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DEA cross efficiency evaluation in presence of linear and nonlinear data

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Abstract Cross-efficiency is an effective approach for evaluation of DMUs which can be performed with different secondary goals. DEA and cross-efficiency view all variables as behaving in a linear fashion and regardless of the amounts of a variable held by DMUs, DEA apply a same multiplier to those various amounts. But in certain situations, this linearity assumption is not appropriate, and the conventional models need to be altered to accommodate nonlinear representations. This paper proposed a modified cross efficiency structure of Liang et al. that captures certain form of nonlinear behavior. A numerical example is provided to illustrate the approach.

Keyword: Data Envelopment Analysis (DEA); Cross Efficiency; Nonlinear Value Function.

1 Introduction

Data envelopment analysis (DEA) developed by Charnes et al. [1] is a methodology for measuring the best relative efficiency of a group of decision making units (DMUs) that consume multiple inputs to produce multiple outputs. Since decades, DEA was obtained the dominant role of evaluating, and improving the performance of service operations and it has been extensively applied as a product-oriented analysis method through schools, hospitals, bank branches, production plants and etc., where the main goals are the evaluation; see [2-6].
DEA models were discussed for measuring efficiency score, but it wasn't enough for evaluating the performance of DMUs and as it mentioned in [7] and [8], this is because of the unrestricted weight flexibility problem in DEA. Therefore, for being the discrimination power of DEA more realistic, cross efficiency evaluation has been suggested by Sexton et al. [9], in DEA context. DMUs are mostly evaluated through cross efficiency evaluation considering both self and peer evaluation, whereas the peer evaluation requests each DMU to be evaluated with the weights determined by other DMUs. Finally, the overall efficiency of that DMU is the average of its self-evaluation efficiency and peer evaluation efficiencies and proves to have strong discrimination power and can usually provide a full ranking for the DMUs to be evaluated. That is the reason why efficiency evaluation is found a dominant application in various fields; see [10-18].

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However, a problem which reduces the usefulness of cross efficiency evaluation method is that, cross efficiency scores may not be unique because of alternative optimal solutions to the DEA programs and it is due to this reason that some approaches have been suggested as a remedy for the issue of non-uniqueness of weights; See [19-39].

On the other hands, in DEA and cross efficiency formulations, often, the weights assigned to the outputs are considered as prices assigned by the DMU itself to the outputs. Thus the total virtual output of a unit can be considered as an overall value function of the outputs, which is additively separable with linear partial value functions. The interpretations of inputs are similar, too; see [40]. But, due to the fact that this linearity assumption might be unjustifiable, Cook and Zhu [41], Cook et al. [42] and Despotis et al. [40] relaxed the linearity assumption for input/output weights by using a piecewise linear representation of the value function.

In this paper, we propose a modified cross efficiency structure of Liang et al. [22] that captures certain forms of nonlinear behavior. The rest of the paper is organized as follows. In Section 2, we introduce cross efficiency concept and secondary goal formulation of Liang et al. [22]. In Section 3, we define nonlinear inputs/outputs and their conditions. In Section 4, we introduce cross efficiency for nonlinear data and in Section 5, we apply it for sample of maintenance patrol which introduced by Cook et al. [42], [43]. Finally, conclusion and suggestions are depicted in Section 6.

2. Cross efficiency evaluation

Consider n DMUs to be evaluated with m inputs and s outputs. Denote by x_{ij} and y_{rj} the input/output values of DMU_j, whose self-efficiency can usually be measured by the CCR fractional model (1), where θ^*_{oo} is called CCR-efficiency score of DMU_o . DMU_o is considered to be efficient if and only if it is equal to one. Moreover, this model can be transformed to the LP model (2).

$$
\begin{aligned}
& Max\ \theta_{00} = \frac{\sum_{r=1}^{s} u_{r0} y_{r0}}{\sum_{i=1}^{m} v_{i0} x_{i0}} \\
& s.t. \\
& \qquad \frac{\sum_{r=1}^{s} u_{r0} y_{rj}}{\sum_{i=1}^{m} v_{i0} x_{ij}} \le 1 \quad j = 1,\ldots,n, \\
& \quad u_{ro} \ge 0 \qquad r = 1,\ldots,s, v_{io} \ge 0 \qquad i = 1,\ldots,m.
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
& Max\ \theta_{oo} = \sum\nolimits_{r=1}^{s} u_{ro} y_{ro} \\
& s.t. \\
& \qquad \sum\nolimits_{i=1}^{m} v_{io} x_{io} = 1, \\
& \qquad \sum\nolimits_{r=1}^{s} u_{ro} y_{rj} - \sum\nolimits_{i=1}^{m} v_{io} x_{ij} \le 0 \qquad j = 1,\ldots,n, \\
& \qquad v_{io} \ge 0 \qquad i = 1,\ldots,m, u_{ro} \ge 0 \qquad r = 1,\ldots s.
\end{aligned} \tag{2}
$$

As it is mentioned, the self-evaluation allows each DMU to be evaluated with its most favorable input/output weights so that θ_{oo}^{*} is referred as the optimistic efficiency can be achieved for each DMU_o, whereas the peer evaluation requests each DMU to be evaluated with the weights determined by other DMUs. In other words, peer evaluation of DMU_j using the most favorable weights of DMU_o is calculated based on the formula (3):

$$\theta_{jo}=\frac{\sum_{r=1}^{s}u_{ro}^{*}y_{rj}}{\sum_{i=1}^{m}v_{io}^{*}x_{ij}} \quad j=1,\dots,n \tag{3}$$

And finally formula (4) is referred as the cross efficiency score for DMU_j, which is simply the mean of the self and peer evaluations.

$$E_j=\frac{\sum_{k=1}^{n}\theta_{jk}}{n} \tag{4}$$

However, optimal weights obtained from model (2) are usually not unique. As a result, the cross efficiency score is arbitrarily generated depending on optimal solution arising from the particular software in use. Hence, this non-uniqueness of input/output weights would damage the use of cross efficiency evaluation.

To resolve this problem, one remedy suggested by sexton et al. [9] and was later investigated by Doyle and Green [44] is to introduce a secondary goal which optimizes the input/output weights while keeping unchanged the CCR efficiency score. They were the first who developed aggressive and benevolent formulations of cross efficiency to deal with the non-uniqueness issue. For example, in the benevolent approach, which is more appropriate from the standpoint of the DEA evaluation framework, an attempt is made to identify the optimal weights that maximize the average cross efficiency of other DMUs while keeping unchanged the CCR efficiency score of a particular DMU under evaluation.

Similar thoughts also appeared in the article of Lim [45], since it seeks the optimal weights that minimize (or maximize) the cross efficiency of the best (or worst) performing DMU by incorporating a minimax or a maximin objective into cross efficiency evaluation. A different idea can be found in Wu et al. [46]. They proposed a weight balanced model to solve the non-uniqueness of the optimal weights in DEA models where each DMU makes its own choice of weights without considering the effects on the other DMUs.

In an effort to extend the model of Doyle and Green [44], Liang et al. [22] presented slightly different secondary objective functions by showing that the CCR model can also be expressed equivalently in the deviation variable form (5),

$$\begin{aligned}
&Min \quad \alpha_o\\
&s.t.\\
&\sum_{i=1}^{m}v_{io}x_{io}=1\\
&\sum_{r=1}^{s}u_{ro}y_{rj}-\sum_{i=1}^{m}v_{io}x_{ij}+\alpha_j=0 \qquad j=1,\dots,n,\\
&\alpha_j\ge 0 \qquad j=1,\dots,n,\ u_{ro}\ge 0 \qquad r=1,\dots,s,\ v_{io}\ge 0 \qquad i=1,\dots,m.
\end{aligned} \tag{5}$$

Where α_o is the deviation variable for DMU_o , α_j is the deviation variable for $DMU_j\,(j=1,\dots,n)$, and if DMU_o is inefficient then its efficiency score is $1-\alpha_o^{*}$. Hence, DMU_o is efficient if and only if $\alpha_o^{*}=0$ (6)

Based on this model, a reasonable secondary objective function is to treat α_j as goal achievement variable to minimize total deviation from the ideal point. In this manner, for each DMU_o, Liang et al. [22] derived a multiplier set which with the efficiency score same as to the CCR efficiency score, minimizes the sum of α_j variables as model (7):

$$\begin{aligned}
&Min \quad \sum_{j=1}^{n} \alpha'_j \\
&s.t. \\
&\sum_{i=1}^{m} v_{io} x_{io} = 1 \\
&\sum_{r=1}^{s} u_{ro} y_{rj} - \sum_{i=1}^{m} v_{io} x_{ij} + \alpha'_j = 0 \qquad j = 1, \ldots, n, \\
&\sum_{r=1}^{s} u_{ro} y_{ro} = 1 - \alpha^*_o \\
&u_{ro} \geq 0 \qquad r = 1, \ldots, s,\ v_{io} \geq 0 \qquad i = 1, \ldots, m,\ \alpha'_j \geq 0 \qquad j = 1, \ldots, n.
\end{aligned} \tag{7}$$

3 Nonlinear inputs and outputs

Cook et al. [42] and Despotis et al. [40] presented a DEA approach for measuring the relative efficiencies of a set of maintenance patrols with nonlinear inputs and outputs. Efficiency evaluation has considerable benefit for highway departments and maintenance units and, from the perspective of top management.

One way of expressing the nonlinear inputs/outputs is to replace the single linear expression by the nonlinear function. A piecewise linear function proposed by Despotis et al. [40], by relaxing the linearity assumption overall value of the input vector $X_j = (x_{1j}, \ldots, x_{mj})$ of unit j, can be given by the following additive function $V(X_j) = v_1 x_{1j} + \cdots + v_m x_{mj}$ where $v_1, \ldots, v_m$ are assumed nonlinear partial value function. Then, to deal the nonlinear function $V(X_j)$, the partial value functions $v_i, i = 1, \ldots, m$ in a piecewise linear fashion suggested as follows:

Let $[l_i, h_i]$ be the range of input i over the entire set of DMUs, where

$$l_i = \min_j \{x_{ij}\}, \quad h_i = \max_j \{x_{ij}\} \tag{8}$$

Segmenting the interval $[l_i, h_i]$ by considering the p_i break points

$$l_i = L_i^1, \ldots, L_i^k, \ldots, L_i^{p_i} = h_i \tag{9}$$

Then for each $x_{ij} > l_i$ there is one interval, such that $x_{ij} \in (L_i^{k_j}, L_i^{k_j+1}]$ and then

$$x_{ij} = L_i^1 + (L_i^2 - L_i^3) + \cdots + (L_i^{k_j} - L_i^{k_{j-1}}) + (x_{ij} - L_i^{k_j}) \tag{10}$$

$$\alpha_{i1}^j = L_i^1, \alpha_{i2}^j = L_i^2 - L_i^1, \ldots, \alpha_{ik_j+1}^j = x_{ij} - L_i^{k_j}, \alpha_{ik_j+2}^j = 0, \ldots, \alpha_{ip_i}^j = 0 \tag{11}$$

Without loss of generality, assume that the inputs $i = 1, \ldots, t$ have linear property and nonlinear assumption is applicable only for particular inputs like $i = t+1, \ldots, m$. Then we have:

$$V(X_j)=\sum_{i=1}^{t} x_{ij}v_{io}+\sum_{i=t+1}^{m}(\alpha_{i1}^{j}+\alpha_{i2}^{j})v_{io}^{1}+\sum_{k=3}^{p_i}\alpha_{ik}^{j}v_{io}^{k-1} \tag{12}$$

Furthermore, we can use this manner for nonlinear outputs where we assume that linear outputs are $r=1,\ldots,d$ and nonlinear outputs are $r=d+1,\ldots,s$.

Based on the above discussion, the CCR model (15) obtained with a nonlinear input matrix (13) and a nonlinear output matrix (14).

$$\hat{X}=\begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & & \\ x_{t1} & \cdots & x_{tn} \\ (\alpha_{t+1,1}^{1}+\alpha_{t+1,2}^{1}) & \cdots & (\alpha_{t+1,1}^{n}+\alpha_{t+1,2}^{n}) \\ \vdots & & \\ \alpha_{t+1,p_{t+1}-1}^{1} & \cdots & \alpha_{t+1,p_{t+1}-1}^{n} \\ \vdots & & \\ (\alpha_{m1}^{1}+\alpha_{m2}^{1}) & \cdots & (\alpha_{m1}^{n}+\alpha_{m2}^{n}) \\ \vdots & & \\ \alpha_{m,p_m-1}^{1} & \cdots & \alpha_{m,p_m-1}^{n} \end{bmatrix} \tag{13}$$

$$\hat{Y}=\begin{bmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & & \\ y_{d1} & \cdots & y_{dn} \\ (\beta_{d+1,1}^{1}+\beta_{d+1,2}^{1}) & \cdots & (\beta_{d+1,1}^{n}+\beta_{d+1,2}^{n}) \\ \vdots & & \\ \beta_{d+1,p_{d+1}-1}^{1} & \cdots & \beta_{d+1,p_{d+1}-1}^{n} \\ \vdots & & \\ (\beta_{s1}^{1}+\beta_{s2}^{1}) & \cdots & (\beta_{s1}^{n}+\beta_{s2}^{n}) \\ \vdots & & \\ \beta_{s,p_s-1}^{1} & \cdots & \beta_{s,p_{s-1}}^{n} \end{bmatrix} \tag{14}$$

$$\begin{aligned} & Max \quad \hat{u}_o\hat{y}_o \\ & s.t. \\ & \hat{v}_o\hat{x}_o=1, \\ & \hat{u}_o\hat{Y}-\hat{v}_o\hat{X}\le 0, \\ & \hat{u}_o\ge 0, \quad \hat{v}_o\ge 0. \end{aligned} \tag{15}$$

4. Cross efficiency with linear and nonlinear data

The conventional DEA models are made on the assumption that input/output data are linear. Dispotis et al. [40] addressed the some cases that inputs/outputs must be nonlinear and if the DEA model doesn't have nonlinear supposition, it can't reflect the correct efficiency for

DMUs. Similar is the interpretation of cross efficiency model which doesn't have nonlinear supposition. Then, along the same line of thought, we present an approach that actually accords with that by Liang et al. [22] and Despotis et al. [40], in the sense that we propose a modified cross efficiency structure of Liang et al. [22] that captures certain forms of nonlinear behavior of Despotis et al. [40]. Here and by incorporating secondary goal introduced in model (5) to nonlinear inputs/outputs concept of model (15), we obtain model (16).

$$
\begin{aligned}
&Min \quad 1\hat{\alpha}' \\
&s.t. \\
&\hat{v}_o \hat{x}_o = 1, \\
&\hat{u}_o \hat{Y} - \hat{v}_o \hat{X} + I\hat{\alpha}' = 0, \\
&\hat{u}_o \hat{y}_o = 1 - \alpha_o^*, \\
&\hat{u}_o \geq 0, \quad \hat{v}_o \geq 0, \quad \hat{\alpha}' \geq 0.
\end{aligned} \tag{16}
$$

$\hat{X}$ and $\hat{Y}$ obtain from same rule discussed in Section 3. Moreover, for diminishing marginal value concept proposed by Cook et al. [42], the multipliers which are assign nonlinear inputs should form a non-increasing sequence. Then, we impose assurance region restriction of Thomson et al. [47] $v_{io} \leq \gamma_o, \quad \gamma_o < 1,$ and derive following weight restriction for nonlinear inputs.

$$
\frac{v_{io+1}}{v_{io}} \leq \gamma_o \quad \Rightarrow \quad v_{io+1} - \gamma_o v_{io} \leq 0 \tag{17}
$$

In addition we choose γ_o as follows, where D_o is the width of subinterval O.

$$
\gamma_o < \frac{D_o}{D_{o+1}} \tag{18}
$$

We can define weight restriction for nonlinear outputs in the form of a non-decreasing sequence, too. Then, model (19) with weight restriction obtains as a secondary goal to resolve the problem of non-uniqueness of inputs/outputs weights. By solving it, we can derive a multiplier set for cross efficiency evaluation of linear and nonlinear input/output data which with a same efficiency score as later efficiency score, minimizes the sum of deviation variables.

$$
\begin{aligned}
&Min \quad 1\hat{\alpha}' \\
&s.t. \\
&\hat{v}_o \hat{x}_o = 1, \\
&\hat{u}_o \hat{Y} - \hat{v}_o \hat{X} + I\hat{\alpha}' = 0, \\
&\hat{u}_o \hat{y}_o = 1 - \alpha_o^*, \\
&\hat{v}_o P \leq 0, \quad *
\end{aligned} \tag{19}
$$

$\hat{u}_o Q \leq 0, \quad **$

$\hat{u}_o \geq 0, \quad \hat{v}_o \geq 0, \quad \hat{\alpha}' \geq 0.$

Where * and **are weight restriction Constraints. Efficiency scores obtain from inserting optimal solution of model (19) in (3) and (4). Advantage of our method in comparison with the other methods is using nonlinear supposition for inputs/outputs to introduce both peer and self-evaluation in order to compute the efficiency score, which is more realistic than the CCR efficiency score in some situations. Moreover, the new efficiency score provides complete ranking for all DMUs and based on its results, we can select the most efficient DMU, which is an important task in decision sciences.

5. Illustrative example

To measure the relative efficiencies of highway maintenance patrols, which introduced by Cook et al. [42], we compute cross efficiency scores for that system which has linear and nonlinear inputs.

Table 1 Data for highway maintenance patrols

Crew no	Input1(MEX)	Input2(CEX)	Input3(CLF)	Input4(PCR)	Output1(ASF)	Output2(ATS)	Output3(RCF)
1	585	284	715	60	404	267	184
2	610	245	525	65	551	324	175
3	485	425	680	65	506	284	193
4	345	380	660	70	335	255	180
5	288	325	665	75	455	325	190
6	396	322	604	78	565	350	205
7	336	388	712	70	400	235	177
8	367	413	668	60	433	325	202
9	356	325	678	77	457	202	177
10	535	312	677	63	335	256	248
11	599	248	715	68	421	277	194
12	612	275	525	80	554	364	185
13	465	425	690	83	556	294	173
14	325	390	670	68	317	265	190
15	308	305	665	89	485	345	178
16	366	342	604	92	516	369	200
17	346	378	722	83	423	325	197
18	327	433	678	88	413	235	196
19	236	365	688	85	487	302	197
20	545	322	678	74	385	276	238

The outputs are: Size of the system (ASF), Average traffic serviced (ATS) and Accidents (ACC), and the inputs are: Maintenance expenditure (MEX), Capital expenditure (CEX), Climatic factor (CLF) and Pavement condition rating (PCR); See Table 1.

The scale of maintenance and rehabilitation expenditures as inputs greatly depend upon the road condition prevailing at the time work is being done. Then, maintenance and capital expenditures as behaving in a linear fashion, such is likely not true of the PCR. For this reason we replace this input with piecewise linear function and we assume the PCR in a few subintervals with different values. Moreover, the factor PCR, as an input, should be valued in a diminishing marginal value sense. We assumed the PCR range [0,100] is split into three subintervals [0,60] , [60,80] , [80,100]. Thus, $L_4^1 = 0,\quad L_4^2 = 60,\quad L_4^3 = 80,\quad L_4^4 = 100$. First PCR, second PCR and third PCR are shown in three last column of the input matrix (20).

$$\hat{X} = \begin{bmatrix} 585 & 610 & 485 & 345 & 288 & 396 & 336 & 367 & 356 & 535 & 599 & 612 & 465 & 325 & 308 & 366 & 346 & 327 & 236 & 545 \\ 284 & 245 & 425 & 380 & 325 & 322 & 388 & 413 & 325 & 312 & 248 & 275 & 425 & 390 & 305 & 342 & 378 & 433 & 365 & 322 \\ 715 & 525 & 680 & 660 & 665 & 604 & 712 & 668 & 678 & 677 & 715 & 525 & 690 & 670 & 665 & 604 & 722 & 678 & 688 & 678 \\ 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 & 60 \\ 0 & 5 & 5 & 10 & 15 & 18 & 10 & 0 & 17 & 3 & 8 & 20 & 20 & 8 & 20 & 20 & 20 & 20 & 20 & 14 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 9 & 12 & 3 & 8 & 5 & 0 \end{bmatrix} \quad (20)$$

Table 2 Efficiency scores and ranking

Crew no	CCR efficiency	Cross efficiency	Rank
1	0.9313	0.871	12
2	1	0.911	3
3	1	0.915	2
4	0.87	0.838	18
5	1	0.901	8
6	1	0.905	6
7	0.8693	0.881	11
8	1	0.909	4
9	0.8801	0.858	14
10	1	0.896	9
11	1	0.849	16
12	1	0.903	7
13	0.9841	0.791	19
14	0.944	0.843	17
15	1	0.777	20
16	1	0.923	1
17	0.9655	0.907	5
18	0.9291	0.852	15
19	1	0.862	13
20	0.9979	0.891	10

For example, $PCR = 88$ for DMU_{18} in Table 1. Then, $PCR_1 = 60$ in $[0,60]$, $PCR_2 = 20$ in $[60,80]$ and $PCR_3 = 8$ in $[80,100]$, and we have the subinterval widths as $D_1 = 60, D_2 = 20, D_3 = 20$. For each subinterval different values or weights are attached and $\frac{v_{ik+1}}{v_{ik}} \le \gamma_k, \gamma_1 = 0.75, \gamma_2 = 0.5$

Table 2 and Figure 1 show the efficiency scores from model (16) and cross efficiency scores from model (19). Moreover, rank values of DMUs by cross efficiency scores are given in the third column of Table 2.

As shown in Figure 1, most of DMUs are efficient with Despotis et al.'s model [40] (blue bar). For this reason, that model is not suitable for ranking. Then, we introduced model (19) as a secondary goal of cross efficiency evaluation (orange bar). Various scores obtained for nonlinear data using the new model and it seems that new scores are better than previous one. Note that, for example DMU_{17} is CCR inefficient, but it has cross efficiency score better than some CCR efficient DMUs. Moreover, DMU_{16} is the most efficient unit. Indeed, the purpose of Despotis et al.'s model [40] is only computing the efficiency scores, but our model is an extension of their model.

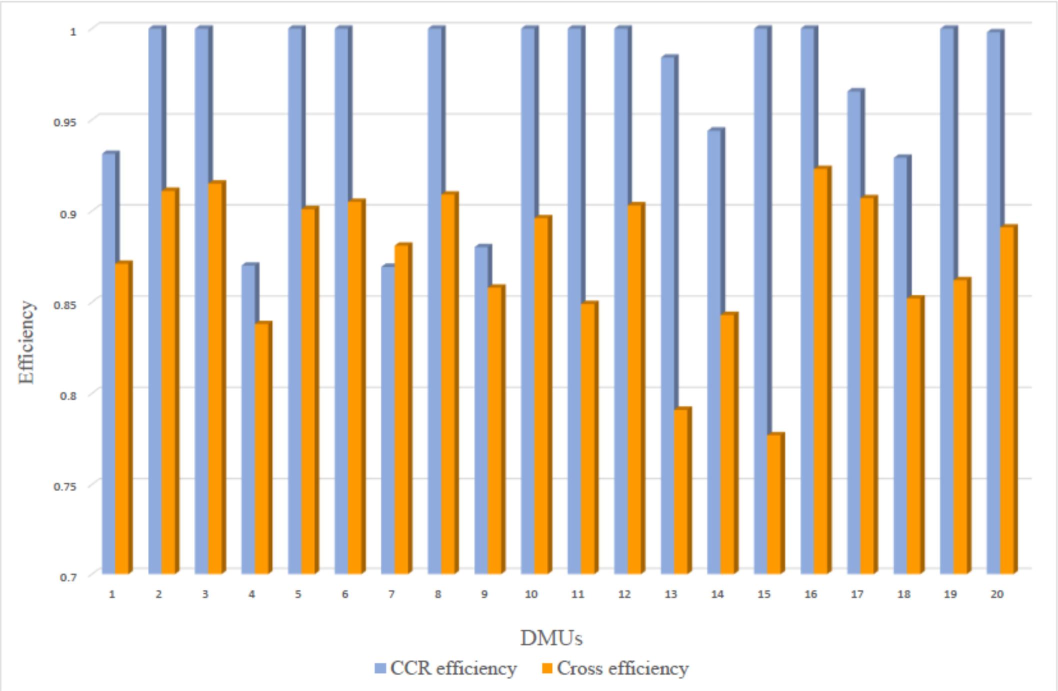

Fig. 1 Efficiency scores

6. Conclusion

Cross efficiency evaluation has been considered to be a powerful extension of DEA, and it can be used for various purposes, e.g. ranking efficient units. The DEA and cross efficiency models traditionally rely on the linearity assumption for the virtual inputs and outputs (i.e. the weights coupled with the ratio scales of the inputs and outputs imply linear value functions). In this paper, we present a general modeling approach for dealing with nonlinear virtual inputs/outputs in cross efficiency concept, which traditional models generally lack this feature. This investigation is an extension of the model introduced by Despotis et al. [40] for nonlinear inputs/outputs, to the cross efficiency method proposed by Liang et al. [22].

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