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# Determining a common set of weight by reducing the flexibility of weight profile

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**Abstract** Data Envelopment Analysis (DEA) is non-parametric mathematical programming for measuring the performance of a set of homogeneous decision-making units (DMUs). Standard DEA models usually result in several efficient units, so, picking the best unit among efficient units has been one of the most challenging subjects in DEA literature. With reference to various researchers, the common set of weights (CSW) approach has been intriguing among them. This paper discusses a mechanism for detecting a common set of weights which is managed to be always positive and prevents weights dissimilarity. Employing this common set of weights can determine the efficiency score of each unit and finally rank them based on their obtained efficiency score. Equally, the proposed model not only provides the closest targets, but also minimizes the deviations of actual DMUs and extreme efficient units. In order to verify the proposed approach an empirical example of Iranian electricity distribution companies is explained.

**Keywords:** Data Envelopment Analysis (DEA), Common Set of Weight (CSW), Efficiency Score, Deviation and Weight dissimilarity.

## 1 Introduction

Data envelopment analysis (DEA) is concerned with a comparative assessment for evaluating the efficiency of decision-making units (DMU). One of the most challenging issues in DEA literature is the concept of input /output weights. Standard DEA models produce more than one efficient unit and determine the weights of inputs and outputs separately for each DMU. So, the flexibility in choosing weights has been questioned. From the practical point of view, picking one or more efficient units looks imprecise and impossible. Consequently, the different sets of weights can lead to different efficiency measures for DMUs. So, the flexibility in selecting input/output weight applying different sets of weights can be argued. This means that employing a common set of weights can reduce flexibility. To overcome a common set of weights (CSW) problem, many researchers have been established and extended in DEA literature. For instance, Ganley,

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and Cubbin [1] determined the common set of weights by maximizing the summation of unit performance. Roll, Cook, and Golany [2] and Roll and Golany [3] proposed several approximations for common weights production. The authors implemented unbounded DEA models for generating various sets of weights. By taking a weighted average on efficiency scores as the weights, the proposed models maximize the average efficiency of units and the number of efficient DMUs. Although, according to order of importance, various factors can be ranked by their model. As the last step, low weights can be assigned to less important factors and a maximal feasible weight goes to important ones. Sinuany- Stern et al. [4] suggested a two stage linear discriminate analysis approach to produce the common weights. Sinuany-Stern, and Friedman [5] argued a nonlinear discriminate analysis to provide the common weights. Kao and Hung [6] presented the comparison solution approach to generate a common set of weights under the DEA framework. The idea behind the model is searching for common set of weights achieving the shortest distance between the efficiency score calculated from the corresponding weights and the targets. This target is the efficiency score calculated from the standard DEA model. Liu and Peng [7] just focused on efficient DMUs and proposed an approach to identify a common set of weights for the performance indices. Wang et al. [8] introduced a methodology by imposing an appropriate minimum weight restriction on all inputs and outputs that rank all DMUs. In another attempt, Wang et al [9] proposed an alternative method based on regression analysis to search a common set of weights for fully ranking DMUs. Sun et al. [10] suggested two different models with reference to ideal and anti- ideal DMUs to conduct common weight for efficiency scores then ranking units. Surveying these researches, this paper proposes a model for determining a common set of weights which evaluates the absolute efficiency of each unit. The contribution of the paper is three folded. First, the proposed method provides the closest targets on the efficient frontier for each input and output. Especially, the targets on the efficient frontier satisfy the update characteristics and composed of extreme efficient units at the same time. Second, the proposed model allows minimizing the deviations of actual inputs and outputs with the determined target. Third, the model generates positive input/output weights and prevents weights' dissimilarity simultaneously. As a non- parametric technique, the proposed model does not require the initial information on input /output's weights. Top of all, in efficiency evaluation and production estimate, the results of the proposed model is more trust able.

The structure of this paper unfolds as follows. The following section is briefly speaking about traditional DEA models and discusses some necessary properties. Section 3 extends our proposed methodology for determining a common set of weights. In Section 4, a real -world example of electricity distribution companies in Iran is analyzed to illustrate the applicability of the proposed approach. The Conclusion will end the paper.

## 2 Preliminaries

Consider that there are $n$ production units ($DMU$) that can be evaluated in terms of $m$ inputs and $s$ outputs. Let $x_{ij} (i=1,...,m)$ and $\mathbf{y}_{rj}$ $(r=1,...,s)$ be the input and output vectors of $DMU_j$ $(j=1,...,n)$. Also, imagine $u_r$ $(r=1,...,s))$ is the weight vector given to $r$-th output and $v_i$ $(i=1,...,m)$ plays the role of the weight vector given to $i$-th input. According to Charnes et.al


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[11], the best relative efficiency of each unit can be measured by following CCR* model that was named by the acronym of the three authors:

$$Max \quad Eff_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

$$s.t. \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,...,n, \qquad (1)$$

$$u_r, v_i \ge 0, \quad for\ all\ r,\ i.$$

In the above model, $DMU_o (o = 1,...,n)$ refers to the $DMU$ under evaluation. If the optimal value of the objective function for $DMU_o$ equals to unity, then the under evaluated unit is efficient. Otherwise, it is called as inefficient. For more description, consider the dual format of Model (1). The dual formulation of Model (1) can be stated as follows.

$$Min \quad \theta_o$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o x_{io}, \quad i = 1,...,m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r = 1,...,s \qquad (2)$$

$$\lambda_j \ge 0, \quad j = 1,...,n.$$

In the model above, $\theta_o$ indicates the efficiency score of the under evaluated unit. Also, $\lambda_j (j = 1,...,n)$ shows the intensity variable of each unit. The standard CCR model, model (1), applies the unit invariant property. This property has been structured by Lovell and Pastor [12] which employs to normalize the weights. Since the main interest of our study is to prevent weight dispersion; this valuable property can be implemented on model (1) as constraints (3-2) and (3-3) admits. In other words, there is a scale of data leading to the equivalent form of model (1) as follows:

$$Max \quad Eff_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \qquad (3)$$

$$s.t.$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,...,n, \quad (3-1)$$

$$0 \le u_r \le 1, \quad r = 1,...,s \qquad (3\text{-}2)$$

$$0 \le v_i \le 1, \quad i = 1,...,m \qquad (3\text{-}3)$$

* Charnes, Cooper and Rhodes (CCR)



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Suppose that $u_r^*\ (r=1,...,s)$ and $v_i^*\ (i=1,...,m)$ are optimal solutions of model (1) and φ defined as the maximum of optimal weight values of $u_r^*\ (r=1,...,s)$ and $v_i^*\ (i=1,...,m)$. If the weights of input and output of the model (1) are divided to φ, an answer for model (3) is assessed. So, models (1) and (3) are equivalent. As it can be seen the objective function and first constraint of models (1) and (3) are similar. But nonnegative variables in the model (1) have been replaced with the bounded variables in the model (3). The bounded variables in the model (3) can prevent dissimilar weights. This property is the advantage of the model (3) in contrast to the model (1). In the following section, a developed model is proposed which can support the generation of nonnegative weights. Model (3) has been taken from the article by Pourhabib et al [13].

## 3 The Proposed Approach

Once we obtain the optimal solution of model (1), each unit selects its best weights to maximize the efficiency score. However, some questions are raised. First, different sets of weights may result in different efficiency scores. Hence, comparing the scores and ranking the units is disputable. Secondly, the standard DEA models always generate more than one efficient unit which leads to a lack of discrimination among units. In order to tackle with these shortcomings, a common set of weight (CSW) approach has been proposed to reduce the flexibility in weight selection. Based on the idea behind the model (3), an alternative CSW mechanism is developed in this section. Again suppose that there are $n$ production units $DMU_j\ (j=1,...,n)$ that consume varying amount of $m$ inputs $x_{ij}(i=1,...,m)$ to produce $s$ outputs $y_{rj}\ (r=1,...,s)$. The production possibility set (PPS) T can be described as:

$$T=\left\{(x,y), x\geq\sum_{j=1}^{n}\lambda_j x_j, y\leq\sum_{j=1}^{n}\lambda_j y_j, \lambda_j\geq 0\right\}.$$

This set has set up on the constant return to scale (CRS) assumption for the production technology and employs to evaluate the efficiency of all $DMUs$. As a matter of fact, each unit should catch its target on the efficient frontier which formed by some extreme efficient $DMUs$. However, the aim is to set the closest targets which can be achieved, especially for inefficient $DMUs$. In doing so, the proposed model employs a mechanism that can provide the closest target after the linear combinations of extreme units are allowed. So, the proposed model seeks to minimize the deviations of actual inputs and outputs from the given targets. Model (4) is then formulated as follows:

$$Eff^{new} = Min \quad \frac{\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\left(x_{ij} - xh(i)\right) + \sum_{r=1}^{s}\left(y_{rj} - yh(r)\right)\right]}{\varphi}$$

$s.t.$

$$xh(i) = \sum_{\substack{j=1 \\ j\in E}}^{n} \lambda_j x_{ij}, i = 1, \dots, m \qquad (4-1)$$

$$yh(r) = \sum_{\substack{j=1 \\ j\in E}}^{n} \lambda_j y_{rj}, r = 1, \dots, s, \qquad (4-2)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \; j = 1, \dots, n, \qquad (4-3)$$

$$\sum_{r=1}^{s} u_r yh(r) - \sum_{i=1}^{m} v_i xh(r) = 0, \qquad (4-4)$$

$$\varphi \le v_i \le 1, \quad i = 1, \dots, m, \qquad (4-5)$$

$$\varphi \le u_r \le 1, \quad r = 1, \dots, s, \qquad (4-6)$$

$$u_r, v_i, \varphi, \lambda_j \ge 0 \; for \; all \; i, \; j \; and \; r, \qquad (4-7)$$

Clearly, Model (4) is a nonlinear program and focuses on two main features. According to constraints (4-5) and (4-6) the model prevents weight dissimilarity. These constraints screw all weights to settle between two bounds: the common lower bound φ and the upper bound of unity. In this way, it prevents the dispersion of input and output weights. The second feature of the model (4) is determining positive common set of weights. The model (4) assumes that all observation belong to the production frontier as indicated by the constraint $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \;,\; j = 1, \dots, n$ . Also, this constraint can define all hyper planes of the efficient frontier. The first two constraints (4-1) and (4-2) represent the linear combinations of extremely efficient units. In fact, the presence of constraint (4-4) attempts to catch such a set of weights conducted by extreme efficient units. This model attempts to pick up a set of weights conducted by extremely efficient units. This strength of the model leads to set positive and dissimilar weights. In addition, guarantees that we can select a common set of weights among the multipliers of supporting hyper planes. The objective function of the model (4) can support the idea of minimum deviation. Looking closely, deviation of each unit and the virtual one is minimized. As model (4) admits, the virtual unit is composed of linear combination of extreme efficient units. Note that, setting this minimum deviation in the objective function, the lower bound $\varphi$, can be increased. What's more, by proper choice for $\varphi$ , among all feasible multipliers, our proposed model can effectively avoid weight dissimilarity and generates positive common weight and at the same time. Notably, this process in common set of weight choice does not require additional information about the unit under evaluation. The initial information about inputs and outputs is sufficient. Likewise, this process has an influence on optimal solutions. That is to say: the common set of weights can be



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selected by jointly restricting the input and output weights with a single bound. However, this common lower bound of input/output weights, $\varphi$, may not be sufficient to achieve optimal weights. Hence, this model can be extended to a general model that can restrict input and output weights separately. To address this issue, the following linear fractional programming problem is proposed.

$$Min \quad \frac{\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\left(x_{ij} - xh(i)\right) + \sum_{r=1}^{s}\left(y_{rj} - yh(r)\right)\right]}{\varphi}$$

$$s.t.$$

$$xh(i) = \sum_{\substack{j=1 \\ j\in E}}^{n} \lambda_j x_{ij}, i = 1,\dots,m \qquad (5-1)$$

$$yh(r) = \sum_{\substack{j=1 \\ j\in E}}^{n} \lambda_j y_{rj}, r = 1,\dots,s, \qquad (5-2)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \; j = 1,\dots,n, \qquad (5-3)$$

$$\sum_{r=1}^{s} u_r yh(r) - \sum_{i=1}^{m} v_i xh(r) = 0, \qquad (5-4)$$

$$Z_I \le v_i \le 1, i = 1,\dots,m, \qquad (5-5)$$

$$Z_O \le u_r \le 1, r = 1,\dots,s, \qquad (5-6)$$

$$Z_I \ge \varphi, \qquad (5-7)$$

$$Z_O \ge \varphi, \qquad (5-8)$$

$$u_r, v_i, \varphi, Z_I, Z_O \lambda_j \ge 0 \; for\ all\ i, j\ and\ r,$$

The idea behind the model (5) is as same as that one in the model (4). But there exists a great divide between model (4) and model (5). The constraint (5-5) forces all input multipliers to screw between the lower bound $Z_I$ and unity. The constraint (5-6) also makes all output multipliers lie down between the lower bound $Z_O$ and the upper bound 1. The constraints (5-7) and (5-8) restrict the lower bound of input and output's weights to the positive variable φ. That is to say, input and output weights are restricted separately. The objective function of the model (5) is also established to maximize φ while the distance between $Z_I$ and $Z_O$ with their upper bounds is reduced. Thus, the model supplies the weights with the least dissimilarity, which is the main interest of the study. The following two theorems emphasize that both proposed models are always feasible. Besides, positive weights can be assessed by implementing these models.

**Theorem 1:** The proposed model (4) is feasible and generates positive weights in optimality.

**Proof:** Imagine $DMU_o$ is evaluated by the CCR model (model (1)). Also, $DMU_d$ plays the reference unit of $DMU_o$. Thus, we have $u_r^d > 0$ and $v_i^d > 0 (i = 1,\cdots,m, r = 1,\cdots,s)$. As a result,



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$u_r^d, v_i^d, \varphi = \underset{i,r}{Min}\{u_r^d, v_i^d\}$ is a feasible solution for evaluating $DMU_o$ by the model (4). Also, note that that φ>0.

**Theorem 2:** The proposed model (5) is always feasible and in optimality φ>0.

**Proof:** the proof is the same as Theorem (1).

In order to highlight the models' applicability, two different examples are distinguished.

## 4 Numerical example

To verify the applicability of the proposed models, two various examples are executed. For the first try, the proposed model is compared with those proposed by Kao and Hung's [6] models. The authors have proposed three models employing different distance functions to generate common weights. For more information, refer to Kao and Hung [6].

### Example 1

This example consists of 12 flexible manufacturing systems (FMSs) with two inputs and four outputs. These data are derived from Shang and Sueyoshi [14] and are recorded in Table 1. The inputs include annual operating and depreciation costs ( $x_1$) and the floor space requirements of each specific system ( $x_2$). Outputs signify the improvement of qualitative benefits ( $y_1$), work in process ( $y_2$), average number of tardy jobs ( $y_3$), and average yield ( $y_4$ ).

**Table 1** Data set for 12 flexible manufacturing system (FMSs)

| FMS | $x_1$ | $x_2$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ |
|---|---|---|---|---|---|---|
| 1 | 17.02 | 5 | 42 | 45.3 | 14.2 | 30.1 |
| 2 | 16.46 | 4.5 | 39 | 40.1 | 13 | 29.8 |
| 3 | 11.76 | 6 | 26 | 39.6 | 13.8 | 24.5 |
| 4 | 10.52 | 4 | 22 | 36 | 11.3 | 25 |
| 5 | 9.50 | 3.8 | 21 | 34.2 | 12 | 20.4 |
| 6 | 4.79 | 5.4 | 10 | 20.1 | 5 | 16.5 |
| 7 | 6.21 | 6.2 | 14 | 26.5 | 7 | 19.7 |
| 8 | 11.12 | 6 | 25 | 35.9 | 9 | 24.7 |
| 9 | 3.67 | 8 | 4 | 17.4 | 0.1 | 18.1 |
| 10 | 8.93 | 7 | 16 | 34.3 | 6.5 | 20.6 |
| 11 | 17.74 | 7.1 | 43 | 45.6 | 14 | 31.1 |
| 12 | 14.85 | 6.2 | 27 | 38.7 | 13.8 | 25.4 |

The proposed model (4) and Kao and Hung [6] models were implemented on this data set. Table 2 shows the results. The first three rows of Table 2 demonstrate three different results (based on different distance functions) and the last row shows the results of the proposed model (4). As Table (2) records the weights for output 3 in Kao and Hung [6] model are always zero, whilst the proposed model (4) gives strictly positive weights for all outputs. Note that the results of Kao and Hung [6] models are taken from Sun et al [10].



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**Table 2** Common weights Derived by different models

| Model | $I_1$ | $I_2$ | $O_1$ | $O_2$ | $O_3$ | $O_4$ |
|---|---|---|---|---|---|---|
| P=1 | 0.076680 | 0.026510 | 0.021330 | 0.004024 | 0 | 0.011944 |
| P=2 | 0.928986 | 0.386831 | 0.225722 | 0 | 0 | 0.254175 |
| P=∞ | 0.896384 | 0.415631 | 0.150772 | 0.05559 | 0 | 0.238215 |
| Proposed model(4) | 0.928542 | 1.00000 | 0.177336 | 0.191191 | 0.060618 | 0.127374 |

For detailed analysis, the efficiency scores of different models are depicted in Table 3. The second column is calculated by the traditional CCR model (1), the rest three columns present the efficiency score of Kao and Hung [6] models. The last column includes the efficiency scores of the proposed model (4).

**Table 3** The efficiency scores calculated by different models

| DMUs | CCR | P=1 | P = 2 | P = ∞ | $Eff^{new}$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 0.9654 | 0.9111 | 0.98990 |
| 2 | 1 | 0.9788 | 0.9616 | 0.9026 | 0.96880 |
| 3 | 0.9824 | 0.9488 | 0.9132 | 0.9021 | 0.95387 |
| 4 | 1 | 1 | 1 | 1 | 0.99998 |
| 5 | 1 | 1 | 0.9641 | 0.9663 | 1 |
| 6 | 1 | 0.9624 | 0.9866 | 0.9872 | 0.81451 |
| 7 | 1 | 1 | 1 | 1 | 0.87604 |
| 8 | 0.9614 | 0.9614 | 0.9423 | 0.9203 | 0.91813 |
| 9 | 1 | 0.7528 | 0.8462 | 0.8760 | 0.55643 |
| 10 | 0.956 | 0.8334 | 0.8041 | 0.8295 | 0.81175 |
| 11 | 0.9831 | 0.9507 | 0.9160 | 0.8591 | 0.89740 |
| 12 | 0.8012 | 0.7943 | 0.7750 | 0.7602 | 0.81340 |
| Average | 0.9734 | 0.9317 | 0.9228 | 0.9095 | 0.883351 |

The notification $Eff^{new}$ indicates the performance score of the proposed model (4). The second column of Table 3 shows the CCR efficiency scores. The traditional CCR model (1) evaluates seven out of the twelve units as DEA efficient. This subject leads to a lack of discrimination power of the CCR model (1). The rest three columns of Table 3 derive the efficiency scores of Kao and Hung [6] models which are based on different distance functions. It can be seen, there are four efficient units in P=1and two efficient units in P = 2and P = ∞. As the last column of Table 3 presents, the proposed model (4) gives only one efficient unit. Therefore, the number of efficient units is reduced from 7 to 1 in the proposed model (4). From the statistical point of view, the average efficiency of all mentioned methods is listed in the last row of Table 3. The average of efficiencies in the proposed model (4) is 0.8833 while this quantity is recorded as 0.9734 in the CCR model (1) and 0.9317 in Kao and Hung [6] with p = 1. The other two models of Kao and Hung [12] with P = 2 and P = ∞, the average of efficiency scores are depicted as 0.9228 and 0.9095, respectively. As expected, the results show that the proposed model (4) outperforms the existing model. Notably, the proposed model (4) not only results in strictly positive weights compared to other models, but also avoids weight dissimilarity. Generally, model (4) reduces efficiency scores and the number of efficient units.



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## Example 2

In this section, we apply our proposed models to analyze a real example of Iranian electricity distribution companies. Since electricity distribution has been pointed out in most of the researches, a summary of some studies has been given. Yuzhi and Zhangan [15] have studied the input-output efficiency of distribution systems from the more different aspects. Performance analysis of 21 Turkish electricity distribution companies was conducted by Celen [16]. Omrani et al. [17] employed a mixed methodology of bargaining game, principal component analysis, and DEA to evaluate the efficiency of electricity distribution in Iran. The ranking of the electricity distribution in Iran was carried out by Tavassoli et al. [18] with a view to strong complementary slackness conditions. All of these studies have focused on calculating the efficiency of electricity distribution companies. This section struggles to evaluate electricity distribution companies in Iran employing the proposed models. In this study, 39 electricity distribution companies in Iran with 14 variables including 6 inputs and 8 outputs are selected. Input variables include: transformer capacity or maximum amount of power that can be transformed by the transformer and denoted as ($x_1$), Number of Transformers in circuits is ($x_2$), Low voltage network or voltage levels less than 1 KV ($x_3$), Medium voltage network or voltage levels greater than 1kV and less 100 kV ($x_4$), Number of employees ($x_5$) and Area (x6). Output variables can be listed as the Energy delivery ($y_1$), Energy consumption of other customers or the total amount of energy used except industrial and household consumption ($y_2$), Industrial energy consumption ($y_3$), Household energy consumption ($y_4$), Number of other customers ($y_5$), number of industrial customers ($y_6$), Number of household customers ($y_7$), and Number of Lights of a street lighting ($y_8$). Table 4 represents the data set for these 39 companies.

**Table 4** The data set for 39 electricity distribution companies

| Company | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_{5,7}$ | $y_6$ | $y_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tabriz | 1686 | 5590 | 5317 | 2966 | 572 | 4770 | 3816 | 1066 | 1154 | 1300 | 864 | 9 | 189 |
| Azarbayejansharghi | 1731 | 15422 | 89871 | 13737 | 700 | 40722 | 3296 | 1162 | 853 | 867 | 744 | 5 | 298 |
| Azarbayejangharbi | 2330 | 17345 | 11443 | 14485 | 725 | 37412 | 5059 | 1819 | 803 | 1666 | 1097 | 5 | 322 |
| Ardebil | 853 | 5992 | 5923 | 7060 | 299 | 17867 | 1640 | 555 | 262 | 586 | 477 | 3 | 148 |
| Ostan Esfahan | 5069 | 29865 | 16827 | 19131 | 512 | 91000 | 9564 | 3287 | 3552 | 1941 | 1262 | 17 | 446 |
| Esfahan | 2498 | 9806 | 7938 | 5066 | 295 | 16104 | 5409 | 1833 | 1425 | 1810 | 1030 | 10 | 215 |
| Chaharmahal-o-bakhtiari | 971 | 7554 | 4555 | 6296 | 158 | 16411 | 1651 | 754 | 257 | 414 | 313 | 2 | 109 |
| Markazi | 2215 | 14675 | 7982 | 11201 | 315 | 29127 | 4787 | 1625 | 1679 | 917 | 635 | 6 | 173 |
| Hamedan | 2118 | 15021 | 7545 | 9937 | 369 | 19493 | 3521 | 1667 | 326 | 1003 | 658 | 5 | 276 |
| Lorestan | 1797 | 12605 | 6968 | 8866 | 221 | 28306 | 2926 | 1158 | 456 | 917 | 558 | 3 | 149 |
| Alborz | 2659 | 12643 | 7211 | 4836 | 349 | 5142 | 6286 | 1763 | 1539 | 1886 | 1154 | 5 | 203 |
| Tehran | 10756 | 16602 | 22299 | 8469 | 1742 | 1011 | 20512 | 9482 | 1753 | 7811 | 4255 | 12 | 364 |
| Ostan Tehran | 7592 | 38199 | 17487 | 13805 | 720 | 13029 | 12654 | 3786 | 3648 | 3112 | 1940 | 26 | 281 |
| Ghom | 1406 | 5636 | 3569 | 3287 | 250 | 11237 | 3188 | 1087 | 711 | 955 | 481 | 5 | 83 |
| Mashhad | 2596 | 10707 | 9016 | 5440 | 396 | 3168 | 6477 | 2121 | 1516 | 2310 | 1367 | 11 | 258 |
| Khorasanrazavi | 2998 | 22765 | 12939 | 26143 | 572 | 103950 | 7613 | 4565 | 796 | 1552 | 1115 | 6 | 380 |
| Khorasanjonobi | 908 | 8301 | 4848 | 12197 | 189 | 151196 | 1539 | 763 | 267 | 373 | 327 | 2 | 150 |
| Khorasanshomali | 699 | 5582 | 4159 | 5763 | 196 | 28166 | 1205 | 496 | 221 | 389 | 310 | 1 | 103 |
| Ahvaz | 4228 | 12864 | 5531 | 3732 | 419 | 11304 | 8957 | 1819 | 988 | 4338 | 500 | 2 | 132 |
| Khozestan | 7322 | 34824 | 11633 | 17295 | 507 | 57945 | 16195 | 3515 | 1869 | 8066 | 912 | 2 | 255 |
| Kohkiloyeh-o-boyerahmad | 1083 | 6874 | 3318 | 4688 | 174 | 15563 | 1583 | 363 | 269 | 558 | 215 | 1 | 63 |
| Zanjan | 1363 | 8560 | 5437 | 7857 | 232 | 22164 | 3099 | 848 | 1429 | 518 | 390 | 3 | 149 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ghazvin | 1750 | 10799 | 4885 | 6763 | 236 | 15637 | 4412 | 1516 | 1835 | 753 | 526 | 4 | 165 |
| Semnan | 1241 | 7001 | 4202 | 6850 | 183 | 97491 | 2530 | 985 | 904 | 464 | 335 | 4 | 105 |
| Sistan-o-balochestan | 2460 | 19630 | 11369 | 22680 | 671 | 187502 | 5165 | 1702 | 147 | 2473 | 685 | 2 | 193 |
| Kermanshah | 1922 | 15575 | 6487 | 11221 | 296 | 24641 | 3142 | 1131 | 298 | 1072 | 676 | 2 | 151 |
| Kurdestan | 1273 | 10741 | 5226 | 9905 | 232 | 28817 | 2120 | 847 | 165 | 877 | 561 | 2 | 139 |
| Illam | 857 | 5084 | 2493 | 4367 | 112 | 20150 | 1320 | 488 | 74 | 448 | 193 | 1 | 63 |
| Shiraz | 3680 | 24105 | 11488 | 11339 | 530 | 20184 | 5807 | 2503 | 816 | 1629 | 920 | 8 | 202 |
| Fars | 3621 | 33351 | 11613 | 22059 | 356 | 103000 | 6977 | 3757 | 471 | 2105 | 837 | 5 | 361 |
| Boshehr | 2970 | 13161 | 5774 | 7100 | 227 | 23168 | 5486 | 1230 | 141 | 3257 | 387 | 2 | 155 |
| Shomal-e-kerman | 2019 | 14070 | 7483 | 11128 | 380 | 91193 | 4090 | 2172 | 500 | 922 | 543 | 3 | 269 |
| Jonob-e-kerman | 2708 | 23226 | 12331 | 18249 | 368 | 95887 | 5500 | 2679 | 224 | 1780 | 500 | 2 | 162 |
| Gilan | 2890 | 17006 | 18528 | 8648 | 621 | 14711 | 5080 | 1483 | 861 | 2063 | 1250 | 5 | 500 |
| Mazandaran | 3331 | 25825 | 14271 | 10311 | 624 | 14732 | 5941 | 1576 | 1111 | 2210 | 1163 | 9 | 160 |
| Garb-e-mazandaran | 1691 | 11287 | 6110 | 3923 | 232 | 9040 | 2131 | 706 | 205 | 899 | 499 | 3 | 106 |
| Golestan | 2021 | 14843 | 6988 | 7078 | 348 | 20381 | 3258 | 1020 | 414 | 1413 | 633 | 3 | 102 |
| Hormozgan | 3750 | 21016 | 8268 | 13926 | 447 | 66539 | 819 | 2131 | 242 | 4445 | 585 | 2 | 184 |
| Yazd | 1765 | 13385 | 7378 | 9688 | 349 | 74650 | 4491 | 1231 | 2092 | 874 | 569 | 9 | 226 |

Our proposed model (4) and three different distance function models of Kao and Hung [6] are employed on the data set of Table 4. The weights of six inputs and eight outputs are listed in Table 5. It is worth to note that, the first three columns in Table 5 (Kao and Hung [6] models) present the weights as close as to zero. In contrast, the proposed model (4) gives the weights with the least dissimilarity. Also, as the last column of Table 5 draws there is a huge discrepancy between the results of the proposed model (4) and zero.

**Table 5** Common weights derived by different models

| | p = 1 | P = 2 | P = ∞ | $Eff^{new}$ |
|---|---|---|---|---|
| $V_1$ | 0.00293 | 0.0063 | 0.0049 | 0.204230 |
| $V_2$ | 0.0001 | 0.0001 | 0.0001 | 0.469186 |
| $V_3$ | 0.0001 | 0.00041 | 0.0001 | 0.415018 |
| $V_4$ | 0.0001 | 0.0001 | 0.0001 | 0.239133 |
| $V_5$ | 0.00276 | 0.0047 | 0.00205 | 0.001598 |
| $V_6$ | 0.0001 | 0.0001 | 0.00011 | 0.420507 |
| $U_1$ | 0.0005 | 0.00034 | 0.0001 | 0.604785 |
| $U_2$ | 0.0023 | 0.00474 | 0.00288 | 0.831806 |
| $U_3$ | 0.0001 | 0.0001 | 0.0001 | 1 |
| $U_4$ | 0.0001 | 0.00296 | 0.00321 | 1 |
| $U_5$ | 0.0001 | 0.0001 | 0.0001 | 1 |
| $U_6$ | 0.0001 | 0.0001 | 0.0001 | 1 |
| $U_7$ | 0.0001 | 0.0001 | 0.0001 | 1 |
| $U_8$ | 0.0185 | 0.03473 | 0.1907 | 1 |

To find the optimal solution to Kao and Hung's [6] Models, weight restrictions $u_r \geq \varepsilon(r=1,...,s), v_i \geq \varepsilon(i=1,..,m)$ can start with small as $\varepsilon = 0.0001$ the initial point. Turning back to Table (5), the input and output weights in the three first models of Kao and Hung [6] catches the epsilon value. Hence, the weights 'value is dependent on the epsilon value. Changing the epsilon value can reform the quantity of the weights too. On the other hand, the proposed model (4) gives all positive weights and the least dissimilarity. In order to show the advantages of our proposed model (4), we compare it with Kao and Hung [6] models and the CCR model (1)



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based on efficiency scores. Table 6 records the efficiency scores of different methods on the data set of Table (4).

The second column of Table 6 shows the CCR efficiency scores. As it can be seen, CCR model (1) shows 23 of the 39 companies as DEA efficient. The rest columns of Table (6) record the efficiency scores of Kao and Hung [6] distance functions models. There are one efficient unit in P = 1, eight efficient units in P = 2 and three efficient units in P = ∞. The efficiency scores of the proposed model (4) are recorded in the last column of Table 6. Interestingly, there is only one efficient unit in this approach. As the last row of Table 6 displays, the average of efficiency scores in proposed model (4) is the least (0.357989) among the models. The maximum average efficiency is recorded as 0.93469 in the CCR model (1). Kao and Hung [6] models with P=1 and P = ∞ have drawn the second and third rank with0.69670 and 0.731641. The next row is assigned to Kao and Hung [4] models with P=2 with the value of 0. 808232. DMU#12 is the top-ranked company in the proposed model (4). Generally, the proposed model (4) can support the advantages of the least efficient units and the least efficiency scores. Also, the proposed model (4) not only results in strictly positive weights but also can avoid weights dissimilarity and reduces the number of efficiency units.

**Table 6** The efficiency scores calculated by different models

| DMU | CCR | P = 1 | P = 2 | P = ∞ | $Eff^{new}$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0.84811 |
| 2 | 1 | 0.69075 | 0.80934 | 0.701 | 0.18088 |
| 3 | 0.9930 | 0.79342 | 0.92787 | 0.85465 | 0.26452 |
| 4 | 1 | 0.73063 | 0.84573 | 0.78659 | 0.20834 |
| 5 | 1 | 0.66842 | 0.73435 | 0.60994 | 0.23862 |
| 6 | 1 | 0.96111 | 1 | 0.9378 | 0.54676 |
| 7 | 0.8824 | 0.70992 | 0.79691 | 0.68123 | 0.19888 |
| 8 | 0.8702 | 0.73714 | 0.79102 | 0.67624 | 0.29917 |
| 9 | 1 | 0.90926 | 0.99206 | 0.85652 | 0.27843 |
| 10 | 0.8610 | 0.65931 | 0.79523 | 0.69053 | 0.21385 |
| 11 | 1 | 0.9861 | 1 | 0.95951 | 0.79229 |
| 12 | 1 | 0.98946 | 1 | 0.99923 | 1 |
| 13 | 1 | 0.66236 | 065675 | 0.60252 | 0.55421 |
| 14 | 1 | 0.72485 | 0.79227 | 0.73188 | 0.4918 |
| 15 | 1 | 0.9892 | 1 | 0.97743 | 0.87614 |
| 16 | 1 | 0.81264 | 1 | 0.80582 | 0.1861 |
| 17 | 1 | 0.26075 | 0.4084 | 0.28697 | 0.03807 |
| 18 | 0.9261 | 0.55171 | 0.70309 | 0.59791 | 0.12408 |
| 19 | 1 | 0.76535 | 0.9632 | 1 | 0.93274 |
| 20 | 1 | 0.67464 | 0.91614 | 0.91825 | 0.48008 |
| 21 | 0.6250 | 0.51486 | 0.62597 | 0.57307 | 0.21351 |
| 22 | 1 | 0.73067 | 0.77067 | 0.64983 | 0.28877 |
| 23 | 1 | 0.89938 | 0.92514 | 0.78599 | 0.43252 |
| 24 | 0.9246 | 0.36329 | 0.5063 | 0.37371 | 0.08468 |
| 25 | 0.9877 | 0.33332 | 0.55507 | 0.49013 | 0.08836 |
| 26 | 0.7715 | 0.63452 | 0.76186 | 0.67884 | 0.2265 |
| 27 | 0.9233 | 0.58184 | 0.75177 | 0.66774 | 0.17178 |
| 28 | 0.8700 | 0.54577 | 0.703 | 0.60608 | 0.16202 |



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| | | | | | |
|---|---|---|---|---|---|
| 29 | 0.7879 | 0.69571 | 0.73849 | 0.66623 | 0.33423 |
| 30 | 1 | 0.66525 | 0.86281 | 0.70802 | 0.16466 |
| 31 | 1 | 0.7576 | 1 | 0.99733 | 0.44553 |
| 32 | 1 | 0.65895 | 0.84585 | 0.6557 | 0.13007 |
| 33 | 0.9033 | 0.53301 | 0.69768 | 0.59402 | 0.13614 |
| 34 | 1 | 0.99528 | 1 | 1 | 0.37024 |
| 35 | 0.7377 | 0.68044 | 0.74571 | 0.73164 | 0.35526 |
| 36 | 0.6618 | 0.55465 | 0.61504 | 0.59267 | 0.27535 |
| 37 | 0.7277 | 0.53269 | 0.65197 | 0.63335 | 0.26702 |
| 38 | 1 | 0.62215 | 0.90364 | 0.87635 | 0.2882 |
| 39 | 1 | 0.86497 | 0.72151 | 0.57929 | 0.17722 |
| Average | 0.934697 | 0.6967018 | 0.8082326 | 0.7316413 | 0.3579869 |

## 5 Conclusions

In this paper, an alternative approach has been proposed to address the issue of common set of weights (CSW) in DEA literature. The key point of this method is to minimize deviations of real inputs and outputs of units and the virtual inputs and outputs composed of extreme efficient units. The proposed model has several contributions in study of common set of weights (CSW). First, the proposed model result in strictly positive weights and avoids weight dissimilarity simultaneously. Second, the proposed model reduced the efficiency scores and the number of the efficient units. The application illustration revealed that the proposed models can support the idea of choosing dissimilar and positive
Common set of weights.

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