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A Revised Fuzzy - PROMETHEE Method, Using Fuzzy Distance and Similarity Measures

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Abstract PROMETHEE refers to a collection of methods of ranking in the field of multi-criteria decision making. These methods are characterized by conceptual simplicity and practical applicability. However, the nature of phenomena involving decision-making in real world leads us to use fuzzy method of preference ranking. The most common criticism on mathematical ranking procedures is that they tend to defuzzify the problem by calculating a real number for each fuzzy set. In this paper we present a more precise fuzzy preference ranking method in uncertain, fuzzy environment for decision making. The new method allocates a linguistic term to each alternative by using fuzzy distance and fuzzy similarity measures. The linguistic term with the greatest similarity is allocated to a related alternative choice. The alternatives with fuzzy scores are ranked based on their allocated linguistic terms. Accordingly, we can make better decisions because we have verbal forms of the scores. A numerical example of decision making is presented and the results are compared with results of other F-PROMETHEE methods. Selecting nanotechnology application fields in Iran is presented as a real case.

Keywords: Decision Making, Fuzzy-PROMETHEE, Similarity Measure, Fuzzy Distance, Generalized Fuzzy Number, Nanotechnology.

1 Introduction

Multi-criteria decision making (MCDM) comprises of methods such as: the scoring approach i.e. Analytic Hierarchy Process (AHP), the compromising solution approach i.e. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), and the ranking approach i.e. Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE) and Elimination and Choice Translating Reality (ELECTRE).

It is commonly held view that PROMETHEE procedures are superior to the competing approaches. The advantages of using PROMETHEE procedures are as follows: simplicity the mathematical background behind PROMETHEE [1], ability to use qualitative and quantitative data, flexibility of its software package, problem visualization, and ability of

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considering all criteria when some of them are at odds with each other [2]. In fuzzy decision making environments, using fuzzy PROMETHEE, the input data expressed by fuzzy sets. However, the final rankings based on this approach are expressed by fuzzy sets.

Obtaining ranking order of fuzzy sets is not a trivial task. Indeed those researchers who have worked on Fuzzy-PROMETHEE in recent years, have converted fuzzy outputs of the PROMETHEE procedure to crisp and have arranged them in crisp ranking. We believe that "the distance between two fuzzy numbers is fuzzy number", and develop a method with the advantage that it allows us to consider both fuzzy inputs and outputs for ranking.(by using fuzzy similarity measures).

Outline of the paper follows: Section 2, presents a literature review. Section 3 and 4, present PROMETHEE and Fuzzy-PROMETHEE and some principles. A new Fuzzy-PROMETHEE using fuzzy similarity measures is presented in Section 5. A numerical example is described in Section 6. In section 7, as a real application of the method, we use the method in ranking of the projects involving application of nanotechnology in Iran. This new presentation method is compared with some other Fuzzy-PROMETHEE methods in Section 8. Finally, Section 9 presents the empirical results and some future research proposals.

2 Literature review

PROMETHEE stands for **P**reference **R**anking **O**rganization **Meth**od for **E**nrichment of **E**valuations, and the method has evolved from PROMETHEE- I to PROMETHEE- VI since 1982. PROMETHEE- I and II were developed by Brans as partial ranking and complete ranking, respectively [3]. After a few years, Brans and Mareschal developed a ranking based on intervals and a continuous action set extension of PROMETHEE named PROMETHEE-III and PROMETHEE -IV, respectively. PROMETHEE- III was an attempt to enhance indifferences, which happen rarely in practice in PROMETHEE ranking. PROMETHEE- IV was applied where the set of actions is defined by decision variables and constraints, as in mathematical programming [4]. Mareschal and Brans presented GAIA (Geometrical Analysis for Interactive Assistance), which is a graphical representation supporting the PROMETHEE methodology [5].

PROMETHEE -V, MCDA including segmentation constraints, is a procedure for multiple selections of alternatives under constraints, which have been presented by Brans and Mareschal [6]. In addition, PROMETHEE- VI, representation of the human brain, is a sensitivity analysis suggested by them [7].

Fernandez-Castrol and Jimenez have applied PROMETHEE- V to select distribution centers for a firm in 4 regions in Belgium. Twelve alternatives sites were evaluated through 5 criteria at first, and then the optimization problem involved some additional constraints [8].

Mergias and Moustakas and Papadopoulos and Loizidou have showed the best compromise management scheme for end-of-life vehicles by applying PROMETHEE [29]. Behzadian, Kazemzadeh Albadvi and Aghdasi[4], based on a comprehensive literature review, have presented a classification scheme to uncover, classify, and interpret the current research on PROMETHEE methodologies and applications. They have categorized 195 papers into nine areas: Environment Management, Hydrology and Water Management, Business and Financial Management, Chemistry, Logistics and Transportation, Manufacturing and Assembly, Energy Management, Social Sciences, and other topics. The last area covered in the paper deals with the papers published in several fields: Medicine, Agriculture, Education, Design, Government, and Sports. The papers on each topic are summarized in the specific tables in [10]. Hu and Chen have proposed a classification method with concepts from the flows used in PROMETHEE methods which, defines an overall preference index using both concordance and discordance relations for ordinal sorting problems [11].

Since our research was in the field of a fuzzy environment of PROMETHEE, we confined our literature review in fuzzy conditions.

Goumas and Lygerou have applied the PROMETHEE method with fuzzy input data for evaluating and ranking of alternative energy exploitation schemes of a low temperature geothermal field. The performance of each scenario according to each criterion is introduced as a fuzzy number [12].

The linguistic decision-making approach is an approximate way to use natural words to describe human judgment and perception. Linguistic decision analysis, transforms the linguistic description of Decision Makers into a mathematical model for solving decision problems [13,14,15,16]. Herrera and Harrera-Viedma [17] have stated that linguistic decision analysis is an appropriate tool to model qualitative information in multiple real world decision situations [17]. Martinez [18] has utilized the decision analysis techniques in evaluation processes. He has applied linguistic decision analysis to sensory evaluation because the information acquired by personal senses and human perceptions involve uncertainty and vagueness. Aliev Pedrycz , et al. [19] have proposed a decision theory, which is capable of dealing with vague preferences and imperfect information based on a fuzzy-valued non-expected utility model representing linguistic preference relations and imprecise beliefs . Wang , Chen and Chen have applied Linguistic variables and their corresponding fuzzy numbers to determine the priority weight of each criterion. The fuzzy outgoing/leaving flow $\tilde{\phi}^{+}(a)$ and $\tilde{\phi}^{-}(a)$ are calculated by using the maximizing and minimizing set method for defuzzifying. Therefore, the net flow of each alternative will be a crisp number, which occurs in the PROMETHEE method in a non fuzzy environment. This method was applied to evaluate information systems outsourcing suppliers [20]. Liu and Guan have applied linguistic variables and corresponding triangular fuzzy numbers and improved PROMETHEE- II to evaluate the quality of railway passenger service by introducing a defuzzification statement for defuzzifying the net flow [21]. A new PROMETHEE- II method based on generalized fuzzy numbers was presented to consider the fuzziness in the decision data during decision-making process by Wei-Xiang and Bang-Yi. They have introduced a defuzzification function for scoring alternatives based on fuzzy net flows [22].

According to our research, all of the generalized fuzzy numbers scoring is done by defuzzifying the fuzzy numbers, especially works by Chen and Sanguansat [23] and Chen and Chen [24]. Wang [32] has shown that centroid defuzzification and the maximizing as well as minimizing set methods are two commonly used approaches to ranking fuzzy numbers. These methods have been applied when explicit membership functions are not known but alpha level sets are available. A defuzzification using a minimized distance between two fuzzy numbers has been proposed by Asady and Zendehnam [25].

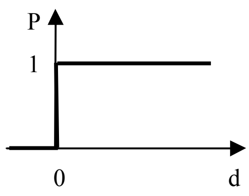
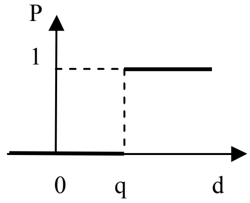
Recent research has applied PROMETHEE in a fuzzy environment for evaluating some alternatives according to certain criteria. They defuzzify the fuzzy net flows for ranking the alternatives. See Table (1).

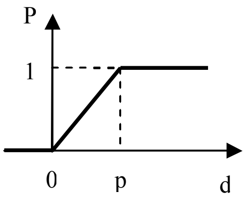
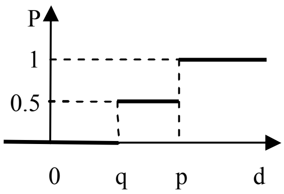
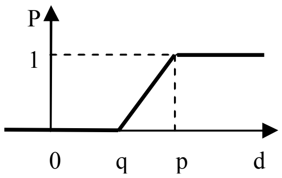
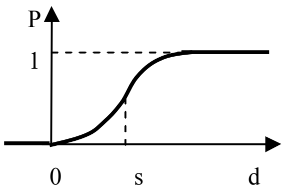
Table 1 Recent researches of F-PROMETHEE

Row	Year of the Research	Researchers	Research Description	Decision Measure	Type of Defuzzification
1	2008	Wang / Chen / Chen	Applying Fuzzy PROMETHEE Method for Evaluating IS Outsourcing Suppliers	$\tilde{\phi}(a)$	Using maximizing set and minimizing set method which was proposed by Chen (1985) for defuzzifying.
2	2009	Liu / Guan	Evaluation Research on the Quality of the Railway Passenger Service Based on the Linguistic Variables and the Improved PROMETHEE-II Method	$\tilde{\phi}(a) = (r_i, r_m, r_u)$	$E(\tilde{\phi}) = \dfrac{(r_i + 2r_m + r_u)}{4}$
3	2010	Wei-Xiang / Bang-Yi	An Extension of the PPOMETHEE II Method Based on Generalized Fuzzy Numbers	$\tilde{\phi}(a) = (a, b, c, d; 1)$ and $STD\tilde{\phi}(a)$	$Score(\tilde{\phi}(a)) = df(\tilde{\phi}(a)).(1-\lambda.STD\tilde{\phi}(a))$ and $df(\tilde{\phi}(a)) = \dfrac{(a+b+c+d)}{4}$

In this paper we present ranking without defuzzifying. The motivation for this reconsideration is derived from question: " if we are not certain about the numbers themselves how can we be *certain* about the distances among them'' [26]. Therefore, we present a reasonable method to compare the fuzzy net flows for ranking alternatives in the PROMETHEE method. Furthermore, we consider the basic table of linguistic variables (terms) and the corresponding generalized fuzzy numbers in Table (3) , which were presented by Zhang [27], and find the similarity measures between each alternatives. The proposed method is described in Section 5, and its comparison with another method can be found in the Discussion section.

Table 2 Types of generalized criteria, (P(d): Preference function) [28]

Generalized criteria		Definitions	Parameters
Type 1: Usual criterion		$P(d) = \begin{cases} 0, & d < 0 \\ 1, & d \geq 0 \end{cases}$	----
Type 2: U-Shape criterion		$P(d) = \begin{cases} 0, & d \leq q \\ 1, & d > q \end{cases}$	q

Generalized criteria		Definitions	Parameters
Type 3: V-Shape criterion		$P(d)=\begin{cases}0, & d\le 0\\ \frac{d}{p} & 0\le d\le p\\ 1, & d>p\end{cases}$	p
Type 4: Level criterion		$P(d)=\begin{cases}0, & d\le q\\ \frac{1}{2} & q\le d\le p\\ 1, & d>p\end{cases}$	p,q
Type 5: V-Shape with indifference criterion		$P(d)=\begin{cases}0, & d\le q\\ \frac{d-q}{p-q} & q\le d\le p\\ 1, & d>p\end{cases}$	p,q
Type 6: Gaussian criterion		$P(d)=\begin{cases}0, & d\le 0\\ 1-e^{-d^2/s^2}, & d\ge p\end{cases}$	s

3 Promethee and fuzzy-promethee

The PROMETHEE method is applied in ranking m alternatives $A=\{a_1,a_2,...,a_m\}$ by considering n criteria $C=\{c_1,c_2,...,c_n\}$. There are 6 preference functions for determining the preference of two alternatives under each criterion, as shown in Table (2).

The importance of each criterion is considered as $W=\{w_1,w_2,....,w_n\}$, subject to $\sum w_j=1$.

Based on this assumption, we develop some equations are as follows:

$$\pi(a_i,a_k)=\sum_{j=1}^{n}P_j(a_i,a_k)\cdot w_j$$

$$P_j : A\times A\to[0,1] \;;\; P_j(a_i,a_k)=F_j[d_j(a_i,a_k)] \;;\; d_j(a_i,a_k)=g(a_i)-g(a_k)$$

$g(a_i)$ and $g(a_k)$ are the values of alternative a_i and a_k under j^{th} criterion respectively.

In this method, three flows must be calculated for each alternative from A: *leaving flow* (positive outranking flow), *entering flow* (negative outranking flow) and *net flow* respectively as follows:

$$\phi^{+}(a)=\frac{1}{m-1}\sum_{x\in A}\pi(a,x) \quad ; \qquad \phi^{-}(a)=\frac{1}{m-1}\sum_{x\in A}\pi(x,a) \quad ;$$

$$\phi(a)=\phi^{+}(a)-\phi^{-}(a)$$

The alternatives are ranked based on the size of their net flowes [28].
By considering fuzzy input data in PROMETHEE II, the following steps are presented.
Step1: When there is a committee of decision-makers, $\tilde{\xi}=\{\tilde{\xi}^{1},\tilde{\xi}^{2},...,\tilde{\xi}^{i},...,\tilde{\xi}^{L}\}$ presents the significance of decision-makers.
Step 2: For every decision maker (DM) in the decision group, we can get a vector of criteria weights such as $\tilde{\omega}^{l}=\{\tilde{\omega}^{l}{}_{1},\tilde{\omega}^{l}{}_{2},...,\tilde{\omega}^{l}{}_{k},...,\tilde{\omega}^{l}{}_{n}\}$. The preference matrix given by 1^{th} DM is written as:

$$\tilde{M}^{l}=\begin{bmatrix}\tilde{M}^{l}_{11} & \cdots & \tilde{M}^{l}_{1n}\\ \vdots & \ddots & \vdots\\ \tilde{M}^{l}_{m1} & \cdots & \tilde{M}^{l}_{mn}\end{bmatrix}$$

; $\tilde{M}^{l}_{ik}$, (i=1,2,…,m; k=1,2,…,n) is the evaluation value of the i^{th} alternative under k^{th} criterion given by 1^{th} DM in terms of linguistic variables.
Step 3: A fuzzy weight for aggregative criterion is as follows:

$$\tilde{\omega}_{k}=\frac{1}{L}\left(\left(\tilde{\zeta}^{1}\otimes\tilde{\omega}^{1}_{k}\right)^{1/2}\oplus\left(\tilde{\zeta}^{2}\otimes\tilde{\omega}^{2}_{k}\right)^{1/2}\oplus...\oplus\left(\tilde{\zeta}^{L}\otimes\tilde{\omega}^{L}_{k}\right)^{1/2}\right)$$

Step 4: The DMs aggregation of the value evaluation of each alternative (such as i) on each criterion (such as k) are a fuzzy number, which is named $\tilde{M}_{ik}$ and is calculated as follows:

$$\tilde{M}_{ik}=\frac{1}{L}\left(\left(\tilde{\zeta}^{1}\otimes\tilde{M}^{1}_{ik}\right)^{1/2}\oplus\left(\tilde{\zeta}^{2}\otimes\tilde{M}^{2}_{ik}\right)^{1/2}\oplus...\oplus\left(\tilde{\zeta}^{L}\otimes\tilde{M}^{L}_{ik}\right)^{1/2}\right)$$

Steps 5 to 8: The procedure of PROMETHEE is followed by defining a defuzzification function for d_j. Then leaving flows, entering flows and net flows are calculated. Score$(\tilde{\phi}(a))$, the scores for fuzzy net flows, are introduced as we mentioned in Table (1). Therefore, the larger the value of Score$(\tilde{\phi}(a))$, the better the ranking of a (for all $a\in A$) [22].

Before we explain our new procedure, it is important to introduce the following principles and definitions.

4 Principles and definitions

4.1 Generalized fuzzy numbers

Trapezoidal fuzzy numbers are introduced as: $\tilde{A}=(a_1,a_2,\beta,\gamma;w)$, where $0<w\le1$ and a_1,a_2 , β and γ are real numbers. The generalized fuzzy number $\tilde{A}$ is a fuzzy subset on the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions:

(1) $\mu_{\tilde{A}}$ is a continuous mapping from R to the closed interval $[0,1]$;
(2) $\mu_{\tilde{A}}(x)=0$, where $-\infty\prec x\prec a_1-\beta$;
(3) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a_1-\beta,a_1]$;
(4) $\mu_{\tilde{A}}(x)=w$, where $a_1\prec x\prec a_2$;
(5) $\mu_{\tilde{A}}(x)$ is strictly decreasing on $[a_2,a_2+\gamma]$;

(6) $\mu_{\widetilde{A}}(x) = 0$, where $a_2 + \gamma \prec x \prec +\infty$;

$\widetilde{A}$ is a normal trapezoidal fuzzy number, where w=1. If $a_1 = a_2$, $\widetilde{A}$ is generalized triangular fuzzy number. If $a_1 = a_2$ and $\beta = \gamma = 0$, w=1, then $\widetilde{A}$ is real number [29],[30].

Assume $\widetilde{A}_1$ and $\widetilde{A}_2$ are two normal trapezoidal fuzzy numbers, where $\widetilde{A}_1 = [a_1, b_1, c_1, d_1; 1]$ and $\widetilde{A}_2 = [a_2, b_2, c_2, d_2; 1]$, some arithmetic operations between two trapezoidal fuzzy numbers are as follows [23] , [31].

1. $\widetilde{A}_1 \oplus \widetilde{A}_2 = [a_1, b_1, c_1, d_1; 1] \oplus [a_2, b_2, c_2, d_2; 1] = [a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; 1]$, for addition.
2. $\widetilde{A}_1 \otimes \widetilde{A}_2 = [a, b, c, d; 1] = [a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2; 1]$, for multiplication.
3. $\widetilde{A}_1 \varnothing \widetilde{A}_2 = \left[\frac{a_1}{d_2}, \frac{b_1}{c_2}, \frac{c_1}{b_2}, \frac{d_1}{a_2}; 1\right]$, for division.
4. $\widetilde{A}_1 \Theta \widetilde{A}_2 = [a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; 1]$, for subtraction.
5. $\lambda \odot \widetilde{A}_1 = [\lambda \cdot a_1, \lambda \cdot b_1, \lambda \cdot c_1, \lambda \cdot d_1; 1]$, for scalar multiplication if $\lambda > 0$.

$\lambda \odot \widetilde{A}_1 = [\lambda \cdot d_1, \lambda \cdot c_1, \lambda \cdot b_1, \lambda \cdot a_1; 1]$, for scalar multiplication if $\lambda < 0$.

In fuzzy theory, a variable whose values are expressed in linguistic terms is called a linguistic variable. A 9-number linguistic term set shown in Table (3) is used in this paper [27].

4.2 Fuzzy distance and similarity measures

Research that presents the distance between two fuzzy numbers is based on the defuzifying concept. So, the distance between two fuzzy numbers is a crisp number. Some notable researches are Voxman [26], Trana and Duckssteinb [32] , Chakraborty and Chakraborty [33]. The fuzzy distance and fuzzy similarity measures, which are applied in this paper is introduced by Guha and Chakraborty [29].

Let us consider $\widetilde{A}_1 = (a_1, a_2, \beta_1, \gamma_1; w_1)$ and $\widetilde{A}_2 = (a_3, a_4, \beta_2, \gamma_2; w_2)$ as two generalized fuzzy numbers, α-cuts of $\widetilde{A}_1$ and $\widetilde{A}_2$ are $[\widetilde{A}_1]_\alpha = [\widetilde{A}_1^L(\alpha), \widetilde{A}_1^R(\alpha)]$ for $0 \prec \alpha \le w_1$ and $[\widetilde{A}_2]_\alpha = [\widetilde{A}_2^L(\alpha), \widetilde{A}_2^R(\alpha)]$ for $0 \prec \alpha \le w_2$, respectively.

The distance between $[\widetilde{A}_1]_\alpha$ and $[\widetilde{A}_2]_\alpha$ for all $\alpha \in [0,1]$ is calculated as follows:

$$\eta([\widetilde{A}_1]_\alpha - [\widetilde{A}_2]_\alpha) + (1-\eta)([\widetilde{A}_2]_\alpha - [\widetilde{A}_1]_\alpha) = [L(\alpha), R(\alpha)]$$

for :

$$\eta = \begin{cases} 1 & \text{if } \dfrac{A_1^L(w_1) + A_1^R(w_1)}{2} \ge \dfrac{A_2^L(w_2) + A_2^R(w_2)}{2} \\ 0 & \text{if } \dfrac{A_1^L(w_1) + A_1^R(w_1)}{2} \prec \dfrac{A_2^L(w_2) + A_2^R(w_2)}{2} \end{cases}$$

where:

$$L(\alpha) = \eta[A_1^L(\alpha) - A_2^L(\alpha) + A_1^R(\alpha) - A_2^R(\alpha)] + [A_2^L(\alpha) - A_1^R(\alpha)]$$

$$R(\alpha) = \eta[A_1^L(\alpha) - A_2^L(\alpha) + A_1^R(\alpha) - A_2^R(\alpha)] + [A_2^R(\alpha) - A_1^L(\alpha)]$$

The α-cut of distance between $[\tilde{A}_1]_\alpha$ and $[\tilde{A}_2]_\alpha$ is denoted by $[d^L_\alpha, d^R_\alpha]$, for $\alpha \in [0, w]$; $w = \min(w_1, w_2)$

$$[d^L_\alpha, d^R_\alpha] = \begin{cases} [L(\alpha), R(\alpha)]; & L(\alpha) \ge 0 \\ [0, |L(\alpha)| \vee R(\alpha)]; & L(\alpha) \le 0 \le R(\alpha) \end{cases} \quad \text{for all } \alpha \in [0, w]$$

Therefore the distance between $\tilde{A}_1$ and $\tilde{A}_2$ is defined by:

$$\tilde{d}(\tilde{A}_1, \tilde{A}_2) = (d^L_{\alpha=w}, d^R_{\alpha=w}; \theta, \sigma)$$

Whereas θ and σ is the following way:

$$\theta = d^L_{\alpha=w} - \max\left\{\int_0^w d^L_\alpha d\alpha, 0\right\} \quad \text{and} \quad \sigma = \left\| \left[\int_0 d^L_\alpha d\alpha - d^R_{\alpha=w} \right] \right\| \quad \text{for } w = \min(w_1, w_2)$$

After normalization the distance and similarity measures between $\tilde{A}_1$ and $\tilde{A}_2$ will be as follows:

$$\tilde{d}(\tilde{A}_1, \tilde{A}_2) = \left(\frac{d^L_{\alpha=w}}{d^R_{\alpha=w} + \sigma}, \frac{d^R_{\alpha=w}}{d^R_{\alpha=w} + \sigma}; \frac{\theta}{d^R_{\alpha=w} + \sigma}, \frac{\sigma}{d^R_{\alpha=w} + \sigma} \right)$$

$$\tilde{S}(\tilde{A}_1, \tilde{A}_2) = \left(1 - \frac{d^R_{\alpha=w}}{d^R_{\alpha=w} + \sigma}, 1 - \frac{d^L_{\alpha=w}}{d^R_{\alpha=w} + \sigma}; \frac{\sigma}{d^R_{\alpha=w} + \sigma}, \frac{\theta}{d^R_{\alpha=w} + \sigma} \right)$$

4.3 Ranking the fuzzy numbers

There is much research that presents the different methods for ranking fuzzy numbers such as [25, 34, 35]. The procedure for ranking fuzzy numbers, which is applied in this research, has been presented by Chen and Sanguansat [23]. This procedure consists of 4 steps. First, the generalized fuzzy numbers are normalized. Then the areas on the negative side and positive side of the normalized fuzzy numbers are calculated. These areas are as follow: for $\tilde{A}_i = [a_i, b_i, c_i, d_i; 1]$

$$\text{Area}^-_{iL} = \frac{(a_i + 1) + (b_i + 1)}{2} \qquad \text{Area}^-_{iR} = \frac{(c_i + 1) + (d_i + 1)}{2}$$

$$\text{Area}^+_{iL} = \frac{(1 - a_i) + (1 - b_i)}{2} \qquad \text{Area}^+_{iR} = \frac{(1 - c_i) + (1 - d_i)}{2}$$

In step3, $XI_{\tilde{A}_i} = Area^-_{iL} + Area^-_{iR}$ and $XD_{\tilde{A}_i} = Area^+_{iL} + Area^+_{iR}$ are calculated.

In the last step, $\text{Score}(\tilde{A}_i)$ is determined based on this relation: $\text{Score}(\tilde{A}_i) = \dfrac{(XI_{\tilde{A}_i} - XD_{\tilde{A}_i})}{(XI_{\tilde{A}_i} + XD_{\tilde{A}_i})}$

This score is between -1 and 1. The larger value of $\text{Score}(\tilde{A}_i)$ is the better ranking of $\tilde{A}_i$ [11].

5 Fuzzy-PROMETHEE with similarity measure framework

By considering $\tilde{M}_{ik} = [m_{ik}, m'_{ik}, \alpha, \beta]$ as a fuzzy number, we will present our suggested method for fuzzifying PROMETHEE II.

As we mentioned in Section 3, we encounter 6 types of preferences functions presented in Table (2), which are introduced as:

$$P_j : A \times A \rightarrow [0,1] \;;\; P_j(a_i, a_k) = F_j[d_j(a_i, a_k)] \;;\; d_j(a_i, a_k) = g(a_i) - g(a_k)$$

$g(a_i)$ and $g(a_k)$ are the values of alternatives a_i and a_k under j^{th} criterion, respectively . These values can be the fuzzy numbers corresponding to linguistic values. P(d) refers to the preference of two alternatives under a criterion. When we allocate value to an alternative under a criterion by linguistic term, the distance between the groups of allocated linguistic terms is considered for preference determination. For example, high is preferred to fairly-high, if the experts consensus is q=1. So P((high)-(fairly-High)) is P(1) and determined based on preference functions, which have been presented in Table (2). Table (3) shows the linguistic terms and their corresponding generalized fuzzy numbers [27,22,16]. The weight of each criterion can be presented with linguistic terms by experts. Arithmetic mean is used to aggregate the expert opinions to obtain fuzzy weights for criteria.

According to above analysis, we discuss the PROMETHEE II next. The main contribution of this research is fuzziness of decisions on fuzzy net flows for ranking the alternatives. We use fuzzy distance and fuzzy similarity measures, which have been introduced by Guha and Chakraborty [29] for determining the ranking of fuzzy net flows of alternatives.

Steps 1 to 4 of new procedure have been presented by wei-xiang and Bang-yi, which we have mentioned in Section 3 [22]. Hence, the algorithm of the new procedure follows:

Step 1: The identified importance of weights vector of decision-makers are determined (see Section 3).

Step 2: For every DM in the decision group, we get a vector of criteria weights. The preference matrix given by each DM is determined (see Section 3). $\tilde{M}_{ik}^{l}$, (i=1,2,…,m; k=1,2,…,n) is the evaluation value of i^{th} alternative under k^{th} criterion given by l^{th} DM in terms of linguistic variables.

Step 3: A fuzzy weight for aggregative criterion is determined (see Section 3).

Step 4: The DMs aggregation of the value evaluation of each alternative (such as i) on each criterion (such as k) is a fuzzy number which, is named $\tilde{M}_{ik}$ and is calculated (see Section3).

Step 5: The DMs aggregation of the evaluation value of alternatives under criteria is named "DMs Preference matrix" in linguistic terms. For each $\tilde{M}_{ik}$, calculated in Step 4, the similarity measures with fuzzy numbers corresponding to linguistic terms are calculated by using the described method in Section 4-2. By using the ranking fuzzy numbers method, which has been introduced in Section 4-3, the maximum score, we can allocate the related linguistic term to $\tilde{M}_{ik}$.

Step 6: $P_j(d_j)$, preference of two alternatives under each criteria, are calculated for each criteria. The distance between two groups of linguistic terms is d. In addition, p and q are determined as a parameter of preference function by DMs.

Step 7: The leaving, entering and net flows are calculated as following:

$$\tilde{\pi}(a_i, a_k) = \sum_{j=1}^{n} P_j(a_i, a_k) \cdot \tilde{w}_j$$

$$\tilde{\phi}^{+}(a) = \frac{1}{m-1}\sum_{x \in A} \tilde{\pi}(a, x) \;;\; \tilde{\phi}^{-}(a) = \frac{1}{m-1}\sum_{x \in A} \tilde{\pi}(x, a) \;;\; \tilde{\phi}(a) = \tilde{\phi}^{+}(a) - \tilde{\phi}^{-}(a)$$

All of these relations are divided by $\sum_{j=1}^{n} \tilde{w}_j$ to contain the summation weights in unit interval.

<u>*Step 8*:</u> Fuzzy similarity measures between fuzzy net flows and fuzzy numbers corresponding to linguistic terms Table (3), are calculated using the described method in Section 4-2.

<u>*Step 9*:</u> The scores of similarity measures are determined by using the method, which has been described in Section 4-3. The linguistic term related to the largest value of Score $(\tilde{\phi}(a))$ allocates to alternative a. So, the alternatives are ranked based on the allocated linguistic terms.

The schematic of the new framework is shown in Figure 1.

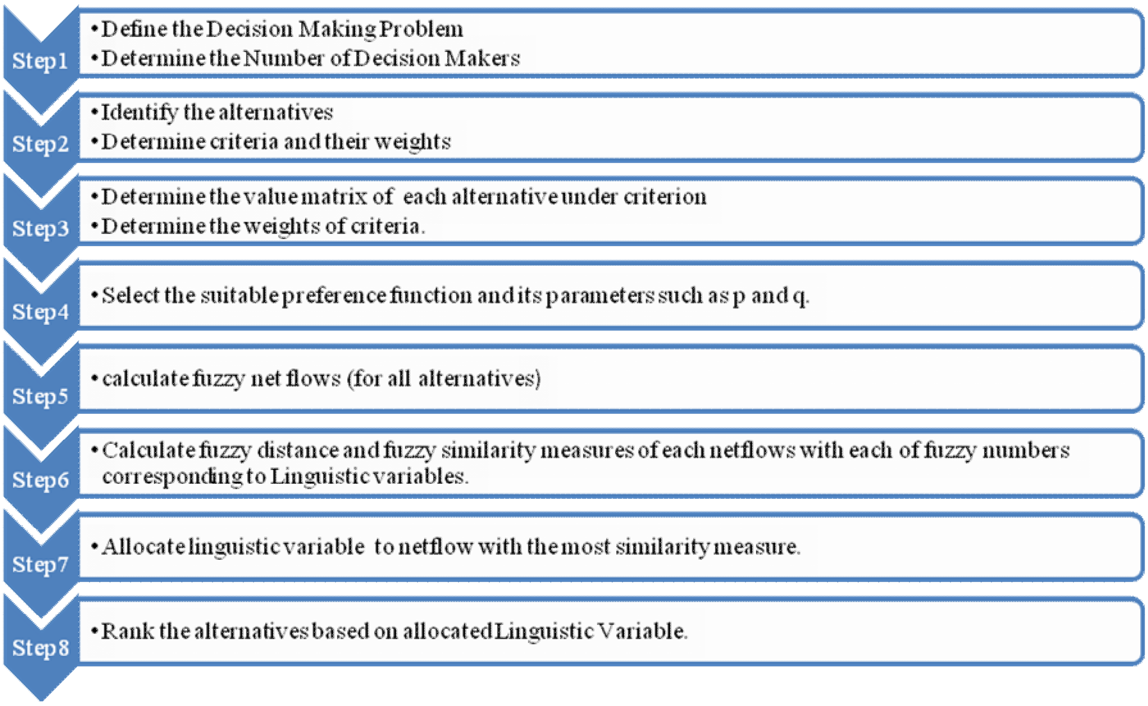


Fig. 1 Schematic of new framework for F-PROMETHEE method

6 Numerical example

As an example, we use our proposed method to solve the problem of evaluating the quality of railway passenger services in China that was presented in Liu and Guan paper by F-PROMETHEE method [21].

In the last step of their procedure, the net flows for three alternatives a_1, a_2 and a_3 have been calculated as follows:

$\tilde{\phi}(a_1)$ =(-2.4739,0.0816,3.0881), $\tilde{\phi}(a_2)$ =(-2.4416,0.8351,3.1667),

$\tilde{\phi}(a_3)$ =(-3.1500,-0.9167,1.8107).

We rank these net flows by considering the similarity measures. The similarity measure of each net flow with each elements of Table (3), must be calculated. The results of the calculations are summarized in Table (4). For example, in the first column of Table (4) similarity measures between $\tilde{\phi}(a_1)$ and each of linguistic variables in Table (3) are found. The most similar linguistic variable to each $\tilde{\phi}(a_i)$ must be found. For determining the most similarity, as the best allocation, we apply the fuzzy ranking method, which has been presented by Chen and Sanguansat [23]. Therefore, a score is allocated to each of the similarity measures. The greatest score presents the most similarity. So, the linguistic term, which relates to the most similarity, is related to the corresponding alternative, as shown in Table (5). So, the ranking for alternatives by Revised Fuzzy-PROMETHEE is: $a_2 \succ a_3 \succ a_1$. Liu and Guan [21] have presented by using defuzzification : $a_2 \succ a_1 \succ a_3$.

Table 3 Linguistic terms and their corresponding generalized fuzzy numbers [27]

Linguistic terms	Generalized fuzzy numbers
Absolutely-low/absolutely-dominated	(0.0,0.0,0.0,0.0;1.0)
Very-low/very-dominated	(0.0,0.0,0.02,0.07;1.0)
Low/dominated	(0.04,0.10,0.18,0.23;1.0)
Fairly-low/fairly-dominated	(0.17,0.22,0.36,0.42;1.0)
Medium/medium	(0.32,0.41,0.58,0.65;1.0)
Fairly-high/fairly-dominating	(0.58,0.63,0.80,0.86;1.0)
High/dominating	(0.72,0.78,0.92,0.97;1.0)
Very-high/very-dominating	(0.93,0.98,1.0,1.0;1.0)
Absolutely-high/absolutely-dominating	(1.0,1.0,1.0,1.0;1.0)

Table 4 Similarity measures of each net flows and generalized fuzzy numbers in Table (3)

Linguistic Terms	Generalized fuzzy numbers	Similarity measures		
		$\tilde{\phi}(a_1)$=(-2.4739, 0.0816, 3.0881)	$\tilde{\phi}(a_2)$=(-2.4416, 0.8351, 3.1667)	$\tilde{\phi}(a_3)$=(-3.1500,-0.9167, 1.8107)
Absolutely-Low	(0.0,0.0,0.0,0.0;1)	(0.95,0.95,0.95,0.051;1)	(0.582,0.582,0.582,0.365;1)	(0.55,0.55,0.55,0.37;1)
Very Low	(0.0,0.0,0.02,0.07;1)	(0.95,1,0.95,0;1)	(0.58,0.59,0.58,0.36;1)	(0.55,0.56,0.55,0.37;1)
Low	(0.04,0.10,0.18,0.23;1)	(0.935,0.988,.0935,0.012;1)	(0.62,0.66,0.62,0.24;1)	(0.5,0.54,0.51,0.38;1)
Fairly-Low	(0.17,0.22,0.36,0.42;1)	(0.84,0.933,0.84,0.066,;1)	(0.66,0.75,0.66,0.23;1)	(0.47,0.53,0.46,0.37;1)
Medium	(0.32,0.41,0.58,0.65;1)	(0.725,0.82,0.725,0.171;1)	(0.75,0.85,0.74,0.15;1)	(0.43,0.5,0.43,0.38;1)
Fairly-High	(0.58,0.63,0.80,0.86;1)	(0.645,0.73,0.63,0.25;1)	(0.89,0.97,.85,0.025;1)	(0.4,0.46,0.4,0.39;1)
High	(0.72,0.78,0.92,0.97;1)	(0.6,0.674,0.61,0.29;1)	(0.95,1,0.95,0;1)	(0.38,0.43,0.38,0.39;1)
Very-High	(0.93,0.98,1.0,1.0;1.0)	(0.58,0.59,0.58,0.35;1)	(0.91,0.92,0.9,0.08;1)	(0.37,0.37,0.37,0.41;1)
Absolutely-High	(1.0,1.0,1.0,1.0;1.0)	(0.58,0.58,0.35,0.39;1)	(0.91,0.91,0.91,0.09;1)	(0.37,0.37,0.37,0.41;1)

Table 5 Score of each similarity measures from Table (4)

Linguistic Terms	Score of each Similarity Measures		
	$\tilde{\phi}(a_1)$	$\tilde{\phi}(a_2)$	$\tilde{\phi}(a_3)$
Absolutely-Low	0.45	0.05	0.01
Very Low	**0.475**	0.06	0.02
Low	0.46	0.1	-0.015
Fairly-Low	0.385	0.2	-0.045
Medium	0.265	0.305	-0.095
Fairly-High	0.18	0.445	**0.165**
High	0.115	**0.475**	-0.185
Very-High	0.055	0.42	-0.24
Absolutely-High	0.18	0.41	-0.24

7 Application of revised Fuzzy-PROMETHEE in Nanotechnology selection in IRAN

The particular attention to nanotechnology in Iran was initiated through the establishment of a dedicated office: Technology Cooperation Office (TCO) in 2001. TCO was made responsible for the development, promotion and coordination of nanotechnology research as well as development and commercialization of nanotechnology products in Iran. TCO has also made a notable attempt to publicize Iranian nanotechnology progress. The National Iranian Nanotechnology Initiative (NINI) was subsequently supported by Iranian Presidential Cabinet in July 2005 [36]. During the past decade, the infrastructure of nanotechnology in Iran has grown to include over 18 university courses, 90 research institutions, 5 incubators, 40 specific laboratories, and 30 special media firms [37]. As a result of these activities, in 2008, Iran was ranked 25th in the worldwide ranking of scientific articles related to nanotechnology. These issues further highlight the importance of determining the application fields within nanotechnology based on which national policies can be determined. Indeed, policy makers in Iran need to know not only *what* the alternatives fields of application in nanotechnology are, but should also be able to prioritize them.

For applying our method in a real case, we decided to rank the nanotechnology application fields in Iran. Identified alternatives and the criteria and sub-criteria for decision-making are considered. We ignored the details of their determination. We intended to rank 13 identified alternatives subject to 15 criteria. The required information was gathered from 14 experts with the sum of the weights of importance attached to the alternatives equals to 1. All of the experts presented their ideas in terms of linguistic terms. The mean arithmetic of their ideas was calculated for aggregation. The aggregated weights for criteria and aggregated value of each alternative under each criterion are presented in Tables 6 and 7, respectively.

Two criteria have quantitative data: 1) Number of Specialists: Number of researchers who have accepted ISI papers in related field. 2) Necessary Research Investment: Necessary investment (million \$) for establishing a primary lab in the related field. A noticeable point is that some criteria include quantitative data, while we had to consider them as qualitative data for other criteria such as: *Necessary Production Investment* or *Price of final product*. Most of such alternatives are not in the production stage, so we have to trust the experts' opinions. Based on this concept, the preferences are considered for all quantitative criteria, q=3 and $q_{\text{number of specialists}}=63$ and $q_{\text{necessary research investment}}=\$0.562$ million. By using this complementary information and considering the U-shape preference function, the calculated fuzzy net flows are presented in Table (8).

Table 6 Fuzzy aggregated importance weights of criteria

Criteria	Sub-criteria	Allocated weights
Existing Infrastructure	Existence proficiencies and instructs in Iran	(0.071,0.105,0.176,0.222)
	Hardware and equipment	(0.178,0.216,0.294,0.334)
	Existence Specialists and experts in Iran	(0.179,0.218,0.304,0.353)
Effectiveness of technology on quality of life improvement	Extent of technology application	(0.738,0.795,0.886,0.916)
	Composition capability with existence technology	(0.738,0.795,0.886,0.916)
	Attractiveness for nongovernmental organizations	(0.334,0.394,0.512,0.577)
Accessibility and localization of technology	Applying capacity of country	(0.193,0.239,0.347,0.405)
	Time between research and market absorption	(0.364,0.426,0.548,0.603)
	Future of technology	(0.306,0.370,0.493,0.548)
	Simplicity of production process	(0.103,0.128,0.196,0.239)

Criteria	Sub-criteria	Allocated weights
	Possibility of international technical and scientific cooperation	(0.171,0.226,0.334,0.391)
	Possibility of entrance to supply chain of products	(0.185,0.247,0.374,0,433)
Market forecasting and economic problems	Research investment	(0.791,0.844,0.927,0.954)
	Investment in production stage	(0.723,0.778,0.883,0.919)
	Product price	(0.659,0.716,0.836,0.878)

Table 7 Aggregated value of each alternative under each criterion in terms of linguistic terms (DMs aggregated matrix)

Criteria / Nanotechnology Application Field	Number of specialists	Existing proficiencies and instructs	Existing hardware	Vastness of technology application	Composition capability with existence technology	Attractiveness for nongovernmental organizations	Applying country capacity	Time between research and market application	Future prospects of the technology	Simplicity of production process	Use of international cooperation	Entrance Possibilities into product supply chain	Necessary Research investment	Necessary Production investment	Price of Final Product
Nanoparticles	107	High	High	Very-High	High	Very-High	Fairly-High	High	High	High	Very-High	Absolutely-High	1.069	Very-High	Very-High
Nanocomposites	111	High	High	Very-High	Very-High	Very-High	High	High	High	High	High	Very-High	0.963	Very-High	Very-High
Nanocrystals	87	High	Medium	Fairly-High	Fairly-High	Fairly-High	Very-High	High	Fairly-High	High	High	Very-High	0.702	Fairly-High	Fairly-High
Nanofibrs	81	Fairly-Low	Fairly-High	Fairly-High	Fairly-High	Fairly-High	Fairly-High	High	High	High	Fairly-High	Fairly-High	0.824	High	High
Nanporous material	83	High	High	Fairly-High	High	High	Fairly-High	Fairly-High	High	Fairly-High	Very-High	Medium	0.804	High	Fairly-High
Nanowire	75	Medium	Medium	Medium	Fairly-Low	Medium	Fairly-Low	Medium	High	Fairly-High	Fairly-High	Fairly-High	0.890	High	High
Nanocapsule	70	Medium	Fairly-High	Fairly-High	Fairly-High	Medium	Fairly-Low	High	Medium	Medium	High	High	0.937	High	High
Nanoorganic structures	87	Fairly-High	High	Fairly-High	High	Medium	Fairly-Low	Medium	Fairly-High	Medium	Fairly-High	Fairly-Low	0.749	Medium	Medium
Nanotubes	85	Fairly-High	Fairly-High	Very-High	High	Very-High	Fairly-High	Fairly-High	Very-High	Fairly-High	High	Medium	0.869	High	Fairly-High
Nanoporous	55	Medium	Low	Fairly-Low	Low	Low	Fairly-Low	Fairly-Low	Medium	Low	Fairly-Low	Medium	0.401	Fairly-Low	Low
Fullerenes	59	Fairly-Low	Medium	Fairly-Low	Medium	Low	Low	Medium	Medium	Medium	Medium	Medium	0.674	High	Medium
Nanoelectromechanic	65	Fairly-Low	Fairly-Low	Medium	Medium	Medium	Low	Low	Medium	Fairly-Low	Fairly-Low	Fairly-Low	0.594	Medium	Medium
Nanoelectronic and optical systems	33	Low	Low	Fairly-Low	Fairly-Low	Low	Low	Fairly-Low	Medium	Low	Low	Low	0.524	Fairly-Low	Fairly-Low

Table 8 Fuzzy net flows of alternatives by using U-shape preference function

Row (i)	Nanotechnology application fields (Alternatives)	Net flows ($\tilde{\phi}$ (a_i))
1	Nanoparticles	(0.1007,0.1254,0.1937,0.2396)
2	Nanocomposites	(0.1024,0.1256,0.1902,0.2335)
3	Nanocrystals	(0.0278,0.0379,0.0679,0.0895)
4	Nanofibers	(0.0108,0.0146,0.0228,0.0285)
5	Nanporous material	(0.0226,0.0305,0.0527,0.0679)
6	NanoWires	(-0.0063,-0.0054,-0.0025,0.0008)
7	Nanocapsule	(0.0093,0.0132,0.0178,0.0202)
8	Nano organic structures	(-0.0058,-0.0025,0.0066,0.0127)
9	Nanotube	(0.0354,0.0439,0.0666,0.0814)
10	Nanoporous	(-0.1820,-0.1526,-0.2264,-0.1910)
11	Fullerenes	(-0.0604,-0.0605,-0.0601,-0.0599)
12	Nanoelectro mechanic	(-0.0566,-0.0544,-0.0487,-0.0451)
13	Nano electronic/optical systems	(-0.141,-0.1363,-0.1264,-0.1207)

The similarity measures between fuzzy net flows and generalized fuzzy numbers corresponding to linguistic terms are presented in Table (10). Because of the nature of our data in terms of linguistic variables, we can calculate the distance between each fuzzy net flow with generalized fuzzy numbers corresponding to linguistic terms, which are around it. So we don't need to calculate all similarities. For example, Very-Low, Low and Fairly-Low are around the net flow of Nanoparticles. So, the fuzzy similarity measure between the net flow of this alternative and the mentioned groups are needed. Table (11) shows the score of the fuzzy similarity measures. For Nanoparticle, the greatest score of fuzzy similarity measures is 0.36. So, we allocate Low as a value for this alternative. In addition, we allocate Absolutely-Low to negative fuzzy net flows. Therefore, there isn't a need to calculate similarity measures for these net flows.

8 Discussion

Based on our suggested method "Very Low", "High" and "Fairy High" are allocated to $\tilde{\phi}(a_1)$, $\tilde{\phi}(a_2)$ and $\tilde{\phi}(a_3)$, respectively in numerical example . Therefore, the ranking of the alternatives is; $a_2 \succ a_3 \succ a_1$. In contrast, the ranking of these alternatives by Liu and Guan [21], whose example is used here, is: $a_2 \succ a_1 \succ a_3$. As can be seen, the ranking of the alternatives by applying fuzzy similarity measures is somehow different from defuzzifying the scores of alternatives. Therefore, it is more appropriate to follow fuzzy net flows as much as possible without defuzzifying them.

By considering the scores from Table (11), all the alternatives in the real case are categorized into 3 groups: the first group contains Nanoparticles and Nanocomposites as the highest ranked group. The second group includes Nanoporous material, Nanotubes, Nanocrystals, NanoWires, Nanofibers, and finally others are placed in the third group. Alternatives are ranked in each group by ranking the fuzzy net flows of them. In this case, Nanoparticles are preferred to Nanocomposites in the first group. The value of this group is "Low". In the second group, alternatives are ranked as: Nanotube, Nanocrystals, Nanoporus material, Nanofibers, and Nanocapsule, respectively. The value of the second group is "Very-Low". The third or lowest group contains, Nano organic structures, Nanowires, Nanoelectro mechanic, Fullerences, Nano electronic/optical systems and Nanoporus. “Absolutely Low” is the value of the third group.

As we have mentioned before, simple mathematical logic, ability to use qualitative and quantitative data, flexibility of its software package, problem visualization, and ability in considering all criteria when some of them are at odds with each other are the advantages of using PROMETHEE procedures in comparing with other methods such as AHP. Fuzzy PROMETHEE contains these advantages as well. For example, fuzzy AHP contains pair-wise comparison. The revised Fuzzy PROMETHEE method has some good features. First, similarity measures are normalized fuzzy numbers between 0 and 1. Second, the similarity measures between normalized fuzzy numbers and linguistic variable haven’t been considered before by researchers in this field. In addition, in this approach the value of alternatives are determined also. For instance, a_2 is in high ranking and a_3 is after that in numerical example, but we can understand that their values are close to each other because of their similarity with linguistic terms, high and fairly high. Moreover, not only does a_1 have the last ranking but it also has a very low value. Awareness of the value of each alternative makes our analyzing results clear. This can be more important when one encounters many alternatives. Alternatives classification has been done better in our proposed method. The alternatives are classified based on their linguistic values. Value of each alternative helps us to consider the number of groups that can be considered. In addition, the importance of each group is determined. Because of considering fuzzy similarity measures for fuzzy net flows, the decisions are fuzzy.

According to the results of our method in comparison with existing methods, some aspects such as ranking base, type of ranking, alternative’s value and alternative classifications are important points for further consideration. A summarized comparison of the methods in these aspects is mentioned in Table (9).

Table 9 A thorough comparative analysis between Revised Fuzzy-PROMETHEE with exist Fuzzy-PROMETHEE

Methods / Characteristics	Exist Fuzzy-PROMETHEE methods	Revised Fuzzy-PROMETHEE method
Ranking base	Fuzzy net flows Defuzzification	According to similarity of fuzzy net flows with linguistic terms
Type of ranking	Crisp	Fuzzy
Alternative’s value	Allocate a crisp score which the alternatives are sorted based on	Allocate a linguistic term (value) to each alternative
Alternatives classifications	Alternatives can be classified based on decision maker idea	Based on allocated linguistic term (value) to each alternative

9 Conclusion

In this paper we present a new framework to the fuzzy PROMETHEE by applying fuzzy distance and similarity measures. The logic behind this approach, which encouraged us to conduct the research, was to ask the question: “when we are not certain about the numbers themselves how can we be *certain* about the distances among them’’ [26]. This implies that when we apply fuzzy numbers in PROMETHEE method, we get the fuzzy net flows. A revised Fuzzy-PROMETHEE method is presented in this paper for selecting or ranking the alternatives, which can be applied in a fuzzy decision making environment.

This research uses fuzzy distance and similarity measures as fuzzy logic concepts in Fuzzy PROMETHEE- II when fuzzy net flows must be valued to rank the alternatives.

Therefore, fuzziness of decisions on fuzzy net flows for ranking the alternatives are considered, which other methods mentioned in literature review above hadn't taken into account.

Two numerical examples are applied to present this new framework. In the first example, we showed our method in ranking the fuzzy net flows of the problem which has been presented by Liu and Guan [21]. In the second example, selection fields of Nanotechnology applications in Iran have been chosen as a real case for our method.

Table 10 Fuzzy similarity measures between alternatives net flows and generalized fuzzy numbers corresponding to linguistic terms.

Row (i)	Net flows ($\widetilde{\phi}(a_i)$)	Fuzzy similarity measures between net flows and linguistic terms.								
		Absolutely-Low (0,0,0,0;1)	Very-Low (0,0,0.02,0.07;1)	Low (0.04,0.1,0.18,0.23;1)	Fairly-Low (0.17,0.22,0.36,0.42;1)	Medium (0.32,0.41,0.58,0.65;1)	Fairly-High (0.58,0.63,0.8,0.86;1)	High (0.72,0.78,0.92,0.97;1)	Very-High (0.93,0.98,1,1;1)	Absolutely-High (1,1,1,1;1)
1	(0.1007,0.1254,0.1937,0.2396)		(0.05,0.52,0.1,0.19;1)	(0.72,1,0.72,0;1)	(0.14,0.89,0.14,0.07;1)					
2	(0.1024,0.1256,0.1902,0.2335)		(0.1,0.55,0.05,0.05;1)	(0.37,1,0.35,0;1)	(0.14.0.89,0.14,0.11;1)					
3	(0.0278,0.0379,0.0679,0.0895)	(0.13,0.50,0.12,0.06;1)	(0.13,0.75,0.12,0.25;1)	(0.09,0.57,0.08,0.11;1)						
4	(0.0108,0.0146,0.0228,0.0285)	(0,0.61,0,0)	(0.04,1,0.03,0;1)	(0.11,0.32,0.11,0.07;1)						
5	(0.0226,0.0305,0.0527,0.0679)	(0,0.62,0;1)	(0.33,0.62,0.33,0;1)	(0.08,0.57,0.08,0.08;1)						
6	(-0.0063,-0.0054,-0.0025,0.0008)									
7	(0.0093,0.0132,0.0178,0.0202)	(1,0,0,0;1)	(1,1,1,0;1)	(.08,0.41,0.08, 0.22; 1)						
8	(-0.0058,-0.0025,0.0066,0.0127)									
9	(0.0354,0.0439,0.0666,0.0814)	(0.07,0.41,0.06,0.06;1)	(0.07,0.67,0.06,0.14;1)	(0.09,0.56,0.09,0.09;1)						
10	(-0.1820,-0.1526,-0.2264,-0.1910)									
11	(-0.0604,-0.0605,-0.0601,-0.0599)									
12	(-0.0566,-0.0544,-0.0487,-0.0451)									
13	(-0.141,-0.1363,-0.1264,-0.1207)									

In this research, the value of each alternative is determined by ranking and analyzing the results clearly by considering the fuzziness of the circumstances of alternatives in terms of linguistic terms. A linguistic term is allocated to each alternative, which has been determined by using fuzzy distance and similarity measures between fuzzy numbers. This framework uses verbal information in terms of linguistic variables as inputs and presents the outputs in linguistic terms as well.

Table 11 Scores of Fuzzy similarity measures in Table (9)

Row (i)	Net flows ($\tilde{\phi}(a_i)$)	Scores of fuzzy similarity measures								
		Absolutely-Low (0,0,0,0;1)	Very-Low (0,0,0.02,0.07;1)	Low (0.04,0.1,0.18,0.23;1)	Fairly-Low (0.17,0.22,0.36,0.42;1)	Medium (0.32,0.41,0.58,0.65;1)	Fairly-High (0.58,0.63,0.8,0.86;1)	High (0.72,0.78,0.92,0.97;1)	Very-High (0.93,0.98,1,1;1)	Absolutely-High (1,1,1,1;1)
1	(0.1007,0.1254,0.1937,0.2396)	------	-0.385	0.36	-0.005	----	-----	-----	-----	-----
2	(0.1024,0.1256,0.1902,0.2335)	----	-0.35	0.195	0.015	----	-----	-----	-----	-----
3	(0.0278,0.0379,0.0679,0.0895)	-0.4	-0.05	-0.325	-----	-----	-----	-----	-----	-----
4	(0.0108,0.0146,0.0228,0.0285)	-0.39	0.025	-0.59	-----	-----	-----	-----	-----	------
5	(0.0226,0.0305,0.0527,0.0679)	-0.38	-0.215	-0.35	-----	-----	-----	-----	-----	------
7	(0.0093,0.0132,0.0178,0.0202)	0	0.5	-0.44	-----	-----	-----	-----	-----	-----
9	(0.0354,0.0439,0.0666,0.0814)	-0.52	-0.022	-0.35	-----	-----	-----	-----	-----	------

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