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Solution of the non-linear first-order differential equations of the ideal gas: leveraging laminar flame speed in innovative modelling techniques for SI engine performance prediction

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Abstract In this paper, we report a four-stroke SI single-cylinder laboratory engine with a capacity of 392 cc and a compression ratio of 9:1. For comparison, two methods were used to estimate the mass fraction burned and the resultant heat release intensity. The first method used an experimental model to relate laminar flame speed to temperature, pressure, and air/fuel ratio. This method uses the value of the air/fuel ratio as experimental data. The second method estimated the mass fraction burned and the resultant energy release using the Wiebe function during combustion. The main step in this thermodynamic single-zone modeling is the simultaneous solution of the non-linear first-order differential equations of pressure, temperature, and volume obtained from the ideal gas equation during the cycle. MATLAB solved the nonlinear first-order differential equations; by applying the initial conditions and geometrical specifications of an SI laboratory engine. At the same time, work and cylinder temperature were also determined by applying the first law of thermodynamics and the ideal gas equation and accounting for heat loss. Finally, the results obtained from the flame speed method and Wiebe function were compared with the laboratory results obtained under the same conditions. The studies have shown an acceptable relative agreement.

Keywords: Laminar Flame Speed, Mass Fraction Burned, Combustion Process, Single-Zone Model, Air/Fuel Ratio.

1 Introduction

Thermodynamic models are the simplest and most common methods used to determine heat release intensity and charge burn rate during the combustion process. Although these models do not provide information about engine pollutants such as NOx and unburned hydrocarbons, thermodynamic analysis is a very valuable method for predicting engine performance parameters at the design stage [1]. All these analyses, from the simplest to the most complex, require models for combustion and predicting how the heat is released inside the engine.

MFB is one of the most important parameters in SI engines. By definition, this quantity is the ratio of the mass of the burned charge to the total mass of the charge inside the cylinder at the beginning of the compression stage, and its value changes between 0 and 1 (in practice, slightly less than 1) during combustion [2].

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Numerous past studies have related pressure changes within the cylinder to the rate of energy release during the combustion process [3]. This method, which is still widely used, determines the difference between the measured combustion pressure and the pressure caused by a polytropic process during combustion in several piston states, and its value is assumed to be proportional to MFB.

In the current study, a single-cylinder, four-stroke spark-ignition (SI) laboratory engine with a capacity of 392 cc and a compression ratio of 9:1 was investigated using two methods to model the released energy of the fuel. In the first method, the Wiebe function was used to estimate heat release during combustion [4].

The second method, namely the experimental method of flame speed, was the main objective of this study. The value of air/fuel ratio was used as experimental data as well as an experimental model of the relationship between laminar flame speed and temperature, pressure, and air/fuel ratio [5]. This model can be related to the burn intensity and the resultant heat release intensity during the combustion process; in other words, the value of air/fuel ratio (as experimental data) and the burn intensity are determined, then the first law of thermodynamics was applied, and the ideal gas equations of the change curves were determined. It should be noted that the fluid movement in the cylinder is turbulent, and reference [6] has shown that turbulence intensity is related to engine speed.

The final step in thermodynamic single-zone modelling is the simultaneous solution of the nonlinear first-order differential equations of pressure, temperature, and volume derived from the ideal gas equation and the first law of thermodynamics. In the flame speed method, a differential equation was used to estimate the burn rate based on temperature, pressure, and engine speed in terms of crank angle. In the Wiebe function, a mathematical exponential function was employed for the same purpose.

The nonlinear first-order differential equations were solved using MATLAB, that is, by applying the boundary conditions and geometric specifications of an SI laboratory engine. The pressure curve inside the cylinder, temperature, and work at the compression, combustion, and expansion stages can be determined according to the crank angle. Additionally, engine performance parameters such as thermal efficiency, mean effective pressure, and specific brake fuel consumption can be calculated.

Finally, the modelling results obtained from the Wiebe function and flame speed model were compared with the laboratory results, revealing a reasonable agreement between them.

2 Thermodynamic model

Single-zone models are extensions of the standard air cycle, in which it is assumed that the pressure, temperature, and gas composition inside the cylinder are homogeneous and change only with the crank rotation angle [7]. In these models, there is no distinction between burned and unburned gases. Another assumption of single-zone models states that heat transfer losses to the cylinder wall depend on the crank angle. By applying the first law of thermodynamics and disregarding gas mass leakage, changes in pressure can be expressed as: [4]:

$$\frac{dP}{d\theta} = -\gamma \frac{P}{V}\frac{dV}{d\theta} + \frac{\gamma - 1}{V}\left(\frac{dQ_{tot}}{d\theta}\right) \tag{1}$$

where

$$\frac{dQ_{tot}}{d\theta} = \frac{dQ_{hr}}{d\theta} - \frac{dQ_{ht}}{d\theta} \tag{2}$$

Q_{hr} refers to the chemical energy released by combustion, and Q_{ht} refers to heat transfer to the cylinder walls. With the geometric dimensions of the engine, we can write:

$$\frac{dV}{d\theta}=\frac{1}{2}S(\frac{\pi B^2}{4})(Sin\theta+\frac{R}{2l}Sin\theta) \tag{3}$$

where B is the cylinder diameter, S is the piston stroke, R is the crank radius, and l is the connecting rod length. The amount of work and temperature are also obtained from the following equations:

$$dW = PdV \tag{4}$$

$$T=\frac{PV}{mR} \tag{5}$$

The amount of heat transfer to the walls can be written as follows:

$$dQ_{ht}/_{d\theta} = h_g A(\theta)(T_g - T_w)/N \tag{6}$$

where h_g is the heat transfer coefficient, (A_θ) is the contact surface with the gases inside the cylinder, T_g is the mean temperature of the gases inside the cylinder, T_w is the mean temperature of the wall of the combustion chamber, and N is the engine speed. The empirical equation of heat transfer to the cylinder walls, derived from reference [8], can be written as follows:

$$dQ_{ht}/_{d_\theta} = A(\theta)(C_1 B^{(m-1)} P^m V^m T^{(0.45-1.6m)}(T_g - T_w) + C_2({T_g}^4 - {T_w}^4) \tag{7}$$

where (A_θ) is the amount of heat transfer in each state of the crank (m^2), P is the pressure inside the cylinder (KPa), and V is the mean speed of the gas inside the cylinder (m/s). The constant coefficients for a four-stroke engine are determined using the following values:

$C_1 = 3.26$
$C_2 = 3.88$
$m = 0.8$

The mean effective pressure of the indicator and brake can be related to each other by the following equation:

$$imep = bmep + tfmep \tag{8}$$

where $tfmep$ is the effective mean pressure of friction and pumping.

The value of $tfmep$ is obtained from the following experimental equation [9]:

$$tfmep(bar) = 0.97 + 0.15(\frac{N}{1000}) + 0.05(\frac{N}{1000})^2 \tag{9}$$

where N is speed per minute.

In addition, the temperature dependence of the specific heat capacity on temperature is estimated by the following equation [10]:

$$\gamma = 1.4 - 7.18\times10^{-5}T \tag{10}$$

3 Modelling the combustion process

3.1 Using the Wiebe function

The Wiebe function is a common method for determining the curve of changes in MFB according to the crank rotation angle during combustion:

$$x_b(\theta) = 1 - \exp[-a(\frac{\theta - \theta_0}{\Delta\theta})^m] \quad (11)$$

where θ is the crank angle, θ_0 is combustion start angle, $\Delta\theta$ is combustion duration, and m and values to model experimental data are selected as 2 and 5, respectively [4].

Combustion efficiency η_c for the fuel of spark engines (gasoline) can be shown in terms of λ (theoretical and stoichiometric air/fuel ratio) as follows [11]:

$$\eta_c = \eta_{c\max}(-1.6082 + 4.6509\lambda - 2.0764\lambda^2) \qquad 0.75 < \lambda < 1.2 \quad (12)$$

where the value of $\eta_{c\max}$ for spark engines are selected equal to 0.9. The numerical value of η_c in the equation above reaches the maximum value when λ is about 12% more than stoichiometry. The heat released as a result of m_f, i.e., Q_R, is the thermal efficiency of combustion:

$$Q_R = \eta_c m_f LHV_0 \quad (13)$$

where $LHV_{\circ}$ is the low heating value of the fuel.

3.2 Experimental method of flame speed

In SI engines, the flame is premixed. By definition, laminar flame speed is the normal component of the unburned gas velocity relative to the flame front. This definition applies to a state where the flame is stable, and the burned gases move away from it. Due to the lower density of burned gases compared to unburned gases, the velocity of the former will be higher than that of the latter [12].

SI engines operate under more realistic conditions when the flame front advances towards the stationary unburned gas mixture (Figure 1). The reciprocating motion of the piston, along with the design of the intake system and combustion chamber, contribute to the turbulent fluid flow within internal combustion engines. As a result of the factors mentioned, the flame transitions from a laminar state to a turbulent state. Under turbulent conditions, the flame's velocity increases substantially compared to the velocity observed in a laminar flame. Moreover, the flame speed is directly related to both the turbulence intensity and the laminar flame speed. [13].

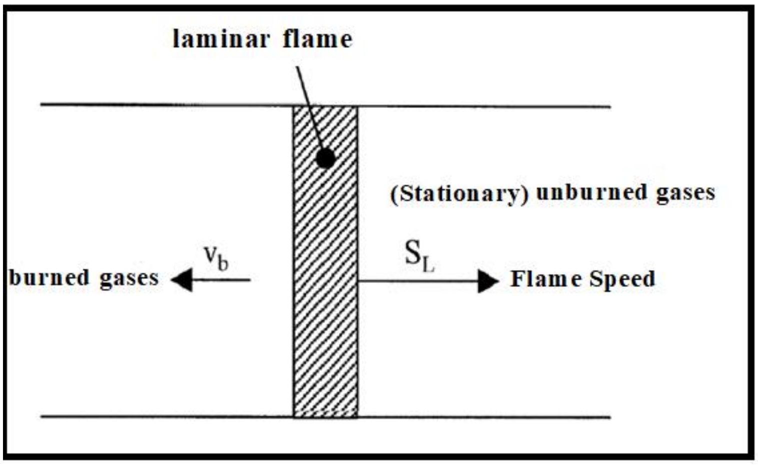

Fig. 1 Laminar flame propagation speed in static gas

The laminar flame speed for the mixture of air and gasoline reaches its maximum value by increasing the temperature of the mixture and reducing the increased pressure in the state where the value of φ is greater than 1 (about 1.1). This value is modeled as follows [12]:

$$S_L = S_{L,O}\left(\frac{T_u}{T_o}\right)^{\alpha}\left(\frac{p}{p_o}\right)^{\beta} \tag{14}$$

where $S_{l,\circ}$ is laminar flame speed, $P_0 = 1atm$, T_u and P are the temperature and pressure of the unburned part in front of the flame front. The parameters α and β depend on φ and are determined experimentally. The value of $S_{l,0}$ for the mixture of air and gasoline is given in [5].

As mentioned earlier, the flame in SI engines is turbulent, and studies have shown that burn intensity, especially in the major part of the heat release, i.e., the angle $\Delta\theta_b$ (related to 90% of the heat release), is not significantly changed by increasing engine speed. Because increasing engine speed increases turbulence intensity, it can be concluded that (according to $\Delta\theta = \omega\Delta t$) the burn time is reduced, which results in increased burn intensity. The rapid burning angle $\Delta\theta_b$ increases only slightly when engine speed is increased, indicating that turbulence intensity increases with engine speed, leading to an increase in flame speed. Experiments showed that the rapid burning angle increases with inlet pressure and the constant φ changes with engine speed almost as an exponential function:

$$\Delta\theta_b \cong N^{0.37} \tag{15}$$

Because the characteristics of the turbulent flow change with engine speed and depend on the engine geometry, the power N is unknown and is shown by η in the experimental results. Therefore [6]:

$$\Delta\theta_b = KN^{\eta} \tag{16}$$

The values K and η are determined experimentally. B is the diameter of the cylinder and s is the coefficient related to the location of the plug. To simplify and with appropriate approximation, it can be assumed that the outer surface of the flame A_f remains almost constant during heat release in the rapid combustion phase [12]. During rapid combustion, the burning rate is assumed to remain constant, and the flame grows at a constant rate. In such cases, the flame level changes are assumed to be equal to a constant value:

$$\frac{dV_f}{dt} = A_f U_f \tag{17}$$

and burn intensity:

$$\approx \frac{1}{AFR} \cdot \frac{P}{RT} . A_f . U_f \quad \frac{dm_f}{dt} = \frac{m_f}{V} \frac{dV_f}{dt} = \frac{m_f}{m_a} \frac{m_a}{V} \frac{dV_f}{dt} \tag{18}$$

where A_f the behaviour of the gas is assumed to be ideal ($PV = m_a RT$). Considering the mass of the fuel to be negligible compared to the air mass, and given that the air/fuel ratio in SI engines with gasoline fuel is approximately AFR=15, this approximation may result in a small error. Considering the aforementioned assumptions, the burn rate can be calculated using the following equation, where represents the laminar flame speed, is the air/fuel ratio, and is the cylinder volume:

$$\frac{dm_f}{d\theta} = bp^{1+\alpha} T^{\beta-1} N^{-\eta} (AFR)^{-1} \tag{19}$$

where is the constant surface of the flame front, and $U_{0,0}$ is the laminar flame speed at normal pressure and temperature [12]:

$$b = \frac{60 A_f U_{0,0}}{2\pi R T_0{}^{\alpha} P_0{}^{\beta} N_0{}^{1-\eta}} \tag{20}$$

4 Data analysis

The two proposed methods were used to model the heat release intensity during combustion in a laboratory single-cylinder gasoline engine having the specifications shown in Table 1. The fuel's heating value was 42.5 kJ/kg. The spark timing was assumed to be 25 degrees before TDC, and the combustion duration was 55 degrees of crank rotation, which falls within the recommended range of 50-80 degrees for SI engines [13]. The pressure of the exhaust gases was considered constant and equal to 1.05 atmospheres at the exhaust stage [13]. The input pressure was calculated based on volumetric efficiency, which is obtained at each engine speed from the intake charge using the ideal gas law. The temperature of the cylinder walls was used to determine the heat loss, assuming a constant value of 400 K [10].

In the flame speed method, performance parameters were determined using Equation (19) for the speed and air/fuel ratio data and applying it to the pressure differential equation (Equation 1), while also considering heat transfer (Equation 7). Note that the non-linear differential equations for pressure, temperature, work, heat transfer, and estimating the fraction of released energy do not have simple, analytical solutions; therefore, they were solved numerically using MATLAB. By applying Equation (8) and the experimental Equation (9), brake performance parameters were obtained for comparison. By taking into account the combustion duration and ignition timing, and similar to what was mentioned in the flame method, the brake performance parameters were determined for this method as well, using the thermodynamic model and other aforementioned equations.

Table 1 p8161 briggs and stratton single cylinder engine

Engine specification	4 Stroke,Air Cooled, Gasoline Engine
Engine Displacement (cc)	392cc (77.8mm Bore)
	82.5mm Stroke
Compression Ratio	9:1
Maximum power	7.46KW at 3600 rpm
Maximum torque	22.7N.m at 2400 rpm

The two applied methods were compared by examining the curves of pressure and temperature changes in relation to the crank rotation angle at an engine speed of 2000 rpm (shown in Figures 2 and 3). As shown, the maximum pressure value obtained using the Wiebe function method was about 20% higher than that obtained using the flame speed method. Similarly, the maximum temperature value calculated using the Wiebe function method was about 6% higher than that calculated using the flame speed method. The results above were compared with laboratory results by conducting a series of experiments at various throttle valve positions (3/4). Based on these experiments, several engine performance parameters, including thermal efficiency, brake mean effective pressure (BMEP), and specific brake fuel consumption, were calculated.

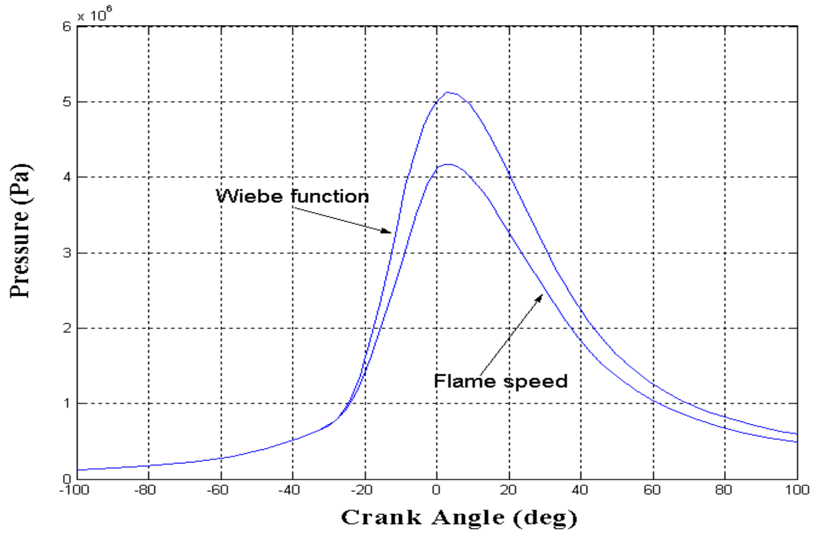

Fig. 2 Comparison of Wiebe function and flame speed method in pressure diagram according to crank angle at 2000 rpm

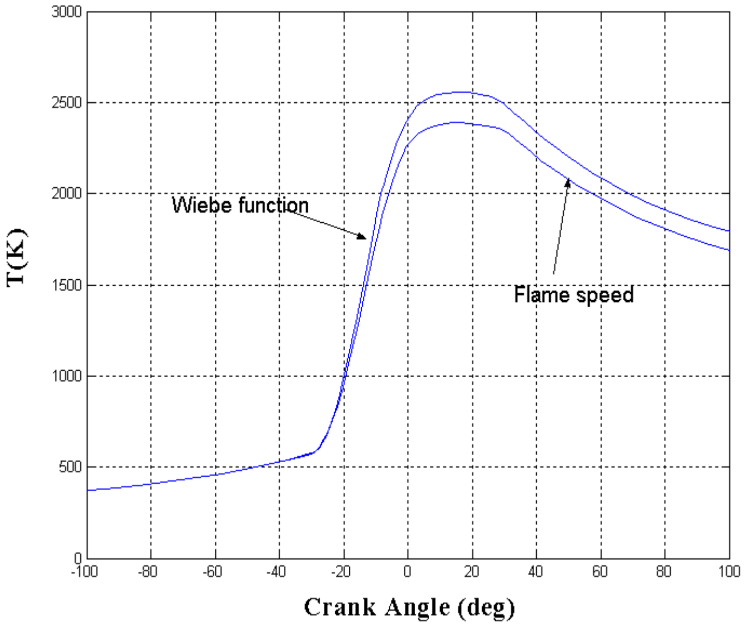

Fig. 3 Comparison of Wiebe function and flame speed in temperature diagram according to crank angle at 2000 rpm

Figure 4 illustrates the changes in thermal efficiency for the flame speed model, Wiebe function, and experimental results at various engine speeds. It is evident that, at all speeds, the flame speed model results are closer to the laboratory values compared to those obtained using the Wiebe function. The maximum error in the flame speed method was 6%, while it reached 18% in the Wiebe function. This trend is also observed in Figures 5 and 6, which depict the changes in brake mean effective pressure and specific brake fuel consumption for the two models in comparison with experimental values, respectively. These results demonstrate that despite the simplifying approximations made in the flame speed model, the error in the results is acceptable when compared to the experimental values.

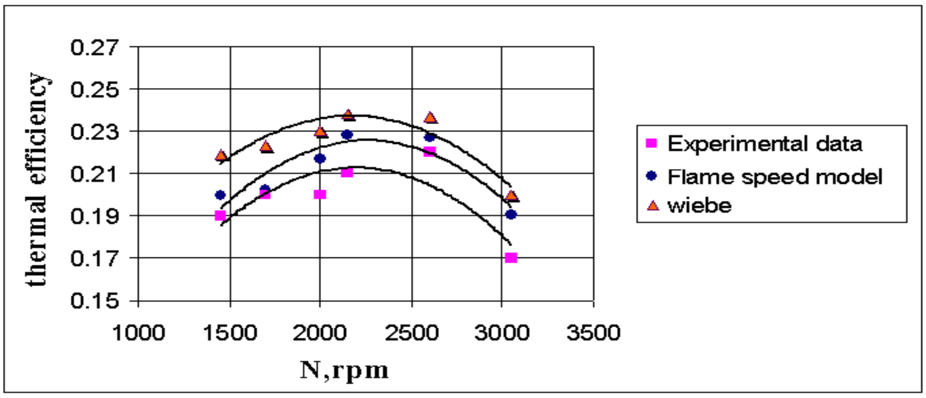

Fig. 4 Comparison of thermal efficiency in different cycles

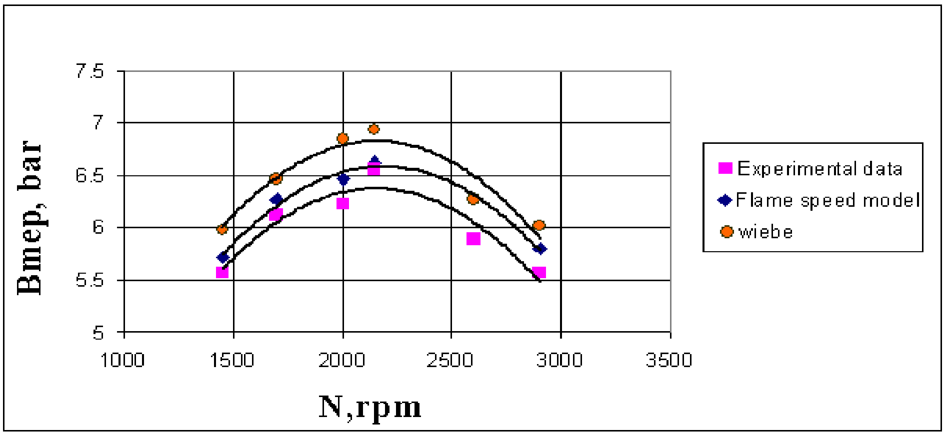

Fig. 5 Comparison of mean effective braking pressure in different cycles

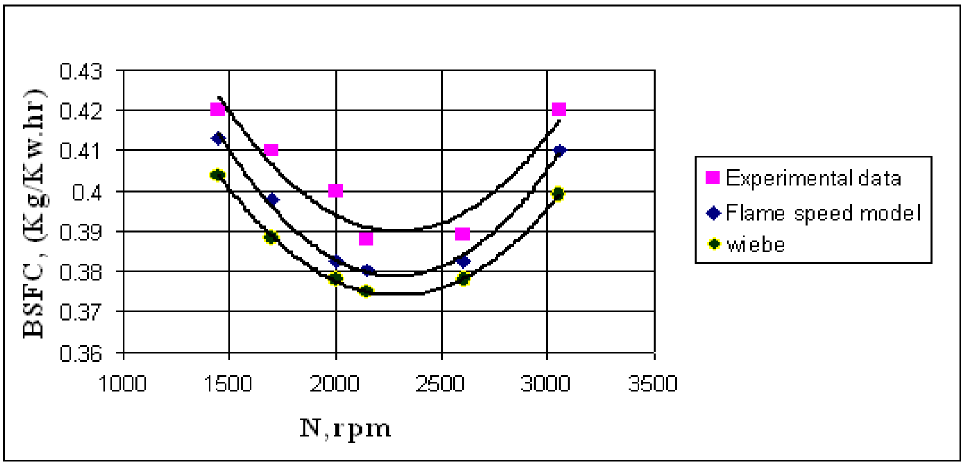

Fig. 6 Comparison of specific brake fuel consumption in different cycles

It was observed that despite the simplifying approximations made in the flame speed model, the error in the results is acceptable when compared to the experimental values.

5 Conclusion

Herein, a new method was introduced to determine the heat release rate at the combustion stage for SI engines. This method employs quantities such as temperature, pressure, and air/fuel ratio to relate the laminar flame speed to the mass fraction burned (MFB) and released energy, which ultimately correlates with the cylinder pressure. Since the actual flame speed in an SI engine depends on turbulence intensity, the turbulence model should also be considered. In this simplified model, turbulence intensity is assumed to be a function of engine speed [14].

The presence of turbulence causes wrinkling and bending in the flame front. In the presence of intense turbulence, the flame front becomes thinner and more distorted, trapping pockets of the unburned mixture. Turbulent gas movements contribute to flame propagation, and the local gas velocities caused by turbulence are significantly higher than the laminar flame speed, resulting in increased turbulence intensity. Of course, the turbulent flame speed is proportional to the laminar flame speed and uniformly increases with turbulence intensity.

This method was slightly modified and used previously to estimate the air/fuel ratio by analyzing pressure changes with respect to the crank angle during combustion. In this study, the model's input data consists of the air/fuel ratio and engine speed, and it was applied to a single-cylinder spark-ignition engine in a laboratory setting. Various engine operating conditions were considered, using experimentally measured values of speed and air/fuel ratio as inputs for the model. Several performance parameters, such as thermal efficiency, brake mean effective pressure, and specific fuel consumption, were calculated. The results demonstrated that, despite certain assumptions, the obtained values for specific fuel consumption, thermal efficiency, and brake mean effective pressure, based on changes in engine speed and spark timing, compare favorably with results obtained from reliable references. The flame speed method employs a simple turbulence model, assuming that the dependence of turbulence intensity on engine speed is a successful approximation for estimating the burn rate.

The results obtained from the flame speed method were compared with the laboratory results and the values obtained using the Wiebe function. The model results were found to be closely aligned with the laboratory results. In five comparisons with the experimental results conducted in this study, the deviation of the model results from the actual values is considered acceptable. It should be noted, however, that part of the observed uncertainty in the experimental values, and another part is related to the simultaneous effect of other basic parameters, along with changes in engine speed, which were not included in this study. Finally, it was concluded that better and more acceptable results would be obtained by further refining the methods and reducing the assumptions.

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