Measuring technological gap ratio of wheat production using StoNED approach to metafrontier

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Abstract The aim of this paper is to use the concept of the metafrontier function to study the determination of efficiency differentials and Technological Gap Ratio (TGR) on wheat production in Khorasan Razavi province. In this study, we used the metafrontier function and group frontier based on the concept of Stochastic Nonparametric Envelopment of Data analysis (StoNED). The data used in this study consisted of a sample of 435 wheat farms in 2011. The data samples collected from wheat farms are divided into three sizes namely small, medium and large farms. The results of estimating group frontier production functions indicate that the mean technical efficiency (TE) for small, medium and large farms are 0.426, 0.606 and 0.365, respectively. The figures were found to be mean TE for small, medium and large farms are 0.286, 0.239 and 0.248 when evaluated based on the metafrontier production function representing that the highest mean TE is devoted to small farms, while medium farms has the lowest mean technical efficiency. The average technological gap ratios for small, medium and large farms were 0.727, 0.463 and 0.725, respectively. Therefore, the medium farms frontier has the most distant to the metafrontier, while the small and large farm frontiers have the closest.

Keywords StoNED, Metafrontier, Technology Gap Ratios, Wheat.

1 Introduction

Technical efficiency (TE) can be defined as the ability of a decision-maker to produce maximum output given a set of inputs and technology [1]. Estimates of technical inefficiency in agricultural production are now commonplace; yet they are suspect so long as variations exist in production technology among sampled farmers. Such variations are the norm rather than the exception, from subtle changes in ways of doing things, such as slight differences in input attributes, to major differences such as use of significantly different production technologies and differences in environmental conditions. The usual methods of dealing with these technology differences risk attributing "technology gaps" between farms to technical inefficiency [2]. A recent methodological advance in estimating technical inefficiency that minimizes this risk by specifying a metafrontier for production allows technology gaps to be distinguished from technical inefficiency [3].
A formal theoretical framework is conducted to make efficiency comparisons across
groups of units. This concept can make into the practical way by measuring efficiency relative
to a common metafrontier. The metafrontier is defined as the area of an infinite production
technology set. In addition, group frontiers are proposed to be the areas of limited production
technology sets [4].

Lack of economic infrastructure and other characteristics of the production environment
are the restrictions. Therefore, the production efficiency in technology assessed relative to the
metafrontier can be classified into two components. The first component is a distance
measurement from an input-output point to the group frontier, namely, technical efficiency
(TE). The second component is a distance measurement between the group frontier and the
metafrontier, namely, Technological Gap Ratios (TGRs). The latter component represents the
restrictive nature of the production environment [4].

Metafrontier analysis is an approach that allows comparison between different
 technologies [5]. The attractive feature of metafrontier model is that it takes into account any
heterogeneity between firms (in this study, different farm size in wheat farming) in the
comparison of efficiency [8]. A metafrontier may be considered as an umbrella (upper or
lower) of all possible frontiers that might arise as a result of heterogeneity between firms [9].
This model therefore produces the maximum output from a given input using the best
technology. Since its introduction, the metafrontier function has been used in a wide range of
studies covering diverse topics, including agriculture [8], hotels [6], hospitals [6] and dairy
farms [9]. The reviewed literature demonstrates that the metafrontier approach is a well-
established tool for evaluating efficiency analysis of non-homogeneous firms.

The main objective of this research is to present how metafrontier function and group
frontier work efficiently based on the concept of Stochastic Nonparametric Envelopment of
Data analysis (StoNED). The StoNED is non-parametric and stochastic approach to efficiency
measurement. Therefore, this research considers the StoNED approach to break down the
difference inefficiency performance of technical efficiency and technical gap effects. In
addition, this paper reports on an analysis of relationship between farm size and technological
gap ratios in wheat farms in Khorasan Razavi province in Iran. Wheat is considered one of the
most important agricultural commodities in Iran in terms of production and consumption. It is
grown on about 43 percent of the total agricultural area and 51 percent of the total cropped
area. It is likewise one of the important sources of income and employment of rural people
[10]. Average production is about 10.6 million tons with an average yield of 1.78 tons per

Per capita consumption of wheat in the country is about 180 kilogram per year, which is
higher than the world per capita consumption of 100 kilogram per year [12]. Iran is the
seventh largest wheat importer in the world because domestic production is still insufficient to
meet domestic demands. The Iranian government encourages farmers to produce more wheat
by increasing farm productivity and efficiency [10].

Zibaei estimated the technical efficiency of wheat farmers in Fars province using the
stochastic frontier production analytical (SFA) approach. A Cobb-Douglas frontier production
function was estimated and average technical efficiency of 68 percent and 80 percent was
obtained for 1989 and 1992 [13], respectively. In a similar study, Shirvanian estimated
technical, allocative and economic efficiency for 73 wheat farms in the District municipality
of Darab, Fars province. Average technical, allocative and economic efficiency indices, were
estimated as 74, 35 and 30 percent, respectively [14]. Karim koshteh calculated technical,
allocative and economic efficiency of wheat farm in Sistan and Baluchestan. A Cob-Douglas
function was estimated using corrected ordinary least squares (COLS) and maximum
likelihood (ML) estimation. Average technical efficiency indices of 50 percent and 62 percent were obtained using COLS and ML, respectively, while average allocative and economic efficiency indices were estimated to be 63 percent and 38 percent [15].

Mehrabi et al. estimated the technical efficiency of wheat farmers of Kerman province with translog production function. They obtained the technology gap, between five main regions which were producing wheat. Results showed that there is a higher technology gap ratio respected to metafrontier in drought regions than others [10].

Mehrabi et al. reported a significant technology gap among three varieties of pistachio with using a selected sample random of 475 pistachio farms of Kerman in 2004 which included three main varieties of pistachio named, Kalleh Ghuchi, Fandoghi and Akbari. After using LR test, they applied the translog production function and the mean value of varietal technology gap ratios were estimated as 0.58, 0.581 and 0.68 for Kalleh Ghuchi, Akbari and Fandoghi varieties, respectively. Results emphasized the importance of taking into account the differences in frontiers imposed by different three varieties [16].

Anyway evidence obtained from earlier studies of technical efficiency in for wheat production in Iran shows a wide range of technical efficiency scores that range from 28 percent to 81 percent.

This paper reports on an analysis of relationship between farm size and Technology Gap Ratio in wheat farms in the Khorasan Razavi province. In this study, a random sample of 434 farmers was selected in 2011. Data samples collected from wheat farms were divided into three sizes: small, medium and large farms.

In this study, we applied the metafrontier model to compare the technological gap and efficiency of wheat farming in different group (different farm size). We apply our results to consider the importance of quantitatively comparing the efficiency of farms that use different technologies.

The paper is organized as follows. At first we present the methods and estimation approaches applied in the study. The results are presented thereafter and the last section concludes the study.

2 Method of Analysis

We can decompose technical inefficiency with respect to metafrontier into the product of technical inefficiency in the specific group and the gap between metafrontier and the group frontier. Previously mentioned authors have suggested either stochastic and parametric or non-stochastic and nonparametric determination of technology frontiers.

With respect to group specific frontiers, the advantage of the metafrontier approaches is that they are able to separate the technical efficiency difference between groups [17]. This is not necessarily a valid assumption when we apply the analysis on a limited number of groups. This is illustrated by Figure 1 which represents a piecewise concave envelopment of the frontier. There is not a joint concave envelopment of the frontier but only a piecewise concave envelopment which is determined by one of the groups in turn [17].
Data envelopment analysis (DEA) can be applied in these comparisons. The virtue of the DEA is that no specific functional form has to be assumed. On the other hand, the conventional DEA does not make any difference between stochastic noise and inefficiency but all deviations from the frontier are interpreted as inefficiencies. The DEA is fairly easy to apply also in the metafrontier approach: we have to solve separate models for each group in order to specify the group-specific technical efficiency (GTE) and one for the joint data set for solving the metafrontier technical efficiency (MTE). Technological gap ratio, the relative productivity of technologies can be obtained by the ratio between MTE and GTE.

I. ESTIMATION OF FRONTIERS

Data envelopment analysis is a non-parametric but non-stochastic method. In this study we applied also the stochastic non-parametric estimation method, which has been developed by Kuosmanen [18]. StoNED model applies a two stage method, which is applied for each group separately. At first a piecewise linear production function is estimated. Concave nonparametric least squares (CNLS) can be written as a quadratic programming problem.

\[
\begin{align*}
\text{Min} & \quad \sum_{i=1}^{n} \hat{\varepsilon}_i^2 \\
\text{s.t.} & \quad y_i = \alpha_i + \beta'_i x_i + \hat{\varepsilon}_i, \quad \forall_i = 1, \ldots, n, \\
& \quad y_h \leq \alpha_i + \beta'_i x_h + \hat{\varepsilon}_h, \quad \forall_h = 1, \ldots, n, \\
& \quad \beta'_i \geq 0, \quad \forall_i = 1, \ldots, n.
\end{align*}
\]

CNLS allows for the intercept and the slope coefficients to vary from one firm to another. Thus, there are (n) different slope vectors \( \beta_i, \ i=1,..,n \). The CNLS regression (1) estimates n tangent hyper planes to one unspecified production function. The slope coefficients \( \beta_i \) represent the marginal products of inputs. The second constraint imposes concavity by applying a system of inequality constraints known as “Afriat inequalities” [19]. The third constraint imposes monotonicity [20]. The CNLS regression provides us with the composite residuals (\( \hat{\varepsilon} \)) which consist of error and inefficiency. To disentangle these two components, we can use the method of moments and calculate the second and third central moments of residual distributions [20].
\[ m_2 = \sum_{i=1}^{n} (\hat{e}_i - \hat{E}(e_i))^2 / n \] (2)

\[ m_3 = \sum_{i=1}^{n} (\hat{e}_i - \hat{E}(e_i))^3 / n \] (3)

These moments are consistent estimators of the true moments \( \mu_2, \mu_3 \) which depend on the variance of the inefficiency term and the error terms in the following manner [20].

\[ \mu_2 = \left( \frac{\pi - 2}{\pi} \right) \sigma_u^2 + \sigma_v^2 \] (4)

\[ \mu_3 = \left( \frac{\sqrt{2}}{\sqrt{\pi}} \right) \left( 1 - \frac{4}{\pi} \right) \sigma_u^3 \] (5)

Thus, the variances \( \sigma_v^2, \sigma_u^2 \) can estimate based on the moments \( m_2 \) and \( m_3 \) [21]. Thus, given the estimated (which should be negative), we can estimate \( \hat{\sigma}_u \) parameter by eq. (6)

\[ \hat{\sigma}_u = \sqrt{\frac{m_3}{\left( \frac{\pi - 2}{\pi} \right) \left( 1 - \frac{4}{\pi} \right) \sigma_u^3}} \] (6)

Subsequently, the standard deviation of the error term \( \hat{\sigma}_i \) is estimated using eq. (7)

\[ \hat{\sigma}_i = \sqrt{\hat{m} - \left( \frac{\pi - 2}{\pi} \right) \sigma_i^2} \] (7)

Given the variance estimates, we can use the conditional estimator for the inefficiency term. Jondrow et al. showed that the conditional distribution of inefficiency \( u_i \) given \( \hat{e}_i \) is a zero truncated normal distribution with mean \( \mu \) and variance \( \sigma^2 \) [22] that presented by eq. (8) and (9).

\[ \mu = -\hat{e}_i \frac{\sigma_u^2}{\sigma_v^2 + \sigma_u^2} \] (8)

\[ \sigma^2 = \frac{\sigma_u^2 \sigma_v^2}{\sigma_v^2 + \sigma_u^2} \] (9)

As a point estimator for \( u_i \), one can use the conditional mean

\[ E(u_i | \hat{e}_i) = \mu_i + \sigma \left[ \frac{\varphi(-\mu_i / \sigma)}{1 - \Phi(-\mu_i / \sigma)} \right] \] (10)
Where $\bar{\Phi}$ is the standard normal density function and $\Phi$ is the standard normal cumulative distribution function [23].

Given the estimated $\hat{\sigma}_u^2,\hat{\sigma}_v^2$ parameters, the conditional expected value of inefficiency can be computed as

$$\hat{E}(u_i|\hat{e}_i) = \frac{-\hat{e}_i \hat{\sigma}_u^2}{\hat{\sigma}_u^2 + \hat{\sigma}_v^2} + \frac{\hat{\sigma}_v^2 \hat{\sigma}_u^2}{\hat{\sigma}_u^2 + \hat{\sigma}_v^2} \left[ \frac{\phi(\hat{e}_i/\hat{\sigma}_v^2)}{1 - \bar{\Phi}(\hat{e}_i/\hat{\sigma}_v^2)} \right]$$  \hspace{1cm} (11)

Where $\hat{e}_i = \hat{v}_i - \hat{\sigma}_u \sqrt{\pi/2}$ is the estimator of the composite error term.

**II. DEA Approach to Metafrontier**

The metafrontier is constructed using a DEA model based on the pooled data for all the units in all the regions. Since we have a total of $L = \sum_{k=1}^{n} L_k$ units, we run the below linear programming model with data for all units. The structure of this linear programming is as follows:

$$\max \quad \phi^*, \lambda^* \phi,$$

s.t.

$$-\phi^* y_i + y^* \lambda^* \geq 0,$$

$$x_i - X^* \lambda^* \geq 0,$$

$$\lambda^* \geq 0.$$  \hspace{1cm} (12)

where

- $y_i$ is the matrix $M^*1$ vector of output quantities for the $i$-th farm;
- $x_i$ is the $N^*1$ vector of input quantities for the $i$-th farm;
- $y^*$ is the $M^*L$ matrix of output quantities for farms;
- $X^*$ is the $N^*L$ matrix of input quantities for farms;
- $\lambda^*$ is the $L^*1$ vector of weights; and
- $\phi^*$ is a scalar provides information on the technical efficiency score for the $i$-th farm.

StoNED is thus applied in estimating the group specific efficiencies. This information is also used in determining the expected value for inefficiency in each group. Technological gap ratios are solved by applying DEA on the joint data, where the original output is replaced by the inefficiency corrected output estimate. In this case the DEA efficiency score shows directly the technological gap ratios [17]. Units are shown to be not more technically efficient when they are assessed against the metafrontier than against the group frontier.

When the metafrontier envelopes all group production frontiers, the efficiency can be decomposed into two components (metafrontier efficiency and group frontier efficiency) and the ratio of these two can be called as technological gap ratios (MTR). Their dependency on each other can be expressed as follow eq. (13).

$$MTE = GTE \times MTR$$  \hspace{1cm} (13)
The data used in this study include a sample of 435 wheat farmers that were taken from census conducted by Jihad Agriculture organization of Iran in 2011. Since the application of StoNED and DEA requires farms that their number of input and output variables must be kept at reasonable level, we consider five important input variables as follows:
X1 = is total area planted area of wheat (in hectares),
X2 = quantity of used chemical fertilizer (in kg),
X3 = represent the cost of machinery (in Rial),
X4 = labor force (in person-day),
X5 = Divisia index to account for other inputs such as seed, irrigation, transportation, and pesticides (in Rial).
And output variable \( y_i \) is the wheat output of the \( i \)th farm (in kg).

3 Results and discussion

Before estimating the metafrontier, we must be sure that the efficiencies of the selected groups really differ between them. For this purpose, we used the Kruskal-Wallis non-parametric test, in which the results were considered as significant if the \( p \) value was equal or smaller than 0.05. From a statistical point of view, the Kruskal–Wallis test does not require a normal distribution of the analyzed variables. This test serves to contrast the hypothesis that \( k \) samples or quantitative groups have been obtained from the same population, provided there are differences between several samples. Therefore, the null hypothesis argues that the \( k \) samples belong to the same population, while the alternative hypothesis says the opposite. The test has been applied to the efficiency scores of the firms included in the three groups selected. The results shown in Table 1 prove that the null hypothesis is rejected with a level of statistical significance of 95%, which implies that the samples are significantly different from each other. Consequently, it is demonstrated that the differences among the three groups are statistically significant and it is appropriate to evaluate their efficiency using metafrontier methodology.

<table>
<thead>
<tr>
<th></th>
<th>Chi-square</th>
<th>Freedom degrees</th>
<th>P - Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>112.822</td>
<td>2</td>
<td>.000</td>
</tr>
</tbody>
</table>

The next step of the analysis is to estimate the stochastic nonparametric envelopment of data for the three group frontiers (small, medium and large farms) were obtained using GAMS software. The results are summarized in table 2. According to table 2, Average technical efficiency from regional frontier \( \text{TE}^k \) and metafrontier \( \text{TE}^* \) and technology gap ratio \( \text{TGR} \) estimates for groups are shown.
Table 2 estimates of TEs and TGRs

<table>
<thead>
<tr>
<th>Model</th>
<th>Item</th>
<th>Farm category</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Small</td>
<td>Medium</td>
</tr>
<tr>
<td>TE</td>
<td>Mean</td>
<td>0.545</td>
<td>0.486</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.026</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.263</td>
<td>0.207</td>
</tr>
<tr>
<td>TE*</td>
<td>Mean</td>
<td>0.426</td>
<td>0.606</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.101</td>
<td>0.122</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.830</td>
<td>0.868</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.156</td>
<td>0.192</td>
</tr>
<tr>
<td>TE'</td>
<td>Mean</td>
<td>0.286</td>
<td>0.239</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.101</td>
<td>0.122</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.640</td>
<td>0.351</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.080</td>
<td>0.052</td>
</tr>
<tr>
<td>TGR</td>
<td>Mean</td>
<td>0.727</td>
<td>0.463</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.201</td>
<td>0.147</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.194</td>
<td>0.223</td>
</tr>
</tbody>
</table>

For group one (small farms), the average technical efficiency score is 0.426, indicating wheat production is increased by about 43% of the potential, given its group frontier. In other words, the technical efficiency score shows that the mean gap between the best farmer and other farmers is about 57% in group one. But the mean technical efficiency of this group is 0.286 when assessed based on the metafrontier and the mean technology gap ratio in this group is 0.727. This means that, the potential wheat production for group one is about 73% of that represented by the metatechnology.

For group two (medium farms), the average technical efficiency score is 0.606, indicating wheat production is increased by about 61% of the potential, given its group frontier. In other words, the technical efficiency score shows that the mean gap between the best farmer and other farmers is about 39% in group two. But the mean technical efficiency of this group is 0.239 when assessed based on the metafrontier and the mean technology gap ratio in this group is 0.463. This means that, the potential wheat production for group one is about 46% of that represented by the metatechnology.

For group three (large farms), the average technical efficiency score is 0.365, indicating wheat production is increased by about 36% of the potential, given its group frontier. In other words, the technical efficiency score shows that the mean gap between the best farmer and other farmers is about 64% in group three. But the mean technical efficiency of this group is 0.248 when assessed based on the metafrontier and the mean technology gap ratio in this group is 0.725. This means that, the potential wheat production for group one is about 72% of that represented by the metatechnology.

Also, results from group frontier show that mean gap between the best producer and other producer is minimum in medium farms while it is maximum in large farms.

Our results about technology gap rate are somewhat different from previous literature study for wheat production. Mehrabi et al. found that there were negative relationship between the size of farms and technology gap rates in Kerman province [10]. Our results indicate that the average technology gap ratio for medium farms is less than small and large farms. This difference in efficiency between medium farms and small / large farms can be
linked to a number of factors. Small farmers tend to invest more labor in their land. That makes it more efficient and the quality of the labor is much better. From this perspective, small farms are more efficient, producing more per unit than medium and large farms. In the wheat farming context it is assumed that large farms are more efficient oriented and achieve larger economy of scale than medium farms ones. Large farms also often find it easier and cheaper to finance investment.

It is no stretch to say that the argument for redistribution of land is bolstered by this study. Giving land to small or large farms will increase overall production, as well as improve the welfare of the small and landless peasantry.

4 Conclusions

Agricultural sector is under pressure to satisfy multiple, often competing demands, such as to produce more crops, pollute less, and fulfill consumer preferences with increasingly scarcity resources. To evolve farming systems that meet all at these demand, productivity growth can play a vital role especially in developing countries. Due to the importance of efficiency in productivity growth, the goal of this paper is to use the concept of the metafrontier function to study the determination of efficiency differentials between farm size and technological gap ratio on wheat in Khorasan Razavi province. The requirement data used in this study consisted of a sample of 435 farms that were taken from farm census conducted by the Jihad Agriculture organization in 2011.

The samples collected from wheat farms are divided into three sizes; small, medium and large. Before estimating the metafrontier, we must be sure that the efficiency of the groups selected really differs between them. For this purpose, we used the Kruskal-Wallis test. Consequently, the Kruskal-Wallis test demonstrated that the differences among the three groups are statistically significant and it is appropriate to evaluate their efficiency using the metafrontier methodology. The results of estimating group frontier production function showed that mean TE for small, medium and large farms are 0.426, 0.606 and 0.365 receptively. This implies that, there are possibilities for either increasing the total production of wheat using the same inputs or decreasing input for the current level of wheat production or a mixture of both by filling the gap between the best farmer and other farmers. The mean TE for small, medium and large farms are 0.286, 0.239 and 0.248 when evaluated based on the metafrontier production function that the highest mean TE is devoted to small farms, while medium farms has the lowest mean technical efficiency. The average technological gap ratio for small, medium and large farms is 0.727, 0.463 and 0.725, respectively. Therefore, the medium farms frontier has the most distant to the metafrontier while the small farms frontier has the closest. The average technological gap ratio for small and large farms is not significantly different from each other and both are higher than the value for medium farms.

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References


