input vector and $\bar{x}_j=\left(c_{1j}x_{1j},...,c_{mj}x_{mj}\right)^T$ is the cost vector. (It is the multiplication of price by input values). Here it is assumed that the matrices X and C are non-negative.

$\left(\bar{x}_0, y_0\right)$ corresponds to the evaluated observed unit of $\left(x_0, y_0\right)$ with input price vector of c_o in PPS (1).

[6] used the Input-Oriented CCR[2] model for evaluation of technical efficiency of $\left(\bar{x}_o, y_o\right)$ in the P_c set.

Technical efficiency of evaluated unit of $\left(\bar{x}_o, y_o\right)$ can be evaluated by the following model:

$$\begin{aligned} \bar{\theta}^* &= \min_{\bar{\theta},\lambda} \bar{\theta} \\ s.t \quad \bar{\theta}\bar{x}_o &\geq \bar{X}\lambda \\ y_o &\leq Y\lambda \\ \lambda &\geq 0 \end{aligned} \tag{5}$$

$\bar{\theta}^*$, which refers to the technical efficiency in the PPS (4), is equivalent to the price efficiency in the P.

$\bar{x}_o^*$ (minimum cost for DMU_O) is the optimal solution of the LP given below:

$$\begin{aligned} e\bar{x}_o^* &= \min_{\bar{x},\lambda} e\bar{x} \\ s.t \quad \bar{x} &\geq \bar{X}\lambda \\ y_o &\leq Y\lambda \\ \lambda &\geq 0 \end{aligned} \tag{6}$$

Where, $\bar{\gamma}^*=\dfrac{e\bar{x}_o^*}{e\bar{x}_o}$ is used as the cost efficiency. The allocative efficiency ($\bar{\alpha}^*$) is then introduced through dividing $\bar{\gamma}^*$ by $\bar{\theta}^*$.

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[2] Charnes Cooper Rhodes

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3.2 Proposed method

Suppose there are n DMUs ($DMU_j, j=1,...,n$) in the cost production-possibility set p, all of which contain input m ($(x_{ij}), i=1,...,m$) for producing output s ($(y_{rj}), r=1,...,s$). First, we calculate the technical efficiency of all the DMUs in the production-possibility set P (1). There are two possibilities: a technically efficient DMU or a technically inefficient DMU. If the DMU in P is technically efficient, we use the specifications of the same efficient DMU in the second stage. However, if the DMU in P is technically inefficient, we use the specifications of the projection points of the DMU in the second stage. Then, in the second stage, similar to [7], we multiply each of the technically efficient points or the projection of the technically inefficient points by their price vectors and create the new cost production-possibility set P_c and calculate the price efficiency of all the DMUs (we denote price efficiency or cost technical efficiency DMU_o by DTE_0). In the third stage, we multiply each DMU by the price vector of the other DMUs. However, this time we have n^2 DMUs and create a new cost production-possibility set (P_c') and calculate the price efficiency of the DMUs in relation to the frontier of the new cost production-possibility. Now, we can compare the price efficiencies obtained for all the DMUs in P_c and P_c'. Since technical inefficiencies regarding input and output were removed in the first stage, all the remaining inefficiencies in this stage are caused by the inefficiency of the price vectors. In other words, this inefficiency is caused by the inappropriate choice of price, meaning a DMU like DMU_o is price inefficient, with its corresponding price vector, but the same DMU_o is either price efficient or superefficient with the price vectors of other DMUs or its price inefficiency is lower compared to when it was evaluated with its own price vector. This suggests that the price vector chosen at first for the DMU_o was inappropriate and this DMU could have been more price efficient with the price vector of another DMU. Thus, we define inefficiency caused by inappropriate choice of price for DMU_o as one, minus the ratio between the observed price efficiency of DMU_o in P_c and maximum price efficiency for the corresponding DMU DMU_o in P_c' and show it as follows.

Definition 1:

Inefficiency caused by inappropriate choice for $DMU_o = 1 - \dfrac{\overline{DTE}_o}{\overline{DTE'}_{\max_o}}$

Where $\overline{DTE}_o$ is the observed price efficiency of DMU_o in P_c and $\overline{DTE'}_{\max_o}$ is the maximum price efficiency for the corresponding DMU_o in P_c'.

4.1 An Empirical example

In this section, in order to investigate the accuracy of provided method in the previous sections, observe new defined inefficiency, numerical example have been brought.

Assume there are 4 DMUs that each of which have two inputs and one output. Table 1 shows the related information to DMUs in production possibility set P.

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Table1 information to DMUs in P

DMU	(x_1,x_2)	Y	(x_1^*,x_2^*)	θ^*
1	(3,5)	1	(1,2.62)	0/25
2	(1,3)	1	(1,3)	1
3	(6,2)	1	(3,1)	0/61
4	(4,1)	1	(4,1)	1

In addition, figure 1 shows data of table 1 in a production possibility set P.

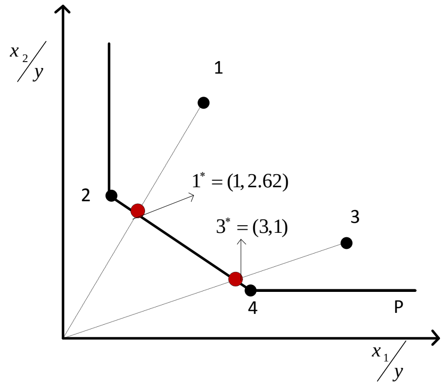

Fig. 1 Production possibility set P

The first stage, an inefficient DMUs means DMU_1 and DMU_3 to projected on the frontier and remove their technical inefficiency to reach $DMU_1^* = (1, 2.62)$ and $DMU_3^* = (3,1)$ points. Hereinafter, DMU_1 and DMU_3 depicted projection points, $DMU_1^* = (1, 2.62)$ and $DMU_3^* = (3,1)$ is used instead of DMU_1 and DMU_3 in the next steps.

The second stage, price vector is considered on DMUs, prices are multiplied in inputs according to the production possibility set of [7], and production possibility set P_c is formed. Finally, price efficiency of all DMUs is measured in P_c. Table 2 shows information to DMUs in P_c.

Table 2 information to DMUs and price efficiency in P_c

DMU	(x_1,x_2)	(c_1,c_2)	Y	$(\bar{x}_1,\bar{x}_2)$	price efficiency
1	(1,2.62)	(5,1)	1	(5,2.26)	1
2	(1,3)	(4,2)	1	(4,6)	1
3	(3,1)	(3,7)	1	(9,7)	0/52
4	(4,1)	(3,6)	1	(12,6)	0/41

Moreover, figure 2 shows data of table 2 in a production possibility set P_c.

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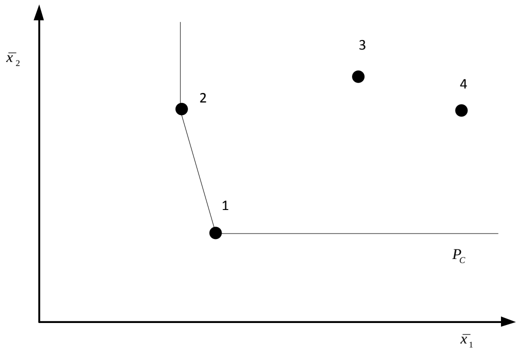

Fig. 2 production possibility set P_c

In the third stage, each DMU is multiplied to price vector of other DMUs. There are $4^2 = 16$ DMUs this time that 4 of them are real (initial DMUs) and we have defined them in table 3 with (•) and the rest are virtual (new DMUs obtained by multiplication of the inputs of a DMU in the price information of other DMUs). Then, we build the production possibility set, P_c'. Table 3 shows the related information to the price efficiency and data of DMUs in P_c', and figure 3 shows both price production possibility sets P_c and P_c' .

Table 3 information to DMUs and cost efficiency in P_c'

DMU	(x_1, x_2)	(c_1, c_2)	$(\bar{x}_1', \bar{x}_2')$	price efficiency
• 1	(1,2.62)	(5,1)	(5,2.62)	1
2	(1,2.62)	(4,2)	(4,5.24)	1
3	(1,2.62)	(3,7)	(3,18.34)	1
4	(1,2.62)	(3,6)	(3,15.72)	1
5	(1,3)	(5,1)	(5,3)	0/96
• 6	(1,3)	(4,2)	(4,6)	0/98
7	(1,3)	(3,7)	(3,21)	1
8	(1,3)	(3,6)	(3,18)	1
9	(3,1)	(5,1)	(15,1)	1
10	(3,1)	(4,2)	(12,2)	0/82
• 11	(3,1)	(3,7)	(9,7)	051
12	(3,1)	(3,6)	(9,6)	0/52
13	(4,1)	(5,1)	(20,1)	1
14	(4,1)	(4,2)	(16,2)	0/72
15	(4,1)	(3,7)	(12,7)	0/40
• 16	(4,1)	(3,6)	(12,6)	0.41

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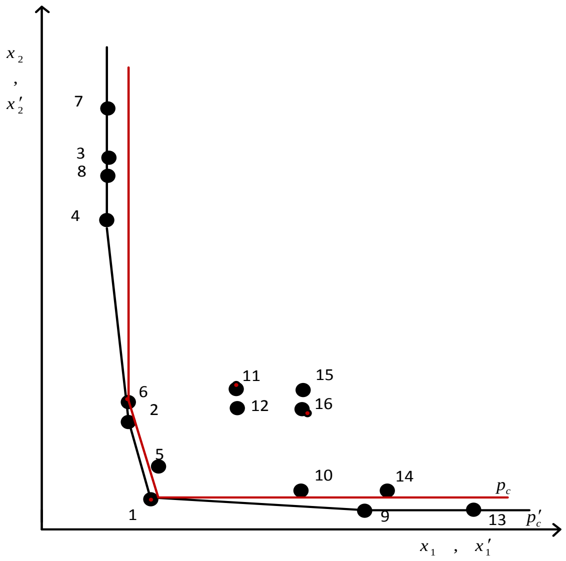

Fig 3 production possibility sets P_c and P_c'

Comparing P_c and P_c' (Fig. 3) in P_c' shows super-efficiency. Having a DMU become efficient on the P_c' frontier shows an inappropriate choice of price vector in the beginning. Otherwise, it should have reached this stage in the beginning. On the other hand, since we have removed the technical inefficiencies of the DMUs, we now know that the deficiency is not associated with input and output and is directly associated with input prices. In fact, this means that the price vector DMU_o is inappropriate and the price vectors of other DMUs are more appropriate for DMU_o .

Now, we calculate the super-efficiency value resulting from the difference between the two production-possibility sets P_c and P_c' . Table 4 shows the price efficiency of all the existing $4^2 = 16$ DMUs in the production-possibility set P_c' compared to P_c frontier.

Table 4 Price efficiency of all the existing DMUs in the production-possibility set P_c' compared to P_c frontier

DMU	$(\bar{x}_1', \bar{x}_2')$	Price efficiency DMUs compared to P_c frontier
• 1	(5,2.62)	1
2	(4,5.24)	1/04
3	(3,18.34)	1/33
4	(3,15.72)	1/33
5	(5,3)	0/97
• 6	(4,6)	1
7	(3,21)	1/33
8	(3,18)	1/33
9	(15,1)	2/26
10	(12,2)	1/13
• 11	(9,7)	0/52
12	(9,6)	0/53
13	(20,1)	2/26
14	(16,2)	1/13
15	(12,7)	0/40
• 16	(12,6)	0/41

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In Table 2, we see, for example, that DMU_3 is not price efficient with its associated price vector in the production-possibility set P_c. However, it is observed that DMU_9 resulting from multiplying DMU_3 input by price vector DMU_1 has the highest price efficiency for the P_c' frontier. This means that the price vector DMU_3 is inappropriate and the price vector DMU_1 is a better fit for DMU_3.

Now, we calculate the inefficiency resulting from inappropriate price for each of the real (primary) DMUs as follows and present its results in Table 5.

Inefficiency resulting from inappropriate price $DMU_1 = 1 - \frac{\overline{DTE}_1}{\overline{DTE'}_{\max_1}} = 1 - \frac{1}{1.33} = 1 - 0.75 = 0.25$

Inefficiency resulting from inappropriate price $DMU_2 = 1 - \frac{\overline{DTE}_2}{\overline{DTE'}_{\max_2}} = 1 - \frac{1}{1.33} = 1 - 0.75 = 0.25$

Inefficiency resulting from inappropriate price $DMU_3 = 1 - \frac{\overline{DTE}_3}{\overline{DTE'}_{\max_3}} = 1 - \frac{0.52}{2.26} = 1 - 0.23 = 0.77$

Inefficiency resulting from inappropriate price $DMU_4 = 1 - \frac{\overline{DTE}_4}{\overline{DTE'}_{\max_4}} = 1 - \frac{0.41}{2.26} = 1 - 0.18 = 0.82$

Table 5 Inefficiency value resulting from inappropriate price for each of real (primary) DMUs

DMU	inefficiency
1	0/25
2	0/25
3	0/77
4	0/82

4.2 Discussion

In this study, first, we calculate the technical efficiency of all the DMUs in the production-possibility set P (1). Then, in the second stage we multiply each of the technically efficient points or the projection of the technically inefficient points by their price vectors and create the new cost production-possibility set P_c and calculate the price efficiency of all the DMUs (we denote price efficiency or cost technical efficiency DMU_o by DTE_0). In the third stage, we multiply each DMU by the price vector of the other DMUs. However, this time we have n^2 DMUs and create a new cost production-possibility set (P_c') and calculate the price efficiency of the DMUs in relation to the frontier of the new cost production-possibility. Then compare the price efficiencies obtained for all the DMUs in P_c and P_c'. Since technical inefficiencies regarding input and output were removed in the first stage, all the remaining inefficiencies in this stage are caused by the inefficiency of the price vectors. In other words, this inefficiency is caused by the inappropriate choice of price. Thus, we define inefficiency caused by inappropriate choice of price for DMU_o as one, minus the ratio between the observed price efficiency of DMU_o in P_c and maximum price efficiency for the corresponding DMU DMU_o in P_c'. But in an article accepted in the journal of new researches in mathematics, The first and second steps are similar to this article but in third stage, it was possible to compare n efficient units of previous stage in production possibility set

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similar to P_c . Then we constructed the third production possibility set that is similar to cost PPS (P_c) and found the radial and allocative inefficiencies in this PPS and reached point $(\tilde{x}_o^{**}, y_o)$ by removing cost technical inefficiency with the least cost. Then we decomposed the actual cost of each DMU as follows:

$C_o = L_o^* + L_{new_o}^* + L_o^{**} + L_o^{***} + C_o^{***}$

Where L_o^* is the difference of these two costs is considered as the cost loss corresponding to the technical inefficiency and $L_{new_o}^*$ is (loss) the lost opportunity due to improper selection of price vector in cost and L_o^{**} is (lost) to cost technical inefficiency and L_o^{***} is (lost) corresponding to the cost allocative inefficiency and finally we reached point C_o^{***} $(\tilde{x}_o^{**}, y_o)$ which has the least cost.

5 Conclusion

In the present paper, based on radial projection points of the input nature in technology and price production-possibility sets, we generated points with price components, based on which the new price-efficient cost production-possibility set P_c' was generated. We also compared the cost production-possibility sets P_c and P_c' introduced the ratio of observed price DMU_o in P_c to maximum price efficiency value for the corresponding DMU DMU_o in P_c as the inefficiency resulting from inappropriate choice of price vector. Since we removed technical inefficiencies in the early stages, the remaining inefficiencies are all associated with the inappropriate prices selected for each unit. Optimizing these vectors can remove or, at least, reduce inefficiencies and create more efficient units.

In this paper, we Using an empirical example, we investigated several DMU in two production possibility sets P_c and P_c'. It was observed that some DMUs are not associated with the price vector associated with themselves in the possibility of producing P_c. However, in Table 4, it is observed that some virtual DMUs that have been product from multiplying each DMU by the price vector of the other DMUs has the highest price efficiency for the P_c' frontier and this means that the price vector first DMU is inappropriate and the price vector another DMUs is a better fit for that.

Although our focus, in the present paper, was on constant returns to scale, the proposed method can also be used for variable returns to scale.

Future studies can focus on developing our proposed method for inaccurate decision-making unit data or prices, and also for network structures in data envelopment analysis and unit performance assessment based on the time factor. In addition, developing the proposed method variable prices and input can also be another topic of interest.

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