Development Model for Supply Chain Network Design by Demand Uncertainty and Mode Selection

I. Rahimi, M. T. Askari*, S. H. Tang, L. S. Lee, S. Azfanizam Binti Ahmad, Adel M. Sharaf

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Abstract It is necessary to consider the impact of demand uncertainty to model the comprehensive approach for supply chain network design. This paper presents four echelons, multiple commodity, and strategic–tactical model for designing supply chain network. Uncertain demand, transportation mode selection with lead time configuration has been considered. A numerical example has been implemented to verify the applicability of model. Finally, the simulation results and sensitivity analysis confirm that the proposed developed model is a suitable decision framework for designing the supply chain network.

Keywords: Supply Chain Network Design, Mode Selection, Demand Uncertainty.

1 Introduction

Facility location has been considered as a suitable research in the field of operations research, which are considered as a branch of decision making policies [1]. According to many research papers and books. Even American Mathematics Society (AMS) has created special codes for facility location and recently European Research Society (EURO) assign special society to this issue. However there are debates about the application of facility models. Conversely, application advantage of logistics has not been considered as an issue. One of the logistic issues which have been considered is supply chain management (SCM). Truly, development of supply chain management (SCM) started from operations research independently, and operations research goes into supply chain management moderately. Melo et al. [2] has reviewed facility location models in supply chain management. Tang et al. [3] have reviewed about integrating supply chain network. Usually there are three levels of decisions in supply chain management:

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Strategic, tactical, and operational. Simchi-Levi et al. [4] mentioned that strategic decision has long-term effects on companies. These decisions include: number (quantity), location, capacity of Strategic, Tactical, and Operational. Simchi-Levi et al. [4] mentioned that strategic decision has long-term effects on companies. These decision includes: number, location, capacity of warehouses and plants, or flow of products in logistic systems [5,6].

**Nomenclature**

**Index sets:**
- I set of plants
- J set of warehouses
- P set of products
- C set of customers
- S set of suppliers
- R set of raw materials
- O set of capacity of warehouses
- E set of capacity of plants
- TR set of available transportation modes

**Parameters**

- INV investment
- F net profit
- BIG M a large number
- $PR_p$ Selling price of a unit P to customers
- $PS_{r,s}$ Price of raw material supplied by supplier s
- $MV_{p, tr}$ Monetary value per unit of lead-time for product p in mode tr
- $CO_i^o$ fixed cost of opening plant I with capacity level o;
- $CO_l^e$ fixed cost of opening warehouse j with capacity level e
- $CU_i^o$ fixed cost of operating plant I with capacity level o;
- $CU_l^e$ fixed cost of operating warehouse with capacity level e;
- $CS_{p,j}$ storage cost of unit p at warehouse j;
- $CD_{r,s,i}$ transportation cost of product p from supplier to plant I;
- $CT_{p,i,j}$ transportation cost from plant I to warehouse j;
- $CF_{p,j,c}$ transportation cost from warehouse to customer;
- $UC1_{i,j}^v, UC2_{i,j}^v, UC3_{i,j}^v$ Unit fixed cost of using transportation mode tr;
- $A(i,j)$ Number of delivery from plant to warehouse;

**Decision variables**

- $X_i^o$ 1 if facility active with capacity level o, 0 otherwise;
- $y_i^e$ 1 if facility active with capacity level e; 0 otherwise;
- $Z_{i,j}^{tr}$ integer decision variable which determines the required number of mode tr for delivery goods between two point a and b;
Usually there are different international transportation modes that include air, rail and water modes. Transportation plays a connection means among several stages that change raw material and resources to finished products. Planning all these functions and sub-functions to a system movement can minimize total cost and maximize services for consumers.

Shapiro [7], Cordeau et al. [26], Wilhelm et al., Sadjady and Davoudpou [27, 28] are some authors that have shown transportation mode as one of the decisions.

Without considering a well transportation system, logistics system could not play an efficient and effective role. Moreover, a well transportation system could provide logistics
efficiency in logistics activities. An improved transportation system needs the effort of both public and private sectors. A well logistic system may increase competitiveness at both public and private sectors.

Demand very short lead-time for customers is another important task must be consider during transportation. Typically customers would like to receive their demands with shortest time. In this paper we will discuss the role of capacity of warehouses for delivery lead time.

Uncertain parameter in supply chain network design models is another important characteristic. The uncertainties can be classified into two groups and they are random or stochastic, and non-random or strategic uncertainties.

Aghezzaf [22], Chan et al.[24], Snyder [29], Longinidis and Georgiadis [30, 31] are some authors which has worked on uncertain demands. In this paper we will discuss about uncertain demand with Monte Carlo method. Monte Carlo methods are based on computing algorithm using repeated random number to compute results. These methods are suitable when we cannot gain exact or result with deterministic model.

This paper considers different transportation mode selection and uncertain demand with the application of Monte Carlo method.

Remainder of this paper is below:

Section 2 proposes model, section 3 explain model with numerical example using Cplex solver with sensitive and scenario analysis. Section 4 implies conclusion.

2 Proposed model

2.1 Problem statement

This study is the extension of the research which was done by Bashiri et al. [32], In this section a mixed integer linear programming model is introduced. This model is four echelons (suppliers, plants, warehouses, and customers). Each supplier provides multi raw materials and sends them to several plants with different transportation modes. Each plant produces multi commodity and then send them to warehouses with different transportation modes. Now warehouses that consider lead-time send different products with different transportation modes to retailers. Demands of retailers are not deterministic and follow Monte Carlo function. Two different decisions are made in this model: strategic and tactical decisions. Two main contributions are the different transportation modes that provide different lead–time and uncertain demand which are considered in Monte Carlo stochastic programming.

2.2 Assumptions

1- An open plant or warehouse cannot be closed during planning.
2- A facility install with its capacity that cannot be changed during planning.
3- Each supplier has limitation on raw materials capacity and availability.
4- Transfers are banned between plants and warehouses.
5- Only on yearly time period that has been considered for planning.

2.3 Decisions

1- Supplier and raw material selection from suppliers to customers
2-Quantity of products that produce at plants and transfer to warehouses, and from warehouse to customers with lead-time consideration.
3-Decision about location of establishing new facilities.
4-Decision about transportation mode between sites.

3 Model formulation

\[
\text{maximize } F = \sum \sum \sum \sum \sum PR_{p,j} f_{p,j}^{r,s} \\
- \sum \sum CO_{i} X_{i}^{o} - \sum \sum CO_{v} y_{v}^{e} - \sum \sum CU_{j} Y_{j}^{v} \\
- \sum \sum \sum CP_{p,j} A_{p,i}^{j} \\
- \sum \sum \sum CS_{p,j} (h_{p,j} + \sum f_{p,j}^{r,s} / 2 A_{i,j}) \\
- \sum \sum \sum CD_{r,s,i} f_{r,s,i}^{r,s} \\
- \sum \sum \sum \sum CT_{p,i,j} f_{p,i,j}^{r,s} \\
- \sum \sum \sum \sum CF_{p,j} f_{p,j}^{r,s} \\
- \sum \sum \sum \sum PS_{r,j} f_{r,j}^{r,s} \\
- \sum \sum \sum \sum MV_{p,v} TPW_{p,j}^{v} f_{p,j}^{r,s} \\
- \sum \sum \sum \sum UCL_{i,j} Z_{i,j}^{v} \\
- \sum \sum \sum \sum UCL_{j}^{v} Z_{j}^{v} \\
- \sum \sum \sum \sum UCL_{s,d}^{v} Z_{s,d}^{v} 
\]

Equations (1) to (12) are related to net income; constraint (1) calculates total revenue of net income. Constraint (2) to (3) shows cost of opening and operating plants and warehouses, and cost of producing products at plants. Storage cost at warehouses has been shown in constraint (4). Constraints (4) to (7) are related to transportation cost from supplier to plant, plant to warehouses and warehouses to customers. Constraint (8) shows raw material supplier cost. Constraint (9) implies delivery lead time cost from warehouse to customer. Constraints (10) to (12) imply fixed cost of using transportation cost from supplier to plant, plant to warehouse, and warehouse to customers.
3.1 Constraints

\[
\sum_{j} f_{p,j,c}^{t} \leq D_{p,c}^{t}
\]

(13)

\[
\sum_{i} f_{p,i,j}^{t} + h_{p,j}^{t-1} = \sum_{c} f_{p,i,c}^{t} + h_{p,j}^{t}
\]

(14)

\[
\sum_{j} f_{r,s,j}^{t} = \sum_{c} b(r,p) q_{p,j}^{t}
\]

(15)

\[
q_{p,j}^{t} = \sum_{j} f_{p,i,j}^{t}
\]

(16)

\[
\sum_{p} v_{p}^{t} (h_{p,j}^{t} + \sum_{j} f_{p,i,j}^{t} / 2 * A(i,j)) \leq \sum_{c} M k_{j}^{t} y_{j}^{e}
\]

(17)

\[
\sum_{i} f_{r,s,i,j}^{t} \leq z_{s,i}^{t} + r_{s,j}^{t}
\]

(18)

\[
\sum_{i} f_{r,s,i,j}^{t} \geq MO_{s,r} z_{s,i}^{t}
\]

(19)

\[
\sum_{a} \sum_{i} CO_{i,j}^{a} x_{i}^{t} + \sum_{c} \sum_{j} CO_{j}^{t} y_{j}^{e} \leq INV
\]

(20)

\[
\sum_{c} \sum_{p} f_{p,i,j}^{t} \leq \sum_{j} y_{j}^{e} \cdot BIGM
\]

(21)

\[
\sum_{a} x_{i}^{o} \leq 1
\]

(22)

\[
\sum_{c} y_{j}^{e} \leq 1
\]

(23)

\[
\sum_{r} f_{r,s,i,j}^{t} \leq cap_{r,i,j}^{l} z_{s,j}^{t}
\]

(24)

\[
\sum_{p} f_{p,i,j}^{t} \leq cap_{i,j}^{l} z_{i,j}^{t}
\]

(25)

\[
\sum_{p} f_{p,i,j}^{t} \leq cap_{i,j}^{l} z_{i,j}^{t}
\]

(26)

\[
\sum_{a} z_{i,j}^{t} \leq A(i,j)
\]

(27)

Constraint (13) implies the demand for each customer and for each transportation mode. There is no need to satisfy all demand requirements of customers. Constraint (14) shows that quantity of products that transfer from plants to warehouses at tactical period plus quantity of product that store at previous tactical period is equal to quantity of product store at the current tactical period plus quantity of product that transfer from warehouses to customer. Constraint (15) states that quantity of product that is transferred from supplier to plant is equal to the requirement of product that is necessary for production at plants. Constraint (16) states that quantity of product produced at each plant is equal to quantity of products transport from each
plant to warehouses for each tactical period and each transportation mode. Constraint (17) illustrates warehouses could not be allowed to store more than their capacity. Supplier does not allow to deliver more than their capacity to plants, constraint (18). Constraint (19) depicts avoiding providing each material less than a prerequisite minimum amount of the quantity which deliver from each supplier to plant. Constraint (20) shows cost of opening plant and warehouses must not be more than its budget. Constraint (21) shows that only open warehouses could transport product to customers. Constraint (22) and (23) illustrates that in each potential site has a maximum of one plant and warehouse location. Constraint (24) to (26) show capacity limits on quantity of product transport between two sites for each mode. Constraint (27) shows that total number of modes between plant and warehouses is less than total number of delivery.

4 Results
4.1 numerical examples

In this step we run our model with different scenario on investment. It is clear from the figure 1 below that, after 4000 investing, profit is constant without any changes. Thus, we can run our model.

![Fig. 1 Investment vs. profit net](image)

In this step we generate the structure of model as below:
Number of supplier: 8
Number of products: 10
Number of raw materials: 10
Number of plants: 10
Number of warehouses: 19
Capacity options: 4
Number of customers: 10
Transportation modes: 2
Data generator commands running in excel are in appendix A. Result show three plants (plants number 7, 9, 10) with three capacities (capacity 1, 3 and 4), and 7 warehouses (warehouse number 2, 3, 6, 10, 11, 12, 16) with three capacities (capacity 1, 2, 3) should be established. These results have been shown in table 2 and 3.

**Table 2** Selected plants with consideration capacity options

<table>
<thead>
<tr>
<th>plant</th>
<th>capacity1</th>
<th>capacity2</th>
<th>capacity3</th>
<th>capacity4</th>
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**Table 3** Capacity options for warehouses

<table>
<thead>
<tr>
<th>WH1</th>
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<th>capacity4</th>
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<tr>
<td>WH19</td>
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</tbody>
</table>

Three raw materials (r6, r7, and r 10) by seven suppliers (s1, s3, s5, s6, s7, s8) have been provided (table 4).
Table 4 raw materials provided by suppliers

<table>
<thead>
<tr>
<th>Column1</th>
<th>supplier1</th>
<th>supplier2</th>
<th>supplier3</th>
<th>supplier4</th>
<th>supplier5</th>
<th>supplier6</th>
<th>supplier7</th>
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<td>r1,r6</td>
<td>r2,r3,r5,r9,r10</td>
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Table 5 Selected raw materials

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Table 5 and 6 implies different raw materials that are provided by different plants from suppliers with two transportation modes.

Table 6 Raw material provided by different suppliers for plants by transportation mode 2

<table>
<thead>
<tr>
<th>Column1</th>
<th>supplier1</th>
<th>supplier2</th>
<th>supplier3</th>
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<th>supplier7</th>
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<tr>
<td>plant7</td>
<td>r10,r6</td>
<td>r8,r4,r7</td>
<td>r1</td>
<td>r2,r3,r5,r9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>plant8</td>
<td>r7</td>
<td>r4,r8</td>
<td>r1,r6</td>
<td>r5,r9,r2,r3,r10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>plant9</td>
<td>r10,r7</td>
<td>r2,r4,r8</td>
<td>r6</td>
<td>r1</td>
<td>r3,r9,r5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>plant10</td>
<td>r10,r7</td>
<td>r2,r4,r8</td>
<td>r6</td>
<td>r1</td>
<td>r3,r9,r5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Since our model run with two statuses (stochastic and deterministic demand), we compare both results in table 7.

Table 7 Deterministic and stochastic demands

<table>
<thead>
<tr>
<th>Demand</th>
<th>Deterministic</th>
<th>Stochastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-52333</td>
<td>3347</td>
</tr>
<tr>
<td>50</td>
<td>-110715</td>
<td>5190</td>
</tr>
<tr>
<td>70</td>
<td>-275671</td>
<td>5921</td>
</tr>
<tr>
<td>80</td>
<td>-352432</td>
<td>6721</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand</th>
<th>Deterministic</th>
<th>Stochastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-59431</td>
<td>3120</td>
</tr>
<tr>
<td>50</td>
<td>-122323</td>
<td>4921</td>
</tr>
<tr>
<td>70</td>
<td>-292531</td>
<td>5721</td>
</tr>
<tr>
<td>80</td>
<td>-342131</td>
<td>6542</td>
</tr>
</tbody>
</table>

It is clear from the aforementioned table 7 that when we run our model under uncertain demand, transportation has increased moderately while decreasing profit. However, with the increment of demand, profit is growing up while demand point increased.

4.2 Scenario and sensitive analysis

During model running, it is seen that some constraints and parameters may affect model. By extracting some of these parameters and constraints, different scenarios could be produced. In this section we try two scenarios:

Scenario 1: Eliminating storage cost
Scenario 2: Eliminating warehouse capacity constraint

To simplify objective functions extract definition as below:

Cost 1 = total transportation cost;
Cost 2 = transportation cost from warehouse to customer;
Cost 3 = delivery lead time cost;
Cost 4 = transportation cost from supplier to plant;
Cost 5 = transportation cost from plant to warehouses;
Cost 6 = fixed cost of using different transportation cost;
Cost 7 = raw material supply cost;
Cost 8 = fixed cost of opening and operating plant and warehouses;
Cost 9 = cost of producing product at plants;
Cost 10 = storage cost at warehouses;

![Fig. 2 scenario 1 results](image)
After scenario 1, it is shown that all cost has been increased while total revenue increases. The main reason for this result is because of the demand. Consequently, benefit of selling prices go up. However, this is not a realistic situation. Result has been shown in figure1.

![Fig. 3 scenario 2 results](image)

After scenario 2 it is seen that cost 1, cost 2, cost 6 decreased slowly, while cost 3 increase moderately. Results have been shown in figure2.

5 Sensitive analysis
5.1 Expansion capacities of warehouses
In this section total performance of model is checked. We expand capacity of warehouses from 1000 to 3500 in a step by step manner to monitor fluctuation of other costs and benefits. The costs are checked based on capacity fluctuations. It is clear from table 8, cost of opening and operating plant and warehouses, and cost of transportation between plant and warehouses are constants. Table 8 depicts the aforementioned results.

<table>
<thead>
<tr>
<th>Table 8 Sensitive Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>cost1</td>
</tr>
<tr>
<td>cost2</td>
</tr>
<tr>
<td>cost3</td>
</tr>
<tr>
<td>cost4</td>
</tr>
<tr>
<td>cost5</td>
</tr>
<tr>
<td>cost6</td>
</tr>
<tr>
<td>cost7</td>
</tr>
<tr>
<td>cost8</td>
</tr>
<tr>
<td>cost9</td>
</tr>
<tr>
<td>benefit</td>
</tr>
</tbody>
</table>

We extend the structure of problem with 4 different classes (Table 9). As it is shown in Table 10, number of variables, discrete variables, constraints and CPU times increase exponentially.
Our model is NP-hard problem. With the increment of size of problem, some heuristics should be designed to reduce time of solution and finding the optimal solution.

Table 9 Structure of the test problem

<table>
<thead>
<tr>
<th>class</th>
<th>supplier</th>
<th>plant</th>
<th>warehouse</th>
<th>transportation mode</th>
<th>customer</th>
<th>raw material</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1</td>
<td>8</td>
<td>10</td>
<td>19</td>
<td>2</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>c2</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>2</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>c3</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>4</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>c4</td>
<td>20</td>
<td>25</td>
<td>25</td>
<td>4</td>
<td>30</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 10 Computational results

<table>
<thead>
<tr>
<th>class</th>
<th>non-zero elements</th>
<th>discrete variables</th>
<th>constraint</th>
<th>cpu time</th>
</tr>
</thead>
<tbody>
<tr>
<td>s1</td>
<td>63412</td>
<td>11816</td>
<td>2163</td>
<td>0.78</td>
</tr>
<tr>
<td>s2</td>
<td>729446</td>
<td>96275</td>
<td>48837</td>
<td>277</td>
</tr>
<tr>
<td>s3</td>
<td>1520999</td>
<td>219690</td>
<td>88027</td>
<td>1001</td>
</tr>
<tr>
<td>s4</td>
<td>669446</td>
<td>93875</td>
<td>39237</td>
<td>2824</td>
</tr>
</tbody>
</table>

6 Conclusion

In this paper four echelons, multi commodity, strategic–tactical mixed integer programming model has been proposed based on the model of Bashiri et al. [32] with stochastic demand and different transportation mode and lead time. A numerical example has been shown to illustrate applicability of model, such as quantity of product which is transferred between different facilities with different transportation mode, decisions about supplier selection, facility location, transportation mode selection, and capacity options. Total profit with deterministic and stochastic demand was compared.

Two different scenario and sensitivity analysis were run to show different results. There is more extension for this research. Uncertainty for other parameter such as cost with different uncertainty concept (e.g. fuzzy environment) is suggested for comparison. As it has been cleared this problem is NP-hard, more heuristic solution need to achieve the optimal solution.

References


Appendix A. Data generator commands

\[
\begin{align*}
TPW_{p, tr, j, c} &= 1 + \text{INT}(\text{RAND}() \times (4-1+1)) \\
MV_{p, tr} &= 1 + \text{INT}(\text{RAND}() \times (3-1+1)) \\
cap_{1tr, i, j} &= 0 + \text{INT}(\text{RAND}() \times (20-0+1)) \\
uc_{tr, i, j} &= 0 + \text{INT}(\text{RAND}() \times (2-0+1)) + 1 + \text{INT}(\text{RAND}() \times (5-1+1)) \times \text{SQRT}('cap_{1tr, i, j}') \\
mo(s, r) &= 10 \\
mk(j, e) &= 500 + \text{INT}(\text{RAND}() \times (1000-500+1)) \\
CK(i, o) &= 200 + \text{INT}(\text{RAND}() \times (500-200+1)) \\
D(c, pt) &= 2000 + \text{INT}(\text{RAND}() \times (4000-2000+1)) \\
b(p, r) &= 1 + \text{INT}(\text{RAND}() \times (3-1+1)) \\
Ai, j &= 10 + \text{INT}(\text{RAND}() \times (15-10+1)) \\
ps(r, s) &= 5 + \text{INT}(\text{RAND}() \times (10-5+1)) \\
co(o, i) &= 0 + \text{INT}(\text{RAND}() \times (90-0+1)) + 100 + \text{INT}(\text{RAND}() \times (500-100+1)) \\
cu(o, i) &= 0 + \text{INT}(\text{RAND}() \times (20-0+1)) + 10 + \text{INT}(\text{RAND}() \times (100-10+1)) \\
co1(e, j) &= 0 + \text{INT}(\text{RAND}() \times (90-0+1)) + 100 + \text{INT}(\text{RAND}() \times (110-100+1)) \\
cu1(e, j) &= 0 + \text{INT}(\text{RAND}() \times (20-0+1)) + 100 + \text{INT}(\text{RAND}() \times (110-100+1)) \\
CP_{p, i} &= 10 + \text{INT}(\text{RAND}() \times (20-10+1)) \\
CS_{p, j} &= 2 + \text{INT}(\text{RAND}() \times (5-2+1)) \\
CT_{p, i} &= 1 + \text{INT}(\text{RAND}() \times (3-1+1)) \\
CD_{r, s, i} &= 1 + \text{INT}(\text{RAND}() \times (3-1+1)) \\
CF1_{p, j, c} &= 1 + \text{INT}(\text{RAND}() \times (3-1+1)) \\
r11(t, s, r) &= 10000 + \text{INT}(\text{RAND}() \times (20000-10000+1))
\end{align*}
\]