International Journal of Applied Operational Research
Vol. 10, No. 1, pp. 11-20, Winter 2020

Journal homepage: ijorlu.liau.ac.ir

Integrated technique for the optimization of healthcare facility problem

I. A. Osinuga*, V. O. Oyebade, W. O. Otun

Received: 2 September 2019 **Accepted:** 20 December 2019

Abstract Access to basic health care services is one of the major issues in developing nations. The location of health care facilities is an important aspect in health care delivery. It is therefore necessary for health care services to be located optimally to serve the demands well. This study develops an integrated methodology in the field of healthcare location which minimizes the weighted average Euclidean distance between existing facilities and new facility in a way that demands will not have to travel a very long distance to access medical facilities. Geographic information systems (GIS) were used to store and handle the coordinates and weights of the existing facilities. While the Weber model and Weiszfeld's algorithm was employed to determine the new healthcare location. The technique was applied to a case study in one of the local government areas in Nigeria. Our results show that the methodology can find a new facility to support demands in the study area. The proposed model can also be applied in studies of a similar nature as in location of library, medical drugs and equipment warehouse and support facility in emergency cases.

Keyword: Optimization, Healthcare Facility, GIS, Weiszfeld's Algorithm, Weber Model.

1 Introduction

Facility location problems have been extensively studied in the public and private sectors. The goals of decision making in the public sector usually include social cost minimization, universality of service, efficiency and equity. While private sector concerns are generally based on maximizing profit or increasing market share from competitors. Facility location is an area of analytical study that has been active for many decades. The area covered by facility location has numerous applications which include: emergency services (such as ambulances, warning sirens, hospitals, fire stations, police stations); warehouse placement, locating automatic teller machines, positioning a computer and communication units, locating National Youth Service Corps (NYSC) orientation camps in the country, locating rail stations, schools, postal facilities, solid waste disposal sites, factory sites, etc. [1].

* **Corresponding Author.** (✉)
E-mail: osinuga08@gmail.com (I. A. Osinuga)

I. A. Osinuga
Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria.

V. O. Oyebade
Department of Mathematics, Tai Solarin University of Education, Ijebu – Ode, Nigeria.

W. O. Otun
Department of Geography and Regional Planning, Olabisi Onanbanjo University, Ago – Iwo, Nigeria.

There is very rich literature on facility location problems. Some of the existing works are classic ones such as [1-5]; some presenting reviews and some discussing important aspects, as model features, types and algorithms [6]. In a number of papers, investigators are very specific for treating a particular type of problem. Examples are papers [7-10]. Other studies focus on specific applications, as in [11-12] while in some instances continuous location problems are considered using different measures of distances, e.g. [8, 13-15], etc.

One of the most important logistic decisions is the location of health care facilities [16]. Health services are a major concern in most developing countries, because of the continued rising costs of precious resources involved. A second concern is that many still do not have access to health care; or to the fact that many patients live in areas with a shortage of medical health care professionals. The study envisages family-oriented health care services for people living in rural and semi-urban communities. These services include primary and preventive health care, outreach and related services. However, the primary level of care, the expected first port of call and bedrock of the health system, is grossly deficient. In rural and semi-urban communities that represent the bulk of the population, primary health centers are dilapidated structures lacking trained manpower, drugs and equipment [17].

This paper addresses this question, by applying Weber model to minimize the weighted Euclidean distance from existing facilities to a new facility in a way that demands will not have to travel a very long distance to access medical facility. More facilities may be cited to reduce weights of demand on existing facilities in the region to reduce spread of diseases and mortality rate.

In the healthcare area, Rahman and Smith [18] developed a review of the location-allocation models in health service planning in developing nations. A number of authors are specific on the type of practical problem. Examples are Chanta *et al.* [19] and Pirkul and Schilling [20] that dealt with emergency medical services. While Sinuany-Stern *et al.* [21] developed a model for the location of hospitals in rural region. Jacobs *et al.* [22] described and analyzed locations and service areas of blood banks in American red cross blood facilities. Others, including authors [23-24] attempted to give new perspectives by discussing the application of classical models to a real health care location problem in different developing nations.

In many papers described, the focus is on developing heuristics that will solve the problems in a reasonable amount of time. However, the work of Griffin *et al.* [26] employed statistical methods while Fo and Mota [23] used the direct approach and four optimization models in solving the problems. In our case, we focus on the Weber model and Weiszfeld's algorithm, implemented within the GIS environment to locate optimally the new health care facility.

The rest of this paper is organized as follows: Section 2 presents the Weber location model and previous works. Section 3 describes the solution algorithm for the location of new healthcare facilities. Section 4 provides an applied example in one of the local government areas in Nigeria. Section 5 presents some concluding remarks and recommendations for future research.

2 Weber location model and previous research

A fundamental challenge in location planning is minimizing the distance between the facility and demand points (patients). Among different distance planning methods for a given number of facilities and locations, the p-median model seeks to minimize the total travelling distance from all clients to their closest serving facilities [3]. According to [27], Sinuany-Stern *et al.* [21] and Mehrez *et al.* [28] have used two different continuous models in identifying sites for a new hospital in Negev. The first was Weber model using three-stage procedures to find the best location, while the second was similar but the square of the Weber objective function. Demir [29] and Demir and Kockal [30] determined the emergency support facility and cement plant locations applying Weber model to Weiszfeld's algorithm with the aid of spatial results respectively. However, the study of Kwarteng [31] located library in Sunyani municipality using planar K-central single-facility Euclidean algorithm similar to Weiszfeld's method. Likewise the authors [32] discussed the optimization of location-allocation problem of pharmacy warehouses using integer programming and GIS while the study of authors [33] stressed the importance of applications of optimization models to locate healthcare facilities. According to Dokmeci and Ozus, as reported in [32] mathematical models and GIS are the most common tools in literature and practice used to arrive at satisfactory decisions for facility location problems. As seen from the studies reviewed above, there is still a dearth of work on the application of mathematical models and GIS to the location problems in healthcare. Therefore, achievements of past studies [29, 30, 32] motivated the study.

1-median model (Weber model) is a variant of the p-median models that have seen relatively little use in the health care location field. For the details on Weber problem, consult a comprehensive review by Wesolowsky [1] and Hakimi [34] for p-median models.

The Weber location problem (also known as Fermat-Weber problem) is a basic model in the location theory whose task is to find the "minisum" point (x^*, y^*) which minimizes the sum of the weighted Euclidean distances from itself to n fixed points with co-ordinates (a_i, b_i). The weights which are associated with the fixed points are denoted by w_i.

Weber problem can be stated as follows:

$$\min W(x) = \sum_{i=1}^{N} w_i d_i (x, a_i) \tag{2.1}$$

where

$W =$ weighted sum of distances of x from $a_1, \ldots, a_N$..

$a_i = (a_i^1, a_i^2, \ldots, a_i^N)$ is the known position of the i^{th} fixed point, $i = 1, \ldots, N$.

$x = (x_1, x_2, \ldots, x_N)^T$ is the unknown position of the new facility;

$w_i > 0$ is a weighting constant for fixed point (customer) i, $i = 1, \ldots, N$; and

$d_i(x, y) = \sqrt{(x_1 - a_i^1)^2 + (x_2 - a_i^2)^2}$ is the Euclidean distance between (x_1, x_2) and (a_i^1, a_i^2).

The Weber problem has been solved with several methods [1], but the most popular method that is used to solve the Weber problem with Euclidean distances is given by a one-point iteration procedure which was first proposed by Weiszfeld. The Weiszfeld iteration procedure is the most widely used procedure for the solution of the single-facility minisum location problem on the plane with Euclidean distances. It is also used in many multi-facility procedures as a step in the solution algorithm. This is presented in [35].

3 Solution algorithm

Among the solution procedures mentioned above, the simplest and, most commonly used technique is the "Weiszfeld procedure", and it is the solution procedure of interest in this research because the weights of demand points have been taken into account in the Weber model formulation.

3.1 Weiszfeld's algorithm

There are a variety of distance metrics used in facility location problems, among which are: l_p weight, l_∞ norm, block norm, round norm, mixed norms, Jaccard, expected, central and mixed gauges (for details, see [6]).

The most common distance metric of all mentioned is the l_p distance metric. When p takes a value of 1, it is known as rectilinear distance metric and when p takes a value of 2, it becomes the Euclidean distance metric. Weiszfeld's algorithm is based on Euclidean distance metric [35].

$$l_2 = \left(\sum_{i=1}^{m} |x_i - y_i|^2 \right)^{1/2} \tag{3.1}$$

Weiszfeld's algorithm locates the new facility among the existing facility locations considering the distance to be Euclidean. This iterative scheme is accepted as a powerful method in practice to identify the optimal location of Weber problem [29]. It solves iteratively the Weber problem based on its objective function as

$$\min\ f(x,y) = \sum w_i \sqrt{(x-a_i)^2 + (y-b_i)^2} \tag{3.2}$$

where

$w_i =$ weights associated with the existing locations,

$x =$ x-coordinates of starting point for the iterative algorithm;

$y =$ y-coordinates of starting point for the iterative algorithm;

$a_i =$ x-coordinates of existing locations and

$b_i =$ y-coordinates of existing locations.

The steps followed in Weiszfeld's algorithm for the solution of planar Euclidean location problem are as listed [31].

Step 1: Input the initial coordinates (x,y)

Step 2: Evaluate every $\beta_i = \dfrac{w_i}{\sqrt{(x-a_i)^2 + (y-b_i)^2}}$ (3.3)

Step 3: Sum for every β_i for $D(x,y)$

$$D(x,y) = \sum \beta_i (x,y) \tag{3.4}$$

Step 4: Determine all $\alpha_i = \dfrac{\beta_i}{D(x,y)}$ (3.5)

Step 5: Evaluate $WF_x = \sum \alpha_i a_i$, $WF_y = \sum \alpha_i b_i$ (3.6)

Step 6: Determine the objective function value f_i^* by summing the individual objective function values for all i using the equation

$$f_i^* = \sqrt{\left(WF_x - a_i^1\right)^2 + \left(WFy - a_i^2\right)^2}$$

where $WF(x_1,x_2)=\sum_{i=1}^{N}\lambda_i(x_1,x_2)(a_i^1,a_i^2)$. Repeat until stopping conditions are met (i.e. when the difference between the ratio of the coordinates of new point and the randomly selected initial point is less than or equal to zero). By iterating the Weiszfeld's values for x, y; we can find the Euclidean close to or equal to the optimal solution (x, y).

4 Numerical implementation and results

4.1 Problem and data analyses

The study area is Ijebu-North local government area, Ogun State, Nigeria, as shown in Fig. 1. The study region is a non-convex polygon with holes and an irregular boundary. The digitization of the area map was carried out on Pentium IV 3.2 GHz personal computer running Windows 7 with 2GB RAM. The Network Analyst in ArcGIS version 10 was utilized to manage and manipulate data layers of demand points, facility locations and links.

It was found out that there are only ten health facilities (Fig. 2) in the local government area expected to service sixty five settlements. The current population of 225,984 has been projected from the 1991 population census (National Population Commission, 1991). Each demand has to travel an average distance of 0.54 km to get to the nearest health facility location as against the 0.5 km stated in the State health policy. In the event of emergency, it is out of the realm of possibility reaching patients by health care provider and vice versa. This is as a result of the care provider serving other patients or having to cover a long distance to reach the patient or vice versa. Hence, the necessity to locate more facilities for enhancing the decision making process cannot be over-emphasized.

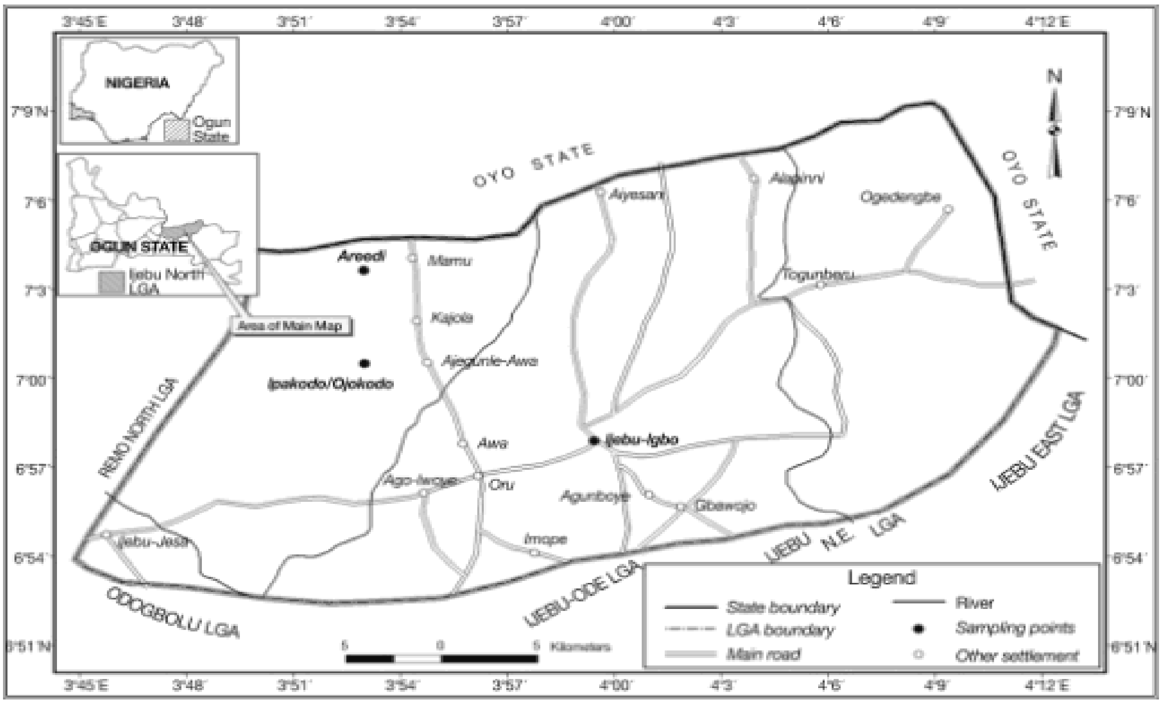

Fig. 1 Map of Ijebu-North local government area, Ogun State, Nigeria.

Table1 Existing health facilities locations

Object ID	Name of location	x and y coordinates	Demand count	Demand weight	Total distance	Total weight
1	Ago-Iwoye Aparaki	3.851, 6.912	11	58945	89.69	28016.80
2	Apoje Labour	3.936, 6.888	1	443	0.00	0.00
3	Camp	4.06, 6.956	3	2597	8.54	2243.71
4	Asigidi	3.974, 6.918	4	4716	12.58	8873.36
5	Falafonmu	3.924, 6.904	1	382	0.00	0.00
6	Ijebu-Igbo	3.953, 6.942	5	123545	33.18	10001.55
7	Imope	3.929, 6.879	1	1191	0.00	0.00
8	Mamu	3.869, 7.054	15	5385	247.01	84911.60
9	Oru	3.902, 6.927	4	21681	6.72	13738.91
10	Osun Bodepo	4.019, 7.018	20	7099	183.59	67417.43

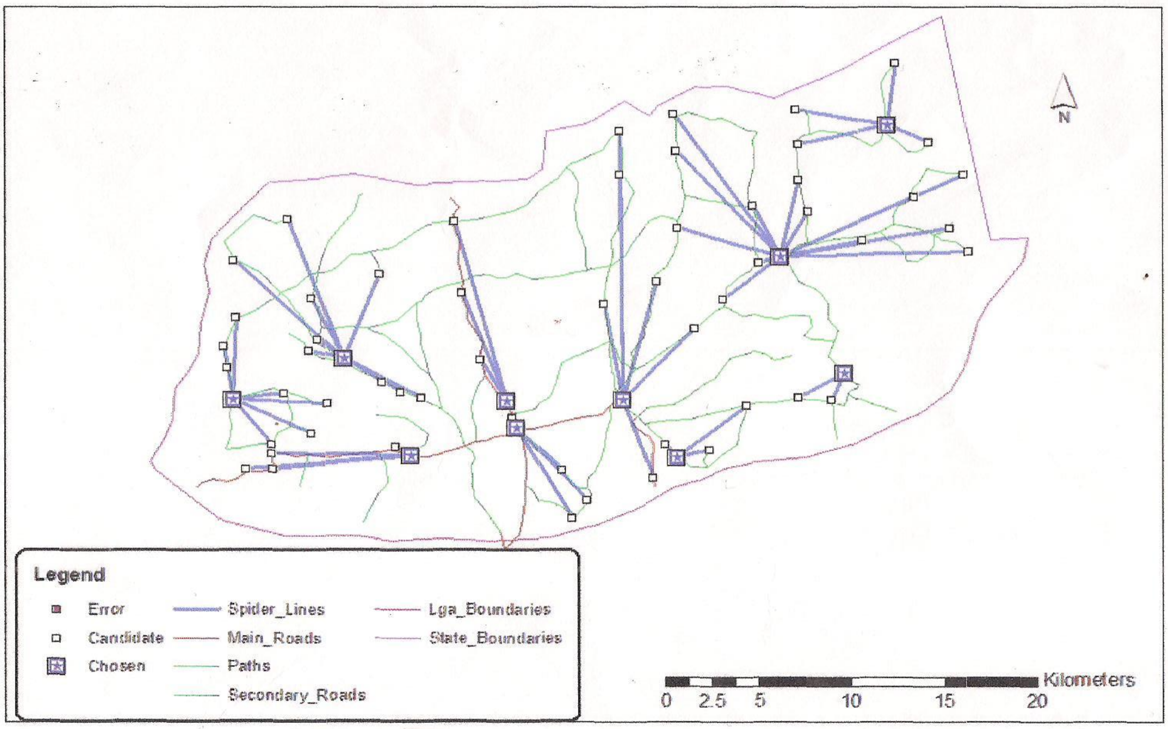

Fig. 2 Location map of existing health facilities and links.

4.2 Numerical experiments and discussions

In this section, we apply the integrated methodology to show its efficiency in the location of additional facilities in healthcare systems. The Weber problem adapted is a weighted planar problem with coordinates listed in Table 1.

(x_n, y_n) gives the initial point with which the algorithm is to start it's iterations. It could be a default value of (0, 0). In this study, the initial point is chosen to be (4, 7). This is an assumed location for which the new facility could be located.

Table 2 Results of example problem for various α values

No	$\alpha - values$
1	0.1322
2	0.0013
3	0.0136
4	0.0213
5	0.0012
6	0.6400
7	0.0033
8	0.0148
9	0.0689
10	0.1034

With the above details, weighted function for x and y coordinates are calculated using the equation 3.6 and presented below (Table 3):

Table 3 The location of the new health care facility

$WF(x_{n+1})$	$WF(y_{n+1})$
3.9433	6.9594

The above procedure was repeated until the following condition

$$\left| \frac{x_{n+1}}{y_{n+1}} - \frac{x_n}{y_n} \right| \le \varepsilon$$

is satisfied.

The Weiszfeld's algorithm was used to strategically locate optimally an additional health care facility. 10 coordinates were generated for 65 settlements in the example problem considered as the input to the algorithm. The optimal results obtained (Table 3) after first iteration based on the termination condition stated above. The algorithm gave one optimal location, i.e. (3.9433, 6.9594). The scatter plot (Fig. 3) indicated the octagonal point as the optimal point. The diamond points are the 10 existing health facilities in the Ijebu North local government, which was used for the work. Note that the optimal point arrived at is a local minimum point. However, the local minimum is the global minimum because the objective function in Weber problem is a convex function. Based on these results, the next section presents conclusions and future work related to the research.

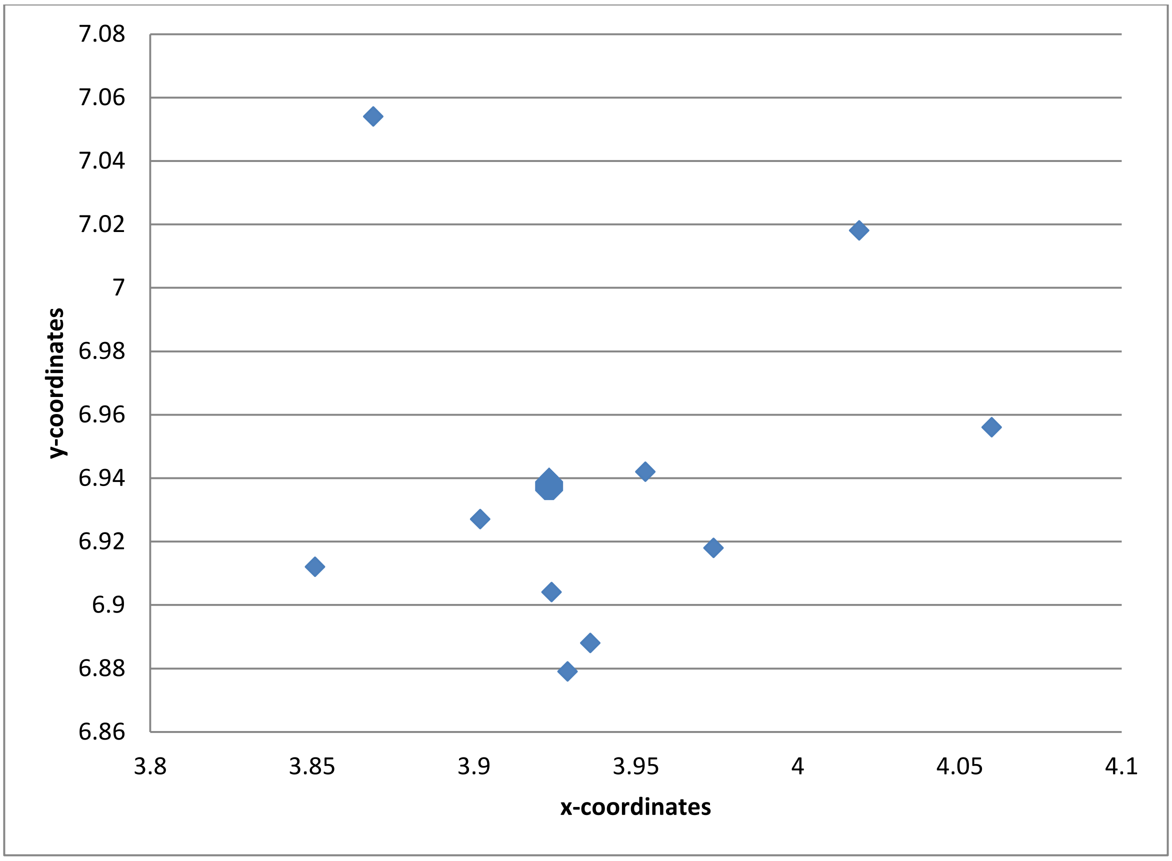

Fig. 3 Graph showing the new facility location with respect to the existing facility locations.

5 Conclusion and future work

The paper presented an experiment based on a real case, in the field of healthcare location, which minimizes the demand weighted average Euclidean distance between existing facilities and new facility. Location of new facilities, in one way will reduce the transportation costs of patients to health care facilities which would maximize the population level increase in accessibility.

Better physical accessibility to care is likely to promote increased utilization of these services. Such increased use may improve the care commonly associated with primary or preventive health care facilities, such as antenatal care and childhood immunization, but may also improve compliance with deadly and chronic diseases, including HIV/AIDS, Ebola virus disease (EVD), etc.

Our contribution includes adapting Weber models to the specific needs of healthcare facility problem in the local government area considered as well as integrating public data sources (captured with the aid of GIS) with Weiszfeld's algorithm to optimally locate new facilities. The study can contribute beyond regional planning to national planning, which is crucial to economic development of a country.

An interesting topic for future research may be to consider other instances of the problem. The Weiszfeld's algorithm is connected with Euclidean distances. Expanding the research a little more, it could be interesting to explore the model to additionally analyze the problem with other performance metrics such as rectangular, lift, etc., in urban region.

References

1. Wesolowsky, G. O. (1983). The Weber Problem: History and perspectives, Location Science, 1, 5-23.
2. Brandeau, M. L. and Chiu, S. S. (1989). An overview of representative problems in location research, Management Science, 35, 1989 645 – 674.
3. Afshari, F. and Peng, O. (2014) Challenges and solutions for location of healthcare facilities, Industrial Engineering and Management, 3:127, DOI 10.4172/2169- 0316.1000127.
4. Revelle, C. S., Eiselt, H. A. and Daskin, M. S. (2008). A Bibliography for some fundamental problem categories in discrete location science, European Journal of Operational Research, 184, 817-848.
5. Osinuga, I. A. and Bamigbola, O. M. (2007). On the minimum norm solution to Weber problem, In: SAMSA Conference Proceedings, Windhoek, Namibia, 2, 54-60.
6. Osinuga, I. A., Stanimirovic, P. S., Kazakovtsev, L. A. and Akinleye, S. A. (2015). A modelingframework on distance predicting functions for location models in continuous space, FactaUniversitatis: Ser. Math. Inform., 30 (4), 419 – 443.
7. Klamroth, K. (2001).Planar location problems with line barrier, Optimization, 49, 517-527.
8. Pfeiffer, B. and Klamroth, K. (2008). A unified model for Weber problem with continuous and network distance, Computers and OR, 35 (2), 312 – 326.
9. Dearing, P. M., Klamroth, K. and Segars Jr., R. (2005). Planar location problems with blockdistance and barriers, Annals of Operations Research, 136,(2005), 117-143.
10. Kafer, B. and Nickel, S. (2001). Error bounds for the approximate solution of restricted planar location problems, European Journal Operational Research, 135, 67-85.
11. Klose A. and Drexl A.(2004). Facility location models for distribution system design, European Journal of Operational Research,DOI:10.1016/j.ejor.2003.10.031.
12. Basar, A.,Catay, B. and Unluyuth, T. (2011).A taxonomy for emergency service station location problem, Optimization Letters, Springer Online, DOI: 10.1007/s11590- 011- 0376-1.
13. Stanimirovic, P. S., Ciric, M. S., Kazakovtsev, L. A. and Osinuga, I. A. (2012). Single - facility Weber location problem based on lift metric, Facta Universitatis: Ser. Math. Inform., 27 (2), 175-190.
14. Osinuga, I. A., Kazakovtsev, I. A. and Stanimirovic, P. S. (2013) Planar Weber problem with French metro metric, Review Bulletin Calcutta Mathematical Society, 21 (1), 7-20.
15. Kazakovtsev, L. A., Stanimirovic, P. S., Osinuga, I. A., Gudyma, M. N. and Antamoshkin, A. N. (2014). Algorithms for location problems based on angular distances, Advances in Operations Research, Art. ID 701267, 12pp. http:dx.doi.org/10.1155/2014/701267
16. Daskin, M. S. (1995). Network and discrete location models, algorithms, and Applications, NewYork: Wiley.
17. Olisaemeka, C. (2014). Towards improved health care delivery, The Punch Newspaper (Monday August 25, 2014), 38 (20734), pp. 24.
18. Rahman, S.-U.and Smith, D. K. (2000). Use of location- allocation models in health service development planning in developing nations, European Journal of Operational Research, 123, 437 – 452.
19. Chanta, S., Mayorga, M. E. and McLay, L. A. (2011). Improving emergency service in rural areas: a bi-objective covering location model for EMS systems**,** Annals of Operations Research, Springer online: DOI: 10.1007/s10479-011-0972-6.
20. Pirkul, H., and Schilling, O. A. (1988). The siting of emergency service facilities with workload capacities and backup service, Management Science, 34(7), 896 - 908.
21. Sinuany-Stern, Z., Mehrez, A., Tal, A.-G.and Shemuel, B. (1995). The location of a hospital in a rural region: The case of the Negev, Location Science, 3, 255 – 266.
22. Jacobs, D. A., Silau, M. N., and Clemson, B. A. (1996). An analysis of alternative locations and service areas of American red cross blood facilities, Interfaces, (3), 40 - 50.
23. Fo, A. and Mota, I. (2012). Optimization models in the location of health care facilities: The case of the Brazil, Journal of Applied Operational Research, 4 (1), 37 – 50.
24. Ammari, F. and Miyagi, T. (1999). Health care facilities location model with genetic algorithm, Journal of the Eastern Society for Transportation Studies, 3 (4): 55–69.
25. Tanser, F. (2005). Methodology for optimizing location of new health care facilities in rural communities: a case study in KwaZulu Natal, South Africa, Journal of Epidemiol Community Health, 60, 846-850. DOI 10.1136/jech.2005.043265.
26. Griffin, P. M., Sherrer, C. R. and Swann, J. L. (2007). Optimization of community health centers locations and service offerings with statistical need estimation. IIE Transactions, 40 (9), 880 – 892.

27. Daskin, M. S. and Dean, D. K. (2004). Location of health care facilities, In Operations Research and Healthcare: A handbook of methods and applications, (Eds.: M. L. Brandeau, F. Sainfort, and W. P. Pierskalla), Springer, New York, US, 43- 76.
28. Mehrez, A., Sinuany-Stern, Z., Tal, A.-G.andShemuel, B. (1996). On implementation of quantitative facility location models: The case of a hospital in the rural region, European Journal of Operations Research, 47, 612 – 625.
29. Demir, A. (2018) Approach for siting a support facility for transport supplies in emergency cases in the Republic of Bulgaria, MATEC Web Conference, 234, 06001, https:doi.org/10.1186/s/3600-018-1651-9, accessed 16 March, 2020).
30. Demir, E and Kochal, N. U. (2019).Iterative methodology on locating a cement plant. Journal of Inequalities and Applications, 174; https:doi.org/10.1186/s/3660-019-2122-7, accessed 10 October, 2019).
31. Kwarteng, S. (2011) Location of a library in Sunyani Municipality using the planar K- centra single facility Euclidean location algorithm. M.Sc. (Ind. Mathematics) Dissertation, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.
32. Ozceylan, E., Uslu, A., Erbas, M., Cetinkaya, C. and Isleyan, S. K. (2017). Optimizing the location allocation problem of pharmacy warehouses: A case study in Gaziantep. An International Journal of Optimization and Control: Theories and Applications, 7 (1), 117-129.
33. Dzator, M. and Dzator, J. (2019) Optimization models to locate health care facilities, 23rd International Congress on Modelling and Simulation, Canbera, ACT, Australia, 1-6, December, 2019.
34. Hakimi, S. L. (1964). Optimal locations of switching centers and absolute centers and medians of a graph, Operations Research,12, 450-459.
35. Morris, J. G. and Verdini, W. A. (1979). Minisum l_p distance location problems solved via perturbed problem and Weiszfeld's algorithm, Operations Research, 27 (6), 1180 – 1188.
36. Weiszfeld, E. (1937). "Sur le point purlequel la somme des distances n points donncesest minimum, Tohoku Mathematical Journal, 43 (1), 335-386.