International Journal of Applied Operational Research
Vol. 2, No. 3, pp. 1-10, Autumn 2012

Journal homepage: www.ijorlu.ir

Heuristic Approach for Specially Structured Two Stage Flow Shop Scheduling to Minimize the Rental Cost, Processing Time, Set Up Time Are Associated with Their Probabilities Including Transportation Time and Job Weightage

D. Gupta*, S. Bala, P. Singla

Received: 26 May 2012 ; **Accepted:** 25 July 2012

Abstract The present paper is an attempt to develop a new heuristic algorithm, find the optimal sequence to minimize the utilization time of the machines and hence their rental cost for two stage specially structured flow shop scheduling under specified rental policy in which processing times and set up time are associated with their respective probabilities including transportation time. Further jobs are attached with weights to indicate their relative importance. The proposed method is very simple and easy to understand and also provide an important tool for the decision maker. Algorithm is justified by numerical illustration.

Keywords Specially Structured Flow Shop Scheduling, Rental Policy, Processing Time, Weight Age of Jobs, Set Up, Transportation Time.

1 Introduction

Scheduling can be defined as the allocation of resources over a period of time to perform a collection of tasks. The goal is to specify a schedule that specify when and on which machine each job is to be executed. All the scheduling models beginning from Johnson's work in 1954 upto the 1980 there is no reference of job weightage in the literature. The scheduling problem with weights arises when inventory costs for jobs are involved. The weights of a job show its relative priority over some other jobs in a scheduling mode. Scheduling theory deals with formulation and study of various scheduling models. Some widely studied classical models comprise single machine, parallel machine, flow shop scheduling, job shop scheduling, open shop scheduling and etc. The objective of flow shop scheduling problem is to find a permutation schedule that minimizes the maximum completion time of a sequence. Scheduling has become a major field with in operation research with several hundred publications appearing each year. Scheduling is a decision making practice that is used on a regular basis in manufacturing and service industries. Its aim is to optimize one or more objectives with the allocation of resources to task over given time periods. The time that a job spends on a machine include three phases viz setup, processing and removal. In the majority

* **Corresponding Author.** (✉)
E-mail: guptadeepak2003@yahoo.co.in (D. Gupta)

D. Gupta
Prof. & Head, Department of Mathematics, M.M.University, Mullana, Ambala,India.

S. Bala, P. Singla
Department of Mathematics, M. M. University, Mullana, Ambala, India.

[Downloaded from ijorlu.liau.ac.ir on 2025-05-25]

of investigation dedicated to production planning and scheduling, set up time considered to be negligible. But considering set up time separate from processing time have great impact on performance measure. As when there exists idle time on the second machine than the setup time for a job on a second machine can be performed prior to the completion time of this job on the first machine. Further the transportation times (loading time, moving time and unloading etc.) from one machine to another are also not negligible and therefore must be included in the job processing. However, in some application, transportation time have major impact on the performance measures considered for the scheduling problem so they need to consider separately. In a flow shop scheduling each job has the same routing throw machines and the sequence of operations is fixed. In a specially structured flow shop scheduling the data is not merely random but bears a well defined structural relation. Gupta J.N.D. [1] gave an algorithm to find the optimal schedule for specially structured flow shop scheduling. Johnson [2] first of all gave a method to minimize the makespan for n-jobs, two machine scheduling problems. Yoshida and Hitomi [3] further considered the problem with set up time. The basic concept of equivalent job for a job block has been introduced by Maggu & Das [4]. Singh T.P. [5] studied the optimal two stage production schedule in which processing time and set up time both were associated with probabilities including job block criteria. The work was developed by Chander Shekheran [6], Bagga [7] and Gupta Deepak et al. [8] by considering various parameters. Miyazaki [9] associated weights with the jobs.

Gupta & Sharma [10] studied 2-stage specially structured flow shop problem to minimize rental cost under the pre-defined rental policy. This paper is an attempt to extend the study made by Gupta & Sharma [10] by introducing transportation time, job weightage & set up time separated from processing time.

Thus the problem discussed in this paper has become wider and very close to practical situation in manufacturing/ process industry. We have obtained an algorithm which gives minimum possible rental cost while minimizing total utilization time.

2 Practical situation

Various practical situations occur in real life when one has got the assignment but does not have one's own machine or does not have enough money to purchase machine. Under such circumstances the machine has to be taken on rent in order to complete the assignment. Rental of various equipments is an affordable and quick solution for a businessman, a manufacturer or a company, which presently constrained by the availability of limited funds due to recent global economic recession. Renting enables saving working capital, gives option for having the equipment and allows up-gradation to new technology

The practical situation of specially structured flow shop scheduling occurs in our day to day working, in banking, offices, educational institutions, factories and industrial concern e.g., in a readymade garment manufacturing plant which has mainly two machines. viz, cutting and sewing , in which the time taken by the 2nd machine (sewing machine) will always be greater then the time taken by first machine (cutting machine).Moreover different quality of garment are to be produced with relative importance i.e. weight of jobs become significant.

[Downloaded from ijorlu.liau.ac.ir on 2025-05-25]

3 Notations

S	:	Sequence of jobs 1, 2, 3, …, n
S_k	:	Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, ..., r.
M_j	:	Machine j, j= 1,2.
a_{ij}	:	Processing time of i^{th} job on machine M_j
s_{ij}	:	Set up time of i^{th} job on machine M_j
p_{ij}	:	Probability associated to the processing time a_{ij}
q_{ij}	:	Probability associated to the processing time s_{ij}
A_{ij}	:	Expected processing time of i^{th} job on machine M_j
S_{ij}	:	Expected set up time of i^{th} job on machine M_j
$t_{i1\to 2}$	:	Transportation time of i^{th} job from machine M_1 to machine M_2
A''_{ij}	:	*Processing* flow time ith job on machine M_j.
$t_{ij}(S_k)$	:	Completion time of i^{th} job of sequence S_k on machine M_j
w_i	:	weight of i^{th} job.
G_i	:	weighted flow time of i^{th} job on machine M_1.
H_i	:	weighted flow time of i^{th} job on machine M_2.
$U_j(S_k)$	:	Utilization time for which machine M_j is required.
Cj	:	Renal cost per unit time of j^{th} machine.
$R(S_k)$	:	Total rental cost for the sequence S_k of all machine

Definition 1.
Completion time of i^{th} *job* on machine M_j is denoted by t_{ij} and is defined as:
$t_{ij} = \max(t_{i-1,j} + S_{i-1,j}, t_{i,j-1} + t_{i1\to 2}) + A_{ij}\,;\ \ j \ge 2.$
where A_{ij}=*Expected* processing time of i^{th} job on j^{th} machine. S_{ij}= Expected set up time of i^{th} job on j^{th} machine.

4 Rental policy (P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2^{nd} machine will be taken on rent at time when 1^{st} job is completed on the 1^{st} machine.

5 Problem formulation

Let some job i (i =1, 2, ..., n) are to be processed on two machines M_j (j = 1,2) under the specified rental policy P. Let A_{ij} & S_{ij} respectively be the expected processing and set up time of i^{th} job on j^{th} machine. Let w_i be weight of the i^{th} job and $t_{i1\to 2}$ be the transportation time of ith job from machine M_1 to machine M_2.Our aim is to find the sequence $\{S_k\}$ of jobs which minimize the rental cost of the machines while minimizing the utilization time of machines.
The mathematical model of the problem in matrix form can be stated as:

[Downloaded from ijorlu.liau.ac.ir on 2025-05-25]

Table 1

Jobs	Machine M_1				$t_{i1\to2}$	Machine M_2				Weight of jobs
I	a_{i1}	p_{i1}	s_{i1}	q_{i1}	$t_{i1\to2}$	a_{i2}	p_{i2}	s_{i2}	q_{i2}	w_i
1	a_{11}	p_{11}	s_{11}	q_{11}	$t_{11\to2}$	A_{12}	p_{12}	s_{12}	q_{12}	w_1
2	a_{21}	p_{21}	s_{21}	q_{21}	$t_{21\to2}$	A_{22}	p_{22}	s_{22}	q_{22}	w_2
3	a_{31}	p_{31}	s_{31}	q_{31}	$t_{31\to2}$	A_{32}	p_{32}	s_{32}	q_{32}	w_3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	a_{n1}	p_{n1}	s_{n1}	q_{n1}	$t_{n1\to2}$	a_{n2}	p_{n2}	s_{n2}	q_{n2}	w_n

Mathematically, the problem is stated as:
Minimize $U_2(S_k)$ and hence

Minimize $R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_j(S_k) \times C_2$

Subject to constraint: Rental Policy (P).
i.e. our objective is to minimize utilization time of machine and hence rental cost of machines.

Theorem 1. If $A_{i1} \le A_{i2}$ *for all* $i, j, i \ne j$, then $k_1, k_2, \ldots, k_n$ is a monotonically decreasing sequence, where $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Proof. Let $A_{i1} \le A_{j2}$ for all $i, j, i \ne j$
i.e., max $A_{i1} \le$ min A_{j2} for all $i, j, i \ne j$

Let $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$ Therefore, we have $k_1 = A_{11}$

Also $k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \le A_{11}$ $(\because A_{21} \le A_{12})$
$\therefore k_1 \le k_2$
Now, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22}$
$= A_{11} + A_{21} - A_{12} + (A_{31} - A_{22}) = k_2 + (A_{31} - A_{22}) \le k_2$ $(\because A_{31} \le A_{22})$
Therefore, $k_3 \le k_2 \le k_1$ or $k_1 \ge k_2 \ge k_3$.
Continuing in this way, we can have $k_1 \ge k_2 \ge k_3 \ge, \ldots, \ge k_n$, a monotonically decreasing sequence.

Corollary 1. The total rental cost of machines is same for all the sequences, if
$A_{i1} \le A_{i2},$ for all i, j, $i \ne j$.

Proof. The total elapsed time $T(S) = \sum_{i=1}^{n} A_{i2} + k_1 = \sum_{i=1}^{n} A_{i2} + A_{11}$.

It implies that under rental policy P the total elapsed time on machine M_2 is same for all the sequences thereby the rental cost of machines is same for all the sequences.

Theorem 2. If $A_{i1} \ge A_{j2}$ for all $i, j, i \ne j$**,** then $K_1, K_2, \ldots, K_n$ is a monotonically increasing sequence, where $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Proof. Let $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Let $A_{i1} \ge A_{j2}$ for all $i, j, i \ne j$ i.e., min $A_{i1} \ge$ max A_{j2} for all $i, j, i \ne j$
Here $k_1 = A_{11}$

[Downloaded from ijorlu.liau.ac.ir on 2025-05-25]

$k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \geq k_1$ ($\because A_{21} \geq A_{j2}$)
Therefore, $k_2 \geq k_1$.
Also, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22} = A_{11} + A_{21} - A_{12} + (A_{31} - A_{22})$
$= k_2 + (A_{31} - A_{22}) \geq k_2$ ($\because A_{31} \geq A_{22}$)
Hence, $k_3 \geq k_2 \geq k_1$.
Continuing in this way, we can have $k_1 \leq k_2 \leq k_3 \leq \ldots \leq k_n$, a monotonically increasing sequence.

Corollary 2. The total elapsed time of machines is same for all the possible sequences, if $A_{i1} \geq A_{j2}$ for all i, j, $i \neq j$.

Proof. The total elapsed time

$$T(S) = \sum_{i=1}^{n} A_{i2} + k_n = \sum_{i=1}^{n} A_{i2} + \left(\sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2} \right) = \sum_{i=1}^{n} A_{i1} + \left(\sum_{i=1}^{n} A_{i2} - \sum_{i=1}^{n-1} A_{i2} \right) = \sum_{i=1}^{n} A_{i1} + A_{n2}$$

Therefore total elapsed time of machines is same for all the sequences.

6 Assumptions

1. Jobs are independent to each other. Let n jobs be processed thorough two machines M_1 and M_2 in order M_1M_2
2. Machine breakdown is not considered.
3. Pre-emption is not allowed.
4. Jobs are independent to each other.
5. Transporting device is always available.
6. Weighted flow time has the following structural relation
   i.e. Either $G_i \geq H_i$
   or $G_i \leq H_i$ for all i

6.1 Algorithm

Step 1: Calculate the expected processing times, $A_{ij} = a_{ij} \times p_{ij}$; $S_{ij} = s_{ij} \times q_{ij}$
Step 2: Compute $A'_{i1} = A_{i1} - S_{i2}$

$A'_{i2} = A_{i2} - S_{i1}$

Step 3 : $A''_{i1} = A'_{i1} + t_{i1\to 2}$ and $A''_{i2} = A'_{i2} + t_{i1\to 2}$

Step 4: Calculate weighted flow time G_i & H_i as follow

If min $(A''_{i1}, A''_{i2}) = A''_{i1}$

Then $G_i = \dfrac{(A''_{i1} + w_i)}{w_i}$ & $H_i = \dfrac{A''_{i2}}{w_i}$

And

If min $(A''_{i1}, A''_{i2}) = A''_{i2}$

Then $G_i = \dfrac{A''_{i1}}{w_i}$ & $H_i = \dfrac{(A''_{i2} + w_i)}{w_i}$

[Downloaded from ijorlu.liau.ac.ir on 2025-05-25]

Step 5: Define a new reduced problem with processing time G_i & H_i as defined in Step 4.

Step 6: Check the structural conditions
Either $G_i \geq H_i$
or $G_i \leq H_i$, for all i
if the structural condition hold good go to Step 6 else reduce the problem in the required structural form.

Step 7: Obtain the job J_1 (say) having maximum processing time on 1st machine and job J_n (say) having maximum processing time on 2nd machine.

Step 8: If $J_1 \neq J_n$ then put J_1 on the first position and J_n on the last position and go to step 11 otherwise go to step 9.
.
Step 9: Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on M_1 call this difference as G_i. Also take the difference of processing time of job J_n on M_2 from job J_{n-1} (say) having next minimum processing time on M_2. Call the difference as G_2.

Step 10: If $G_1 \leq G_2$ put J_n on the last position and J_2 on the first position otherwise put J_1 on 1st position and J_{n-1} on the last position.

Step 11: Arrange the remaining (n-2) jobs between 1st job & last job in any order, thereby we get the sequences $S_1, S_2, \ldots, S_r$.

Step 12: Compute in - out table for any one (say S_1) of the sequence $S_1, S_2, ..., S_r$.

Step 13: Compute the total completion time CT (S_1).

Step 14: Calculate utilization time U_2 of 2nd machine where
$U_2(S_1) = CT(S_1) - A_{i1}(S_1)$;

Step 15: Find rental cost

$$R(S_1) = \sum_{i=1}^{n} A_{i1}(S_1) \times C_1 + U_2(S_1) \times C_2$$

where C_1 & C_2 are the rental cost per unit time of 1st & 2nd machine respectively.

7 Numerical Illustration

Consider 5 jobs, 2 machines problem to minimize the rental cost. The processing times, set up times with their respective probabilities, transportation time and weight in jobs are given in the following table. The rental cost per unit time for machines M_1 and M_2 are 10 units and 5 units respectively.

[Downloaded from ijorlu.liau.ac.ir on 2025-05-25]

Table 2

Jobs	Machine M_1				$t_{i1\to 2}$	Machine M_2				Weight of jobs
I	a_{i1}	p_{i1}	s_{i1}	q_{i1}	$t_{i1\to 2}$	a_{i2}	p_{i2}	s_{i2}	q_{i2}	W_i
1	90	0.3	2	0.2	2	30	0.2	3	0.1	5
2	100	0.2	3	0.2	3	45	0.1	2	0.3	2
3	80	0.2	1	0.3	4	22	0.3	4	0.2	3
4	120	0.2	2	0.2	1	60	0.1	1	0.3	4
5	130	0.1	1	0.1	5	25	0.3	1	0.1	2

Solution: As per step 1: The expected processing time & expected set up times for machines M_1 and M_2 are as follow:

Table 3

Jobs	Machine M_1		$t_{i1\to 2}$	Machine M_2		w_i
I	A_{i1}	S_{i1}		A_{i2}	S_{i2}	
1	27.0	0.4	2	6.0	0.3	5
2	20.0	0.6	3	4.5	0.6	2
3	16.0	0.3	4	6.6	0.8	3
4	24.0	0.4	1	6.0	0.3	4
5	13.0	0.1	5	7.5	0.1	2

As per step 2: Expected flow time for two machines M_1 and M_2 as follow:

Table 4

Jobs	Machine M_1	$t_{i1\to 2}$	Machine M_2	Weight
i	A'_{i1}		A'_{i2}	w_i
1	26.7	2	5.6	5
2	19.4	3	3.9	2
3	15.2	4	6.3	3
4	23.7	1	5.6	4
5	12.9	5	7.4	2

As per step 3: Processing flow time for machines M_1 and M_2 as follow:

Table 5

Jobs	Machine M_1	Machine M_2	Weight
I	A''_{i1}	A''_{i2}	w_i
1	28.7	7.6	5
2	22.4	6.9	2
3	19.2	10.3	3
4	24.7	6.6	4
5	17.9	12.4	2

[Downloaded from ijorlu.liau.ac.ir on 2025-05-25]

As per step 5: New reduced problem with weighted flow time G_i & H_i as follow:

Table 6

Jobs	G_i	H_i
1	5.74	2.52
2	11.2	4.45
3	6.4	4.43
4	6.175	2.65
5	8.95	7.2

Here, $G_i \geq H_i$ for all *i*.
As per step 7: max G_i = 11.2 which is for job 2 i.e. J_1 =2
And min H_i = 2.52 which is for job 1 i.e. J_n = 1.
Since $J_1 \neq J_n$. we put J_1 = 2 on the first position.
And J_n = 1 on the last position.
Therefore the optimal sequences are S_1= 2 – 3 – 4 – 5- 1 .
S_2= 2 – 4 – 5 – 3 – 1, S_3 = 2- 3 -5-4-1, S_4=2 -4-3-5-1, S_5= 2- 5-4-3-1, S_6= 2-5-3-4-1.
Due to our structural conditions the total elapsed time is same for all these 6 possible sequences S_1, S_2; S_3, S_4 ,S_5 , S_6 Find in-out table for any one of these, say for S_1 = 2 – 3 – 4 – 5 – 1 is :

Table 7

Jobs	Machine M_1	Machine M_2
I	In-Out	In-Out
2	0 - 20	23 - 27.5
3	20.6 - 36.6	37.6 - 44.2
4	36.9 - 60.9	62.9 - 68.9
5	61.3 - 74.3	75.3 - 82.8
1	74.4 - 101.4	103.4 - 109.4

Therefore, the total elapsed time = $CT(S_1)$ = 109.4 units
Utilization time of machine $M_2 = U_2(S_1)$ = 86.4 units

Also $\sum_{i=1}^{n} A_{i1}$ =101.4 units.

Therefore the total rental cost for each of the sequence (S_k); k = 1, 2, 3, 4, 5, 6 is
$R(S_k)$ = 101.4 × 10 + 86.4 × 5
=1014 +432
= 1446 units.

8 Remarks

If we solve the same problem by Johnson's methods we get the optimal sequence as S= 5 – 2 – 3 – 4 – 1.

[Downloaded from ijorlu.liau.ac.ir on 2025-05-25]

The in – out flow table is:

Table 8

Jobs	Machine M_1	Machine M_2
i	In - Out	In - Out
1	0-13	14-21.5
2	13.1-33.1	36.1-40.6
3	33.7-49.7	50.7-57.3
4	50.0-74.0	74.2-80.2
5	74.4-101.4	103.4-109.4

Therefore, the total elapsed time = CT(S) = 109.4 units
Utilization time of machine $M_2 = U_2(S)$ = 95.4 units

Also $\sum_{i=1}^{n} A_{i1}$ =101.4 units.

Therefore the total rental cost is
$R(S_k)$ = 101.4 × 10 + 95.4 × 5
= 1014 + 477
= 1491 units .

9 Conclusion

The algorithm proposed here for specially structured two stage flow shop scheduling problem with processing time, setup time associated with their respective probabilities including transportation time and weightage of jobs is more efficient as compared to the algorithm proposed by Johnson [2] to find an optimal sequence to minimize the utilization time of the machines and hence their rental cost.

The study may further be extended by considering various parameters like breakdown effect, job block etc.

The study may further be extended for n job 3 machine specially structured flow shop problem.

References

1. Gupta, J. N. D., (1975). Optimal Schedule for specially structured flow shop. Naval Research Logistic, 22(2), 255-269.
2. Johnson, S. M., (1954). Optimal two and three stage production schedule with set up times included. Naval Research Logistic, 1(1), 61-64.
3. Yoshida and Hitomi, (1979). Optimal two stage production scheduling with set up times separated. AIIE Transactions, 11, (3), 261-269.
4. Maggu, P. L., Das, G., (1977). Equivalent jobs for job block in job scheduling. Opsearch, 14(4), 27.
5. Singh, T. P., (1985). On n×2 shop problem involving job block. Transportation times and Break-down Machine times. PAMS, XXI, 1-2.
6. Chander Shekheran, R., (1992). Two-Stage Flowshop Scheduling Problem with Bicriteria. O.R. Soc., 43(9), 871-84.

[Downloaded from ijorlu.liau.ac.ir on 2025-05-25]

7. Bagga, P. C., (1969). Sequencing in a rental situation. Journal of Canadian Operation Research Society, 7, 152-153.
8. Gupta, D., Sharma, S., (2011). Minimizing rental cost under specified rental policy in two stage flow shop, the processing time associated with probabilities including breakdown interval and Job-block criteria. European Journal of Business and Management, 2(3), 85-103.
9. Miyazaki, S., Nishiyama, N., (1980). Analysis for minimizing weighted mean Minimizing rental cost in two stage flow shop, the processing time associated with probabilies including job block. Reflections de ERA, 1(2), 107-120.
10. Gupta, D., Sharma, S., Bala, S. (2012). Specially Structured Two Stage Flow Shop Scheduling to Minimize the Rental Cost. International Journal of Emerging trends in Engineering and Development, 1(2), 206-215.
11. Anup, (2002). On two machine flow shop problem in which processing time assumes probabilities and there exists equivalent for an ordered job block. JISSOR, XXIII (1-4), 41-44.
12. Chander, S., Rajendra, K., Deepak, C., (1992). An Efficient Heuristic Approach to the scheduling of jobs in a flow shop. European Journal of Operation Research, 61, 318-325.
13. Ignall, E., Schrage, L., (1965). Application of the branch and bound technique to some flow shop scheduling problems. Operation Research, 13, 400-412.
14. Dell'Amico, M., (1980). Shop problems with two machines and time lags, Oper. Res. in flow-shop scheduling. J.O.R. Soc. of Japan, 23, 118-132.
15. Narian, L., Bagga, P. C., (2005). Two machine flow shop problem with availability constraint on each machine. JISSOR, XXIV (1-4), 17-24.
16. Pandian and Rajendran, (2010). Solving Constraint flow shop scheduling problems with three machines. Int. J. Contemp. Math. Sciences, 5(19), 921-929.

[Downloaded from ijorlu.liau.ac.ir on 2025-05-25]