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Interactive multiple objective programming in optimization of the fully fuzzy quadratic programming problems

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Abstract In this paper, a quadratic programming (FFQP) problem is considered in which all of the cost coefficients, constraints coefficients, and right hand side of the constraints are characterized by L-R fuzzy numbers. Through this paper, the concept of α- level of fuzzy numbers for the objective function, and the order relations on the fuzzy numbers for the constraints are considered. To optimize the interval objective function, the order relations represented by decision maker's preference between intervals are defined by the right limit, the left limit, the center and the width of an interval. The maximization (minimization) problem with interval objective function is converted into a bi- objective problem and then the weighting method is applied for solving it and solves the new problem using the Kuhn- Tucker's necessary conditions. The advantages of the approach, referring to covert the fully fuzzy problem into the bi-objective problem, which is significant and being used in an interactive method for achieving the logical and applicable solutions. Finally, a numerical example is given to illustrate the utility, practically and the efficiency of the method.

Keyword: Fully Fuzzy Quadratic Programming, L-R Fuzzy Numbers, α- Level, Weighting Method, Kuhn- Tucker's Optimality Conditions.

1 Introduction

Quadratic programming (QP) is an optimization problem whose objective function is quadratic function and the constraints are linear equalities or inequalities. QP is the problem of optimizing an objective function and is one of the simplest form of non-linear programming. The objective function can contain bilinear or up to second order polynomial terms and the constraints are linear and can be both inequalities and equalities. QP is widely used in real world problem to optimize the portfolio selection problem, in the regression to perform the least square method, in chemical plants to control scheduling, in the sequential quadratic programming, economics, engineering design etc. Since the QP is the most interesting class of the optimization, so it is known as the NP-hard. There are several methods and algorithms for solving the QP problem introduced by Pardalos and Rosen [1]; Horst and Tuy [2] and Bazaraa et al. [3].

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In spite of having a vast decision making experience, the decision maker cannot always articulate the goals precisely. Decision-making in a fuzzy environment, developed by Bellman and Zadeh [4], improved and helped greatly in the management decision problems. The fuzzy set theory and its applications were proposed by Zimmermann [5]. Ishibuchi and Tanka [6] investigated the mathematical programming problem with interval valued objective function. The fuzzy programming with several objective functions, fuzzy sets and systems was proposed by Zimmermann [7]. Quadratic programming with interval coefficients has been extended to the nonlinear interval programming (Jiang et al., [8]; Wu, [9]; and Bhurjee and Panda, [10]). Suprajitno and Mohd [11] introduced solution procedures to obtain the optimum solution in interval form for optimum point and optimum value too. Kheirfam [12] used a fuzzy ranking and arithmetic operations to transform the QP problem with fuzzy numbers in the coefficients and variables into the corresponding deterministic one and solved it to obtain a fuzzy optimal solution.

Allahviranloo and Moazam [13] introduced a new concept of the second power of fuzzy numbers that is providing an analytical and approximate solution for fully fuzzy quadratic equations. Taghi-Nezad and Taleshian [14] proposed a solution method for solving a special class of fuzzy QP problems with fuzziness in relations. Gao and Ruan [15] presented a canonical duality theory for solving quadratic minimization problems subject to either box or integer constraints. Sun et al. [16] investigated the duality gap between the binary quadratic optimization problem and its semidefinite programming relaxation. Gabr [17] presented a comprehensive methodology for solving and analyzing quadratic and nonlinear programming in a fully fuzzy environment. Takapoui et al. [18] proposed an algorithm for approximately minimizing a convex quadratic function over the intersection of affine and separable constraints. Mirmohseni and Nasseri [19] proposed a new approach for deriving the fuzzy objective value of the fuzzy QP problem. Syaripuddin et al. [20] applied a two- level programming approach for solving interval variables quadratic programming problem, where the problem was converted into two classical QP problem, namely the best and worst optimum problems. Shi et al. [21] proposed an effective algorithm for solving quadratic programming problem with quadratic constraints globally.

In this paper, a fully fuzzy quadratic programming is studied. A multi-objective optimization is applied for optimizing the problem after converting it based on the $\alpha-$level of the fuzzy number and the order relations, where the problem is converted into the worst and best problems.

The outlay of the paper is organized as follows: Section 2 presented some preliminaries related to the fuzzy number, $\alpha-$level of the fuzzy number, $L-R$ fuzzy numbers and their arithmetic operations. Section 3 formulates fully fuzzy quadratic programming problem. In Section 4, solution approach is applied for solving the problem. In Section 5, a numerical example to illustrate the efficiency of the solution approach is given. Finally, some concluding remarks are reported in Section 6.

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2 Preliminaries

(Zahed [22]). A fuzzy set $\widetilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be fuzzy numbers if its membership function

$\mu_{\widetilde{A}}(x)$: $\mathbb{R} \rightarrow [0,1]$, have the following properties:

1. $\mu_{\widetilde{A}}(x)$ is an upper semi- continuous membership function;
2. $\widetilde{A}$ is convex fuzzy set, i.e., $\mu_{\widetilde{A}}(\delta\, x + (1-\delta)\, y) \geq \min\{\mu_{\widetilde{A}}(x), \mu_{\widetilde{A}}(y)\}$ for all $x, y \in \mathbb{R}; 0 \leq \delta \leq 1$;
3. $\widetilde{A}$ is normal, i.e., $\exists\, x_0 \in \mathbb{R}$ for which $\mu_{\widetilde{A}}(x_0) = 1$;
4. $\mathrm{Supp}\,(\widetilde{A}) = \{x \in \mathbb{R}: \mu_{\widetilde{A}}(x) > 0\}$ is the support of $\widetilde{A}$, and the closure $\mathrm{cl}(\mathrm{Supp}(\widetilde{A}))$ is compact set.

Definition2. (Dubois and Prade, [23]**).** The α −level set of the fuzzy numbers $\tilde{A}$ is defined as the ordinary set $L_\alpha(\tilde{A})$ for which the degree of their membership functions exceeds the level α: $L_\alpha(\tilde{A}) = \{p: \mu_{\tilde{A}}(A) \geq \alpha\}$.

Definition3. (Kaufmann and Gupta, [24]). A triangular fuzzy number can be represented completely by a triplet $\tilde{A} = (a, b, c)$, and has membership

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a, \\ \frac{x-a}{b-a}, & a \leq x \leq b, \\ \frac{c-x}{c-b}, & b \leq x \leq c, \\ 0, & x > c. \end{cases} \tag{1}$$

Definition4. (Kaufmann and Gupta, [24]). The interval of confidence at level α for the fuzzy number $\tilde{A} = (a, b, c)$ is defined as

$A_\alpha = [(b-a)\alpha + a, -(c-b)\alpha + c];\ \forall \alpha \in [0,1]$.

Definition5. (Sakawa, [25]). A fuzzy number $\widetilde{A} = (x, \alpha, \beta)_{LR}$ is said to be an $L-R$ fuzzy number if

$$\mu_{\widetilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), x \leq m, \alpha > 0 \\ R\left(\frac{x-m}{\beta}\right), x \geq m, \beta > 0 \end{cases} \tag{2}$$

Where m is the mean value of $\tilde{A}$ and α and β are left and right spreads, respectively, and a function $L(.)$ is a left shape function satisfying

1. $L(x) = L(-x)$,
2. $L(0) = 1$,
3. $L(x)$ is nonincreasing on $[0, \infty[$.

It is noted that a right shape function $R(.)$ is similarly defined as $L(.)$.

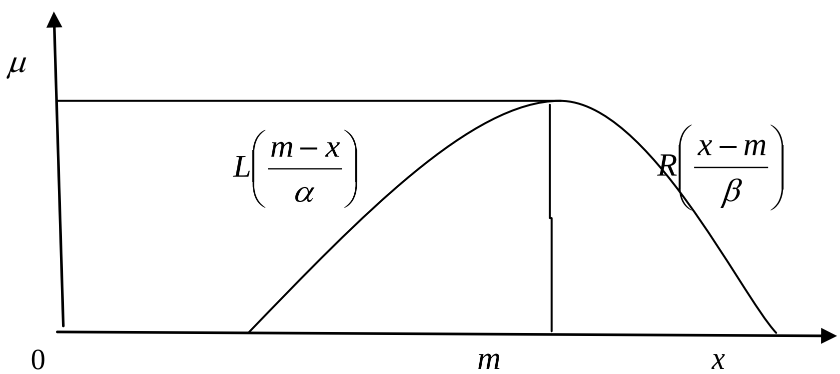

Fig. 1 $\boldsymbol{L-R}$ Fuzzy number

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For two $\mathrm{L-R}$ fuzzy numbers $\tilde{A} = (x, \alpha, \beta)_{LR}$ and $\tilde{B} = (y, \gamma, \sigma)_{LR}$, the arithmetic operations are:

1. Addition: $\tilde{A} \oplus \tilde{B}$
$$(x, \alpha, \beta)_{LR} \oplus (y, \gamma, \sigma)_{LR} = (x + y, \alpha + \gamma, \beta + \sigma)_{LR} \quad (3)$$
2. Opposite: $-\tilde{A}$
$$-(x, \alpha, \beta)_{LR} = (-x, \beta, \alpha)_{LR} \quad (4)$$
3. Subtraction: $\tilde{A} \ominus \tilde{B}$
$$(x, \alpha, \beta)_{LR} \ominus (y, \gamma, \sigma)_{RL} = (x - y, \alpha + \sigma, \beta + \gamma)_{LR} \quad (5)$$
4. Multiplication: $\tilde{A} \odot \tilde{B}$
$$\tilde{A} \odot \tilde{B} = \begin{cases} \text{If } \tilde{A} \succ 0, \tilde{B} \succ 0, \text{then } (x, \alpha, \beta)_{LR} \odot (y, \gamma, \sigma)_{LR} \cong (xy, x\gamma + y\alpha, x\sigma + y\beta)_{LR}, \\ \text{If } \tilde{A} \prec 0, \tilde{B} \succ 0, \text{then } (x, \alpha, \beta)_{LR} \odot (y, \gamma, \sigma)_{LR} \cong (xy, y\alpha - x\sigma, y\beta - x\sigma)_{RL}, \\ \text{If } \tilde{A} \prec 0, \tilde{B} \prec 0, \text{then } (x, \alpha, \beta)_{LR} \odot (y, \gamma, \sigma)_{LR} \cong (xy, -y\beta - x\sigma, -y\sigma - x\gamma)_{RL.} \end{cases} \quad (6)$$

5. Scalar multiplication: $\lambda \odot \tilde{A}$
$$\lambda \odot (x, \alpha, \beta)_{LR} = \begin{cases} (\lambda x, \lambda\alpha, \lambda\beta)_{LR}, & \lambda \succ 0 \\ (\lambda x, -\lambda\beta, -\lambda\alpha)_{RL}, & \lambda \prec 0 \end{cases} \quad (7)$$

6. Partial order relation $(\preccurlyeq)$
$$\tilde{A} \,(\preccurlyeq)\, \tilde{B} \Leftrightarrow x \leq y, x - \alpha \leq y - \gamma, x + \beta \leq y + \sigma. \quad (8)$$
An interval on $\mathbb{R}$ is defined by an ordered pair as
$$A^I = [a^L, a^U] = \{a: a^L \leq a \leq a^U, a \in \mathbb{R}\}. \quad (9)$$
Where, a^L is the left limit and a^U is the upper limit of A. Also, A^I denoted by its center and width as
$$A_I = \langle a^C, a^W \rangle = \{a: a^C - a^W \leq a \leq a^C + a^W, a \in \mathbb{R}\}. \quad (10)$$
Where, a^C is the center and a^W is the width of A. It is clear that the a^C, and a^W are estimated as
$$a^C = \frac{a^U + a^L}{2}, a^W = \frac{a^U - a^L}{2} \quad (11)$$
The arithmetic operations on $A^I = [a^L, a^U], B^I = [b^L, b^U]$:
$$A^I(+)B^I = [a^L, a^U](+)[b^L, b^U] = [a^L + b^L, a^U + b^U]. \quad (12)$$
$$A_I(+)B_I = \langle a^C, a^W \rangle (+) \langle b^C, b^W \rangle = \langle a^C + b^C, a^W + b^W \rangle.$$
$$A^I(-)B^I = [a^L, a^U](-)[b^L, b^U] = [a^L - b^U, a^U - b^L]. \quad (13)$$
$$\lambda A = k[a^L, a^U] = \begin{cases} [ka^L, ka^U], for\ k \geq 0, \\ [ka^U, ka^L], for\ k < 0 \end{cases} \quad (14)$$
$$\lambda A_I = k\langle a^C, a^W \rangle = \langle k^C, |k| a^W \rangle. \quad (15)$$
Definition6. (Ishibuchi and Tanaka [6]). The order relation $(\leq^{LU})$ between $A^I = [a^L, a^U], B^I = [b^L, b^U]$ is defined as

(i) $A^I(\leq^{LU})B^I \Leftrightarrow a^L \leq b^L, and\ a^U \leq b^U$, (16)

(ii) $A^I(<^{LU})B^I \Leftrightarrow a^L \leq b^L, and\ A^I \neq B^I$. (17)

Definition7. (Ishibuchi and Tanaka, [6]). The order relation $(\leq_{CW})$ between $A_I = \langle a^C, a^W \rangle, B_I = \langle b^C, b^W \rangle$ is defined as

(i) $A_I(\leq_{CW})B_I \Leftrightarrow a^C \leq b^C, and\ a^W \leq b^W$, (18)

(ii) $A^I(<_{CW})B_I \Leftrightarrow a^C \leq b^L, and\ A_I \neq B_I$. (19)

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3 Problem formulation and solution concepts

A fully fuzzy quadratic programming problem is formulated as follows:

$$\min \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j \otimes x_j \oplus \frac{1}{2}\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \otimes \tilde{q}_{ij} \otimes x_j\right) \quad (20)$$

s.t.

$$x \in \widetilde{M} = \left\{ \begin{matrix} \sum_{j=1}^{n} \tilde{a}_{ij} \otimes x_j \preccurlyeq \tilde{b}_i, i = 1,2,\dots,m_1, \sum_{j=1}^{n} \tilde{a}_{ij} \otimes x_j \approx \tilde{b}_i, i = m_{i+1},\dots,m \\ x_j \geq, j = 1,2,\dots,n \end{matrix} \right\} \quad (21)$$

Where, $\tilde{C} = (\tilde{c}_1, \tilde{c}_2, \dots, \tilde{c}_n)$, and $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_m)$ are fuzzy cost vector and fuzzy right- hand side vector. $X = (x_1, x_2, \dots, x_n)$ is a vector of variables, and also $\tilde{Q} = [q_{ij}]_{n\times n}$ is a matrix of quadratic form which is symmetric and positive semi definite, and $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$. It is assumed that all of $\tilde{A}, \tilde{b}\ \tilde{C}, and\ \tilde{Q} \in F(R)$, where $F(R)$ denotes the set of all $L - R$ fuzzy numbers.

Definition8. A non-negative fuzzy vector x_j is said to be fuzzy feasible solution for problem (20)-(21) if it satisfies the constraints (21).

Definition9. A fuzzy feasible solution x_j^* is called a fuzzy optimal solution for problem (20)- (21), if

$$\left[\sum_{j=1}^{n} \tilde{c}_j \otimes x_j^* \oplus \frac{1}{2}\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_i^* \otimes \tilde{q}_{ij} \odot x_j^*\right)\right] (\preccurlyeq) \left[\sum_{j=1}^{n} \tilde{c}_j \otimes x_j \oplus \frac{1}{2}\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \otimes \tilde{q}_{ij} \otimes x_j\right)\right], for\ all\ x_j.$$

Let $\tilde{c}_j, \tilde{q}_{ij}, \tilde{a}_{ij}$, and $\tilde{b}_i$ represent by the $L - R$ fuzzy numbers, $(c_j, \alpha_j, \beta_j)_{LR}, (q_{ij}, \delta_{ij}, \varepsilon_{ij})_{LR}, (a_{ij}, \epsilon_{ij,} \theta_{ij})_{LR}$, and $(b_i, \vartheta_i, \rho_i)$, erespectively.

Then problem (20)-(21) becomes

$$\min \tilde{Z} = \sum_{j=1}^{n} (c_j, \alpha_j, \beta_j)_{LR} \otimes x_j \oplus \frac{1}{2}\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \otimes (q_{ij}, \delta_{ij}, \varepsilon_{ij})_{LR} \otimes x_j\right) \quad (22)$$

s.t.

$$x \in \widetilde{M} = \left\{ \begin{matrix} \sum_{j=1}^{n} (a_{ij}, \epsilon_{ij,} \theta_{ij})_{LR} \otimes x_j \preccurlyeq (b_i, \vartheta_i, \rho_i)_{LR}, i = 1,2,\dots,m_1, \\ \sum_{j=1}^{n} (a_{ij}, \epsilon_{ij,} \theta_{ij})_{LR} \otimes x_j \approx (b_i, \vartheta_i, \rho_i)_{LR}, i = m_{i+1},\dots,m \\ x_j \geq, j = 1,2,\dots,n \end{matrix} \right\} \quad (23)$$

Problem (22)-(23) can be rewritten as

$$\min \tilde{Z} = \sum_{j=1}^{n} (c_j - \alpha_j, c_j, c_j +, \beta_j) \otimes x_j \oplus \frac{1}{2}\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \otimes (q_{ij} - \delta_{ij}, q_{ij}, q_{ij} + \varepsilon_{ij}) \otimes x_j\right)$$

Subject to (24)

$$x \in \widetilde{M} = \left\{ \begin{matrix} \sum_{j=1}^{n} (a_{ij}, \epsilon_{ij,} \theta_{ij})_{LR} \otimes x_j \preccurlyeq (b_i, \vartheta_i, \rho_i)_{LR}, i = 1,2,\dots,m_1, \\ \sum_{j=1}^{n} (a_{ij}, \epsilon_{ij,} \theta_{ij})_{LR} \otimes x_j \approx (b_i, \vartheta_i, \rho_i)_{LR}, i = m_{i+1},\dots,m \\ x_j \geq, j = 1,2,\dots,n \end{matrix} \right\}.$$

Based on Definition 4, and the concepts of the partial order ($\preccurlyeq$), problem (24) can be rewritten as

$$\min \tilde{Z}_\alpha = \sum_{j=1}^{n} [(\alpha - 1)\alpha_j + c_j, -(\alpha - 1)\beta_j + c_j] \otimes x_j$$
$$\oplus \frac{1}{2}\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \otimes [(\alpha - 1)\delta_{ij} + q_{ij}, -(\alpha - 1)\varepsilon_{ij} + q_{ij}] \otimes x_j\right), \alpha \in [0,1]$$
$$= [Z_\alpha^L, Z_\alpha^U].$$

s.t. (25)

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$$x \in M = \left\{ \begin{array}{c} \sum_{j=1}^{n} \mathrm{a_{ij}}\, x_j \le \mathrm{b_i}\,, i = 1, 2, \dots, m_1, \\ \sum_{j=1}^{n} \left(\mathrm{a_{ij}} - \epsilon_{\mathrm{ij},}\right) x_j \le \mathrm{b_i} - \vartheta_{\mathrm{i}}, i = 1, 2, \dots, m_1, \\ \sum_{j=1}^{n} \left(\mathrm{a_{ij}} + \theta_{\mathrm{ij}}\right) x_j \le \mathrm{b_i} + \rho_{\mathrm{i}}, i = 1, 2, \dots, m_1, \\ \sum_{j=1}^{n} \mathrm{a_{ij}}\, x_j = \mathrm{b_i}\ \ , i = m_{i+1}, \dots, m, \\ \sum_{j=1}^{n} \left(\mathrm{a_{ij}} - \epsilon_{\mathrm{ij},}\right) x_j = \mathrm{b_i} - \vartheta_{\mathrm{i}},\ i = m_{i+1}, \dots, m, \\ \sum_{j=1}^{n} \left(\mathrm{a_{ij}} + \theta_{\mathrm{ij}}\right) x_j = \mathrm{b_i} + \rho_{\mathrm{i}},\ i = m_{i+1}, \dots, m, \\ x_j \ge, j = 1, 2, \dots, n. \end{array} \right\} \tag{26}$$

Definition10. $x \in M$ is an $\alpha-$solution of problem (25)-(26) if and only if there is no $\hat{x} \in M$ satisfies $\mathrm{Z}(\hat{x}) \le^{LU} Z(x), or\ \mathrm{Z}(\hat{x}) <_{CW} Z.$
Or equivalently, Definition10 can be simplified as in the following definition.

Definition11. $x \in M$ is an $\alpha-$efficient solution of problem (25)-(26) if and only if there is no $\hat{x} \in M$ satisfies $\mathrm{Z}(\hat{x}) \le\le_{C}^{U} Z(x)$**.**
The solution set of problem (25)–(26) can be obtained as the α - efficient solution of the following multi- objective problem

$$\min(Z_\alpha^U(\mathrm{x}),\ \mathrm{Z}_\alpha^C(\mathrm{x}))$$
$$\text{s.t.} \qquad x \in M. \tag{27}$$

Problem (27) may be solved using the weighting method (Chankong and Haimes, [26])

4 Solution approach

In this section, the steps of the proposed approach for solving fully fuzzy quadratic programming are illustrated as:
Step1: Consider the fully fuzzy quadratic problem (20)-(21),
Step2: Convert the problem (20)-(21) into the problem (22)-(23), and then into problem (24),
Step3: Using Definition 4, and the partial order($\preccurlyeq$), problem (24) is converted into problem (25)-(26),
Step4: According to the decision maker's preference, problem (25)-(26) is converted into the bi- objective (27),
Step5: Ask the decision maker the specify the $\alpha-$level $(0 < \alpha < 1)$,
Step6: Transform problem (27) into a single objective problem using the weighting method and then solve it using the Kuhn- Tucker's optimality conditions [3],
Step7: Stop with x^* is the optimal compromise solution, and Z^* is the corresponding optimum value.
The flowchart of the solution approach illustrated as in the following figure

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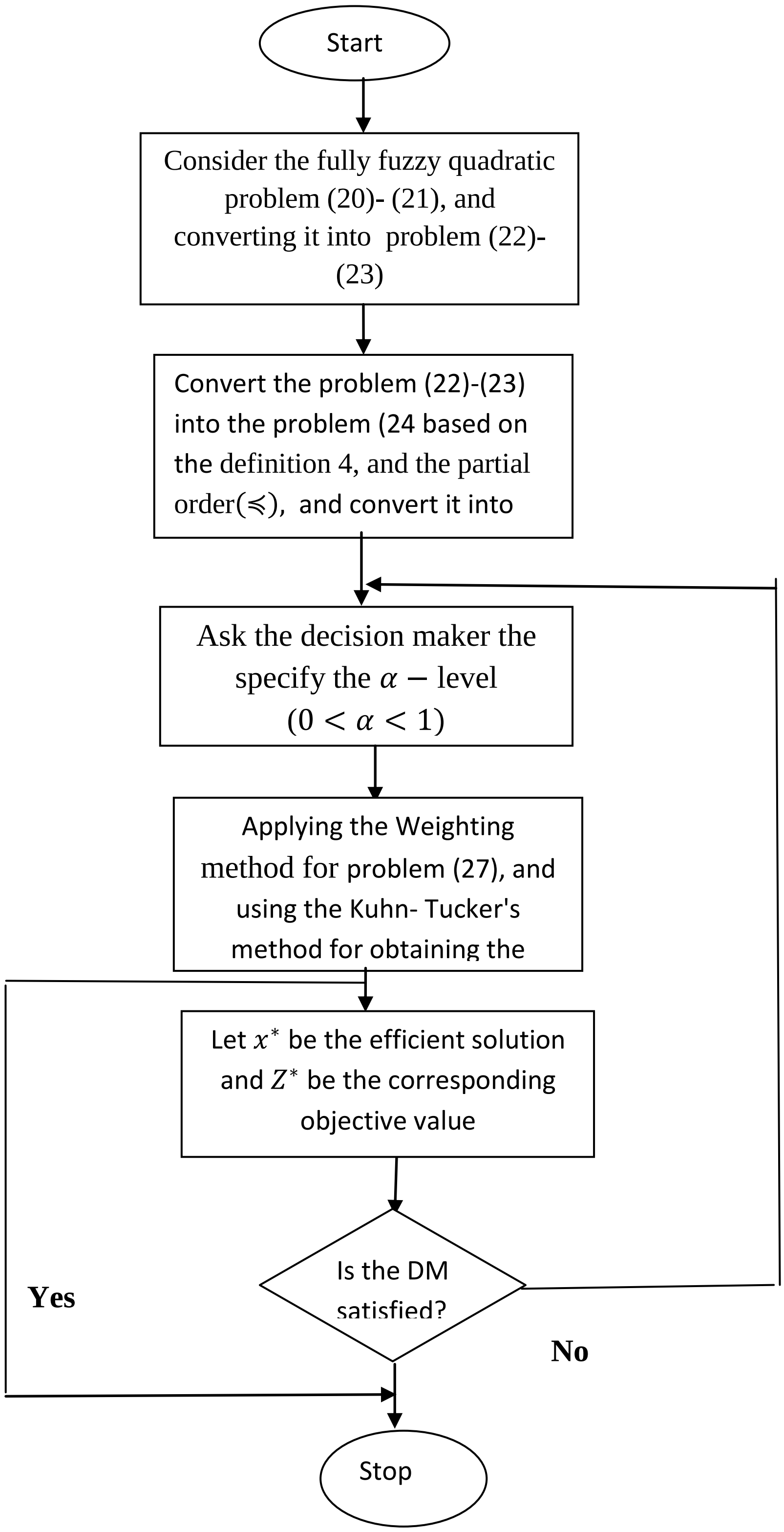

Fig. 2 Flow chart for the solution approach

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5 Numerical example

Consider the following FFQP problem

Step1-2: $\min \tilde{Z} = (-5,1,1)_{LR} \otimes x_1 \oplus (1.5,0.5,0.5)_{LR} \otimes x_2$
$\oplus \frac{1}{2}[(6,2,2)_{LR} \otimes x_1^2 \oplus (-4,2,2)_{LR} x_1 x_2 \oplus (4,2,2)_{LR} \otimes x_2^2]$
Subject to (28)
$(1,0,0)_{LR} \otimes x_1 \oplus (1,0.5,0.5)_{LR} \otimes x_2 \preccurlyeq (2,1,1)_{LR}$,
$(2,1,1)_{LR} \otimes x_1 \oplus (-1,1,0.5)_{LR} \otimes x_2 \preccurlyeq (4\ 1,1)_{LR}$,
$x_1, x_2 \geq 0$.

Step3: According to problem (14), the (19) becomes

$$\min \tilde{Z}_\alpha = [\alpha - 6, -4 - \alpha] \otimes x_1 \oplus \left[1 + \frac{\alpha}{2}, 2 - \frac{\alpha}{2}\right] \otimes x_2 \oplus [2 + \alpha, 4 - \alpha] \otimes x_1^2 \oplus [\alpha - 3, -\alpha - 1] x_1 x_2 \oplus [1 + \alpha, 3 - \alpha] \otimes x_2^2$$

s.t. (29)

$x_1 + x_2 \leq 2;\ x_1 + 0.5x_2 \leq 1; x_1 + 1.5x_2 \leq 3;$
$2x_1 - x_2 \leq 4;\ x_1 - 2x_2 \leq 3;\ 3x_1 - 0.5x_2 \leq 5;$
$x_1, x_2 \geq 0.$

Step4: Problem (29) can be converted into the following bi- objective problem

$$\min Z_\alpha^U = (-4 - \alpha)x_1 + \left(2 - \frac{\alpha}{2}\right)x_2 + (4 - \alpha)x_1^2 + (-\alpha - 1)x_1 x_2 + (3 - \alpha)x_2^2$$
$$\min Z_\alpha^C = -5x_1 + 1.5x_2 + 3x_1^2 - 2x_1 x_2 + 2x_2^2$$

s.t. (30)

$x_1 + x_2 \leq 2;\ x_1 + 0.5x_2 \leq 1; x_1 + 1.5x_2 \leq 3;$
$2x_1 - x_2 \leq 4;\ x_1 - 2x_2 \leq 3;\ 3x_1 - 0.5x_2 \leq 5;$
$x_1, x_2 \geq 0.$

Step5: Suppose the decision maker selects $\alpha = 0.5$. Then

$$\min Z^U = -5.5x_1 + \tfrac{7}{4}x_2 + \tfrac{7}{2}x_1^2 - \tfrac{3}{2}x_1 x_2 + \tfrac{5}{2}x_2^2$$
$$\min Z^C = -5x_1 + 1.5x_2 + 3x_1^2 - 2x_1 x_2 + 2x_2^2$$

s.t. (31)

$x_1 + x_2 \leq 2;\ x_1 + 0.5x_2 \leq 1; x_1 + 1.5x_2 \leq 3;$
$2x_1 - x_2 \leq 4;\ x_1 - 2x_2 \leq 3;\ 3x_1 - 0.5x_2 \leq 5;$
$x_1, x_2 \geq 0.$

Using the weighting method (Chankong and Haimes, [26]) and then the Kuhn-Tucker's optimality conditions [3], we obtain the $\alpha-$efficient solutions of problem (31) from the following problem

$\min(wZ^U + (1 - w)Z^C)$
s.t. (32)
$x_1 + x_2 \leq 2;\ x_1 + 0.5x_2 \leq 1; x_1 + 1.5x_2 \leq 3;$
$2x_1 - x_2 \leq 4;\ x_1 - 2x_2 \leq 3;\ 3x_1 - 0.5x_2 \leq 5;$
$x_1, x_2 \geq 0.$

Table1 The solution of problem (32) at different values of w

Values of w	$x = (x_1, x_2)$	Z
0	$x^a = (0.85, 0.05)$	$Z(x^a) = [-2.90875, -1.26625]$ $= \langle -2.0875, 0.82125 \rangle$
0.5	$x^b = (0.81, 0)$	$Z(x^b) = [-2.81475, -1.34865]$ $= \langle -2.0817, 0.73305 \rangle$
1	$x^c = (0.79, 0)$	$Z(x^c) = [-2.78475, -1.37065]$ $= \langle -2.0777, 0.70705 \rangle$

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6 Conclusion and discussions

Studying fully fuzzy quadratic programming (FFQP) problem leads to its closed connection with real world problems, considered as great importance. At the first, the quadratic problem has considered with fully fuzzy parameters in the objective function and constraints. Hence, the problem has converted into the corresponding interval-valued quadratic programming. Then, according to the decision maker's preference, it coveted into the bi-objective problem, and then by using the weighting method, it transformed into a single objective problem which can be solved by Kuhn-Tucker's optimality conditions. Converting the fully fuzzy problem into a bi-objective problem is significant and being used in an interactive method for achieving the logical and applicable solutions.

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Conflicts

The author declares no conflict of the interest.

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